BOSTON UNIVERSITY

EK 307: Electric Circuits

Fall 2017

Lecture 22

Nov 30, 2017

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Administrivia

- HW9 due today

Last Lecture (21): Chapter 9

1. **Started Chapter 9 – Phasors & Sinusoidal Steady State; Impedance/Admittance**

Phasors

$$z = x + jy = r\underline{/\phi} = r(\cos\phi + j\sin\phi)$$

$$z = x + jy = r\underline{/\phi}, \qquad z_1 = x_1 + jy_1 = r_1\underline{/\phi_1}$$

$$z_2 = x_2 + jy_2 = r_2\underline{/\phi_2}$$

Add in component form.
Multiply in polar form.

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplication:

$$z_1 z_2 = r_1 r_2 \underline{/\phi_1 + \phi_2}$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\underline{/\phi_1 - \phi_2}$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r}\underline{/-\phi}$$

Square Root:

$$\sqrt{z} = \sqrt{r}\underline{/\phi/2}$$

Complex Conjugate:

$$z^* = x - jy = r\underline{/-\phi} = re^{-j\phi}$$

Time and frequency domains

$$\frac{dv}{dt} \quad \Leftrightarrow \quad j\omega \mathbf{V}$$

(Time domain) **(Phasor domain)**

$$\int v \, dt \quad \Leftrightarrow \quad \frac{\mathbf{V}}{j\omega}$$

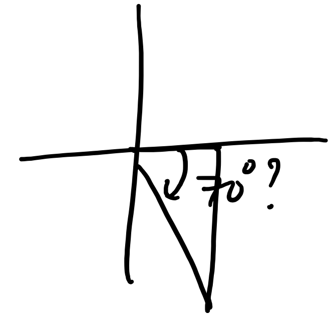
(Time domain) **(Phasor domain)**

v(t)	**V**
Instantaneous/time domain	phasor/frequency domain
Time dependent	Time independent
Always real valued	Complex number in general

Example 9.7

Using the phasor approach, determine the current $i(t)$ in a circuit described by the integrodifferential equation

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75°)$$

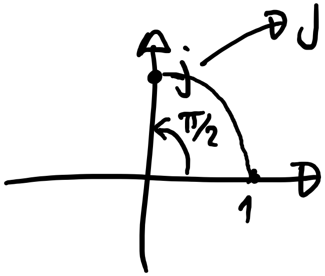


$$4\underline{I} + 8\frac{1}{j\omega}\underline{I} - 3j\omega\underline{I} = 50\angle 75° \quad , \omega = 2$$

$$4\underline{I} + 8\frac{1}{j2}\underline{I} - 3j2\underline{I} = 50\angle 75°$$

$$(4 - j4 - j6)\underline{I} = 50\angle 75° \rightarrow \underline{I} = \frac{50\angle 75°}{4 - j10} = \frac{50\angle 75°}{12\angle -70°}$$

$$j = 1e^{j\frac{\pi}{2}} \rightarrow \frac{1}{j} = e^{-j\frac{\pi}{2}} = -j$$



$$\underline{I} \approx 4\angle 145°$$

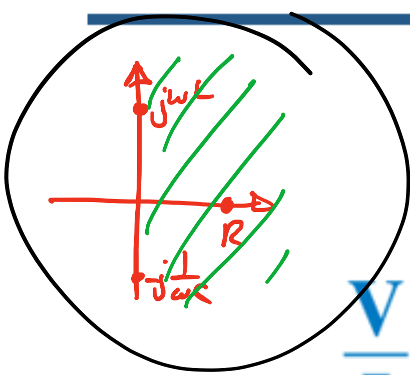
$$\boxed{i(t) = 4\cos(2t + 145°)}$$

Phasor Circuit Laws

TABLE 9.2

Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$



Z

$$\frac{\mathbf{V}}{\mathbf{I}} = R, \qquad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L, \qquad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$$

$= Z_R$ $\qquad = Z_L$ $\qquad = Z_C$

Concept of impedance

$$\frac{\mathbf{V}}{\mathbf{I}} = R, \qquad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L, \qquad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

$$\boxed{\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{Z}\mathbf{I}}$$

The **impedance Z** of a circuit is the ratio of the phasor voltage **V** to the phasor current **I**, measured in ohms (**Ω**).

$$Y = \frac{I}{V}$$

TABLE 9.3

Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \dfrac{1}{R} = G$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \dfrac{1}{j\omega L}$
C	$\mathbf{Z} = \dfrac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

$Z = R + jX$

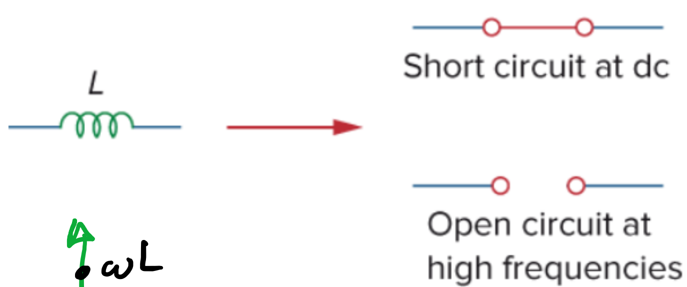
resistance

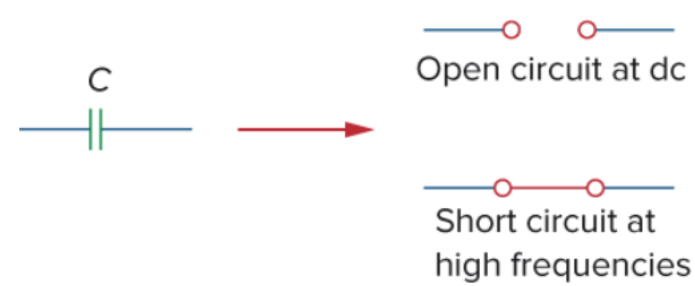
reactance

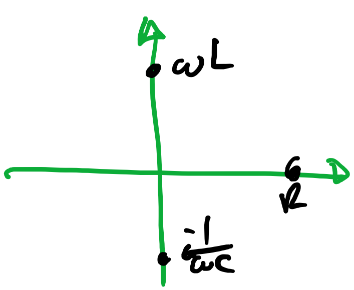
$Y = G + jB$

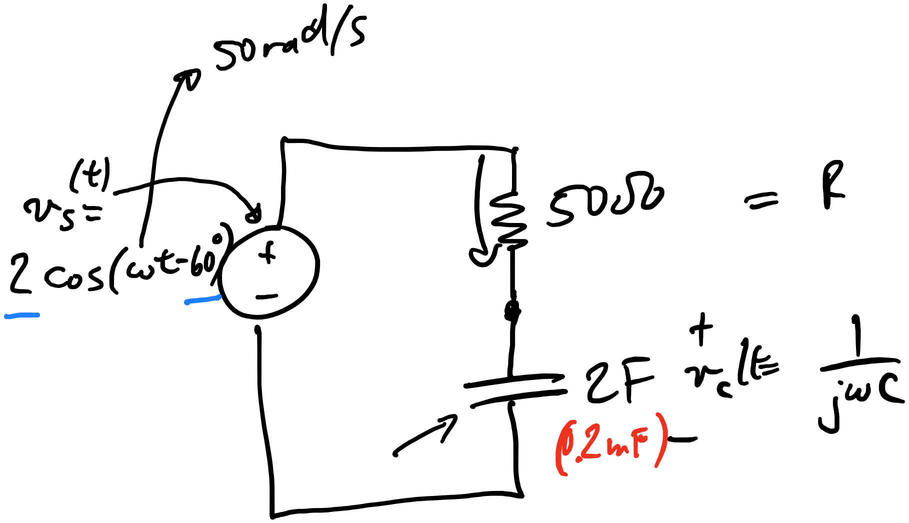
cond

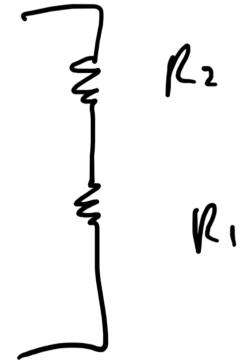
susceptance.

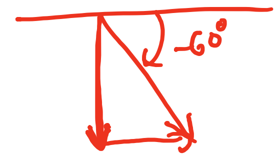












$$v_s \longrightarrow \underline{V_s} = 2\angle -60^\circ \text{ [V]}$$

$$\underline{V_c} = \cancel{\frac{R_1}{R_2+R_1}} V_s = \frac{Z_c}{Z_c + Z_R}\underline{V_s}$$

$$= \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C}+R} V_s = \frac{-j\frac{1}{(50)(2)}}{-j\frac{1}{50\cdot 2}+50} V_s = \frac{-j\,0.01}{-j\,0.01+50}\underline{V_s}$$

$$= \frac{0.01\angle -90^\circ}{50\angle 0^\circ}\underline{V_s}$$

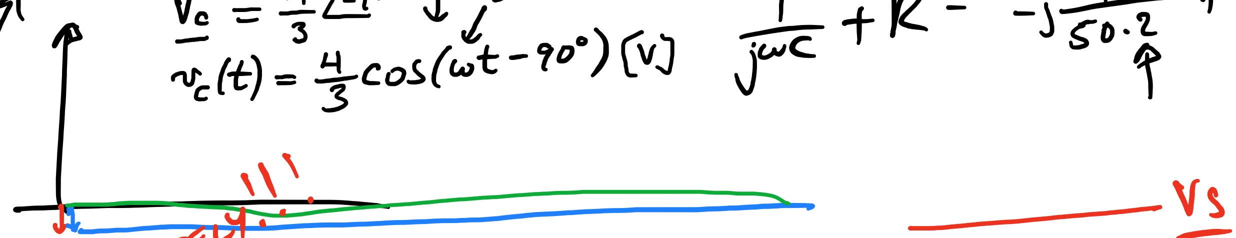
$$\cong 0.0002\angle -90^\circ\, V_s$$

$$\underline{V_c} = \left|\frac{2}{3}\right|\angle -30^\circ\, \underline{V_s} = \frac{2}{3}\angle -30^\circ \cdot 2\angle -60^\circ$$

$$\underline{V_c} = \frac{4}{3}\angle -90^\circ$$

50 rad/s

$$v_c(t) = \frac{4}{3}\cos(\omega t - 90^\circ) \text{ [V]}$$

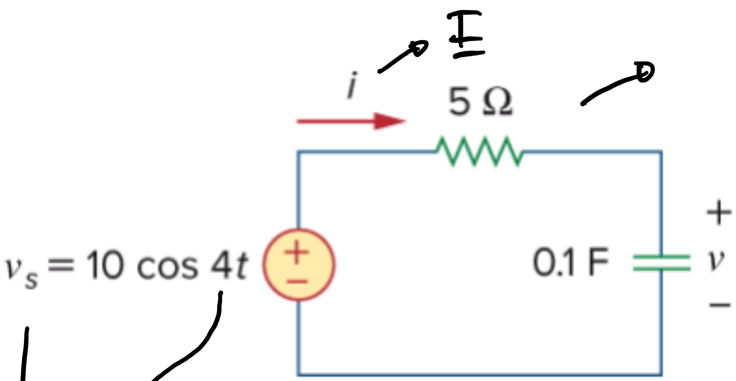


APPROXIMATELY

$$\left|\frac{2}{3}\right|\angle -30^\circ\, \underline{V_s} = \quad V_s \simeq \frac{}{-j100+50}\underline{V_s}$$

Example 9.9

Find $v(t)$ and $i(t)$ in the circuit shown in **Fig. 9.16**.



$$Z = 5\Omega + \frac{1}{j4(0.1)} = 5 - j2.5$$

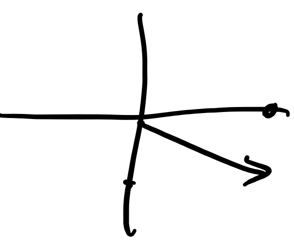
$\omega = 4$

$\underline{V_s} = 10\angle 0^\circ$

$$10\angle 0^\circ = \underline{V_s} = \underline{V_R} + \underline{V_C} = \underline{I}R + \frac{1}{j\omega C}\underline{I}$$

$j\omega c)\ \underline{I}$

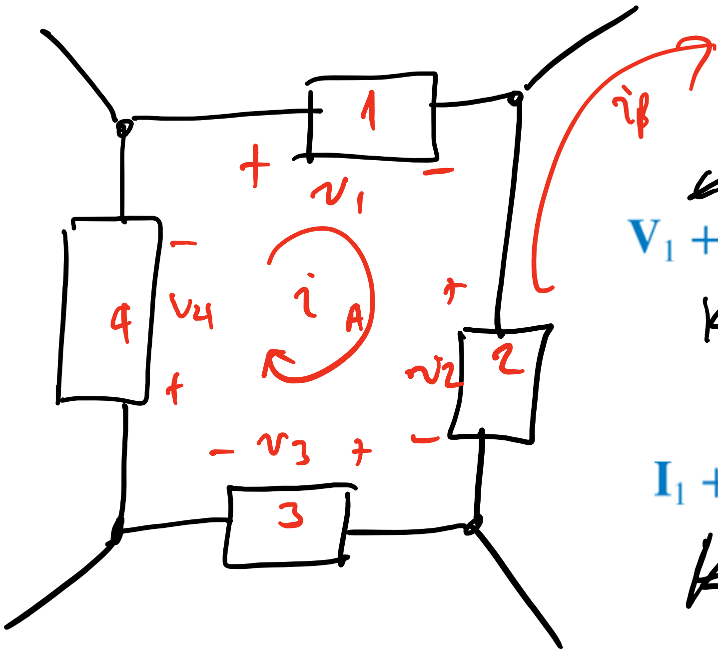
$$\underline{I} = \frac{10\angle 0^\circ}{\left(R + \frac{1}{j\omega C}\right)} = \frac{10}{5 - j2.5} = \frac{10\angle 0^\circ}{6\ \angle -30^\circ}$$



$$\underline{I} \sim 1.8\angle 30^\circ$$

$$\boxed{i(t) = 1.8\cos(4t + 30^\circ)\ \text{A}}$$

KVL and KCL in Phasor Domain



$v_1^{(t)} + v_2^{(t)} + v_3^{(t)} + v_4^{(t)} = 0$

$$\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0$$

KVL

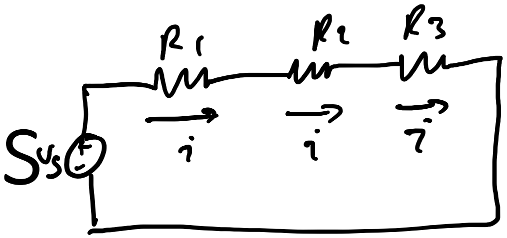
$Re\{\underline{V_1}\, e^{j\omega t}\} + Re\{\underline{V_2}\, e^{j\omega t}\} + \cdots = 0$

$\underbrace{(\qquad\qquad)}_{\underline{V}_{total}=0} e^{j\omega t}\} = 0$

$$\mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n = 0$$

KCL

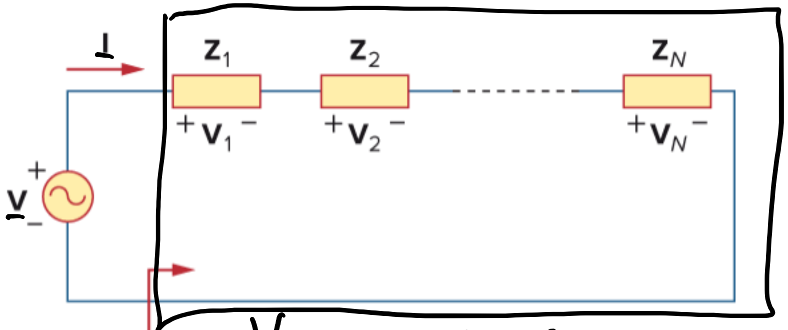
Series elements



$i R_1 + i R_2 + i R_3 - v_s = 0$

$\underbrace{}$

$i(R_{eq}) - v_s = 0$



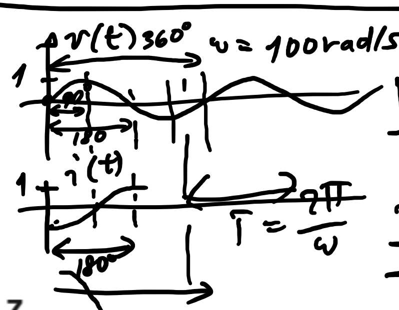
$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N$$

$\mathbf{Z}_{eq} = \dfrac{\underline{V}}{\underline{I}}$

$\underline{V} = Z_{eq} \underline{I}$

$v(t) = 1\cos(\omega t - 90^\circ)$

$\omega = 100\,rad/s$



$T = \frac{2\pi}{\omega}$

$\underline{V} = 1\angle{-90^\circ}\,V$

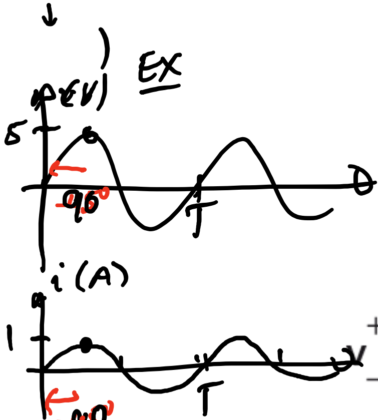
$\underline{I} = \dfrac{1\angle{-180^\circ}\,A}{\cancel{1\angle{90^\circ}}}$ Ω

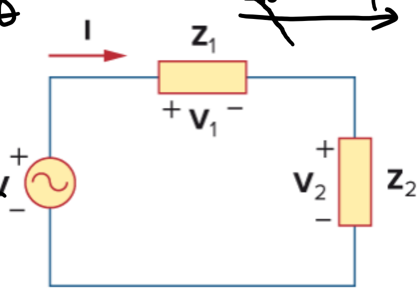
$= j1 = j\omega L$

$\omega L = 1\ \Omega$

$L = 0.01\ H = 10\,mH.$

EX

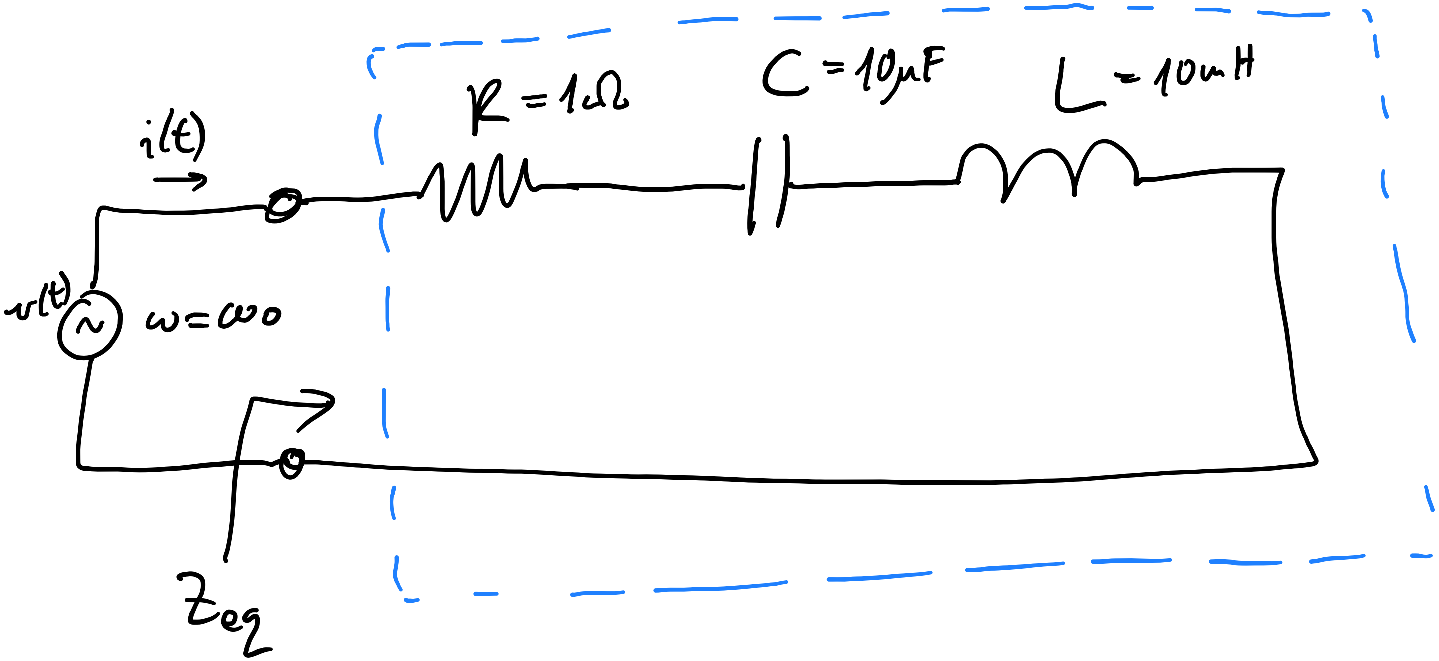




$Z_{eq} = \dfrac{5\angle{-90^\circ}\,V}{1\angle{-90^\circ}\,A}$

$= 5\,\Omega$

$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}, \qquad \mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}$$

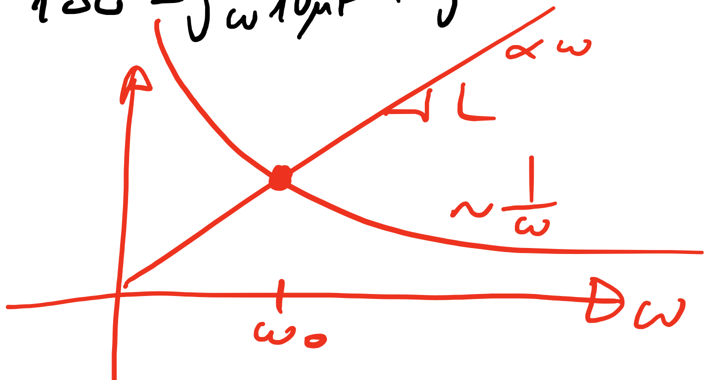


0!

$$Z_{eq} = Z_R + Z_C + Z_L$$

$$= R - j\frac{1}{\omega C} + j\omega L = R + j\boxed{\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= 1\Omega - j\frac{1}{\omega\, 10\mu F} + j\omega\, 10mH = 1\Omega + j\left(\omega\, 10mH - \frac{1}{\omega\, 10\mu F}\right)$$



$$\omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC}$$

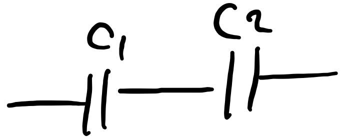
$$\omega = \frac{1}{\sqrt{LC}}$$

$\rightarrow R = R_1 + R_2$

$$\frac{Z}{R}$$

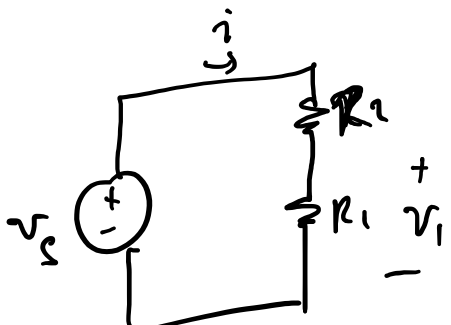
$\rightarrow L = L_1 + L_2$

$j\omega L$



$\rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

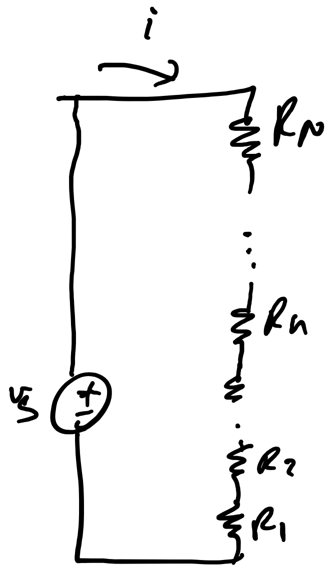
$$\frac{1}{j\omega C}$$



$$i = \frac{v_s}{R_1 + R_2}$$

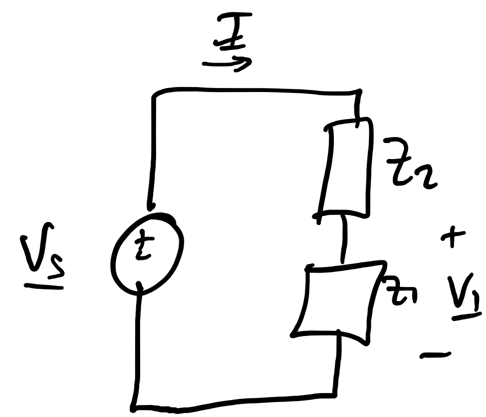
$$v_1 = i R_1$$

$$\boxed{v_1 = \frac{R_1}{R_1 + R_2} v_s}$$



$$i = \frac{v_s}{\sum_n R_n}$$

$$v_n = i R_n \rightarrow v_n = \frac{R_n}{\sum_{m=1}^{N} R_m} v_s$$

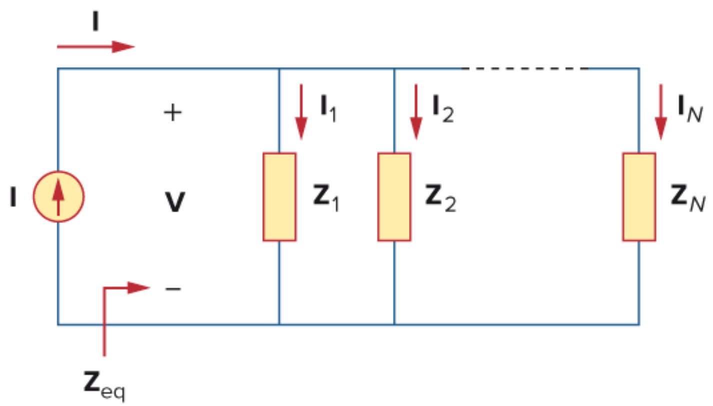


$$\underline{I} = \frac{\underline{V_s}}{Z_1 + Z_2}$$

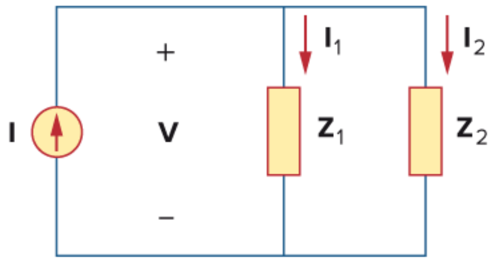
$$\underline{V_1} = \underline{I} \cdot Z_1$$

$$\boxed{\underline{V_1} = \frac{Z_1}{Z_1 + Z_2} \underline{V_s}}$$

Series elements



$$\mathbf{Y}_{eq} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_N$$

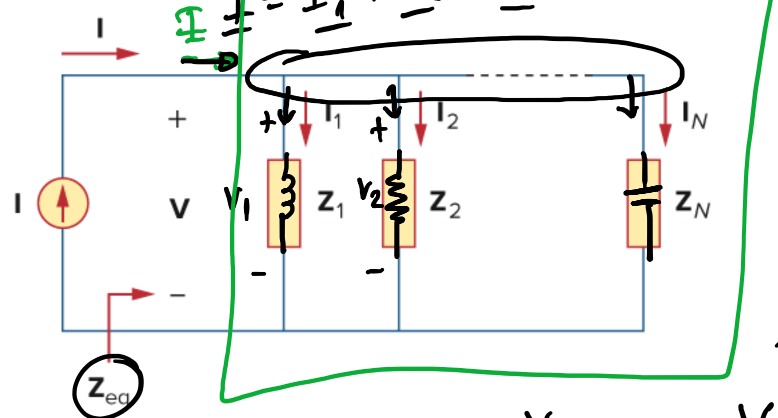


$$\mathbf{I}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}, \qquad \mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}$$

Series elements

$$\boxed{Y = \frac{1}{Z}}$$

$$I = I_1 + I_2 + I_3 + \cdots + I_N$$



$$\mathbf{Y}_{eq} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_N$$

$$V_1 = Z_1 I_1 \rightarrow I_1 = \frac{V}{Z_1}$$

$$V_2 = Z_2 I_2 \rightarrow I_2 = \frac{V}{Z_2}$$

$$V_3 = Z_3 I_3$$

$$V_1 = V_2 = V_3 = \cdots V$$

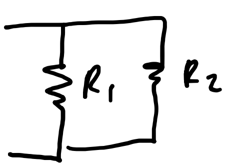
$$V = Z_{eq} I$$

$$= Z_{eq}(I_1 + I_2 + \cdots I_N)$$

$$\cancel{V} = Z_{eq}\left(\frac{V}{Z_1} + \frac{V}{Z_2} + \cdots \frac{V}{Z_N}\right) = \cancel{V}\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots \frac{1}{Z_N}\right)$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots \frac{1}{Z_N}$$

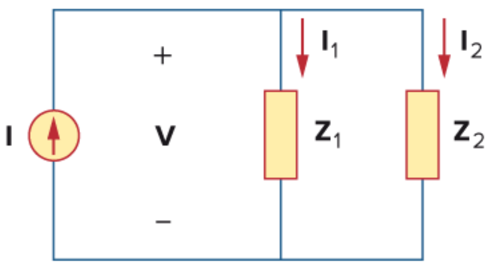
$$\boxed{Y_{eq} = \sum_{k=1}^{N} Y_k}$$



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$G = G_1 + G_2$$

Series elements

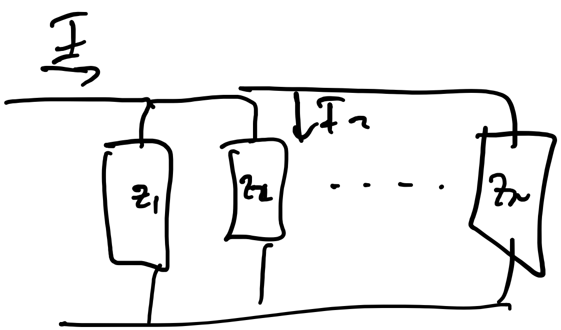


$$= \frac{Y_1}{Y_1+Y_2} I = \frac{\frac{1}{z_1}}{\frac{1}{z_1}+\frac{1}{z_2}} I = \frac{z_2}{z_2+z_1} I$$

$$\boxed{\mathbf{I}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}\mathbf{I}, \qquad \mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2}\mathbf{I}}$$

$$I_1 = \frac{R_2}{R_1+R_2} I_s \quad =$$

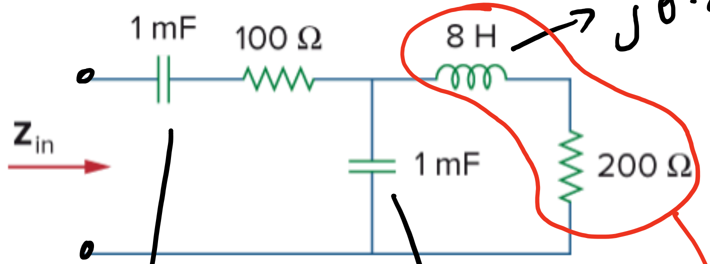
$$\left(V_1 = \frac{R_1}{R_1+R_2} V_s\right)$$



$$\underline{I_2} = \frac{Y_2}{Y_1+Y_2+\cdots Y_N} \underline{I}$$

Practice Problem 9.10

Determine the input impedance of the circuit in **Fig. 9.24** at $\omega = 20$ rad/s.

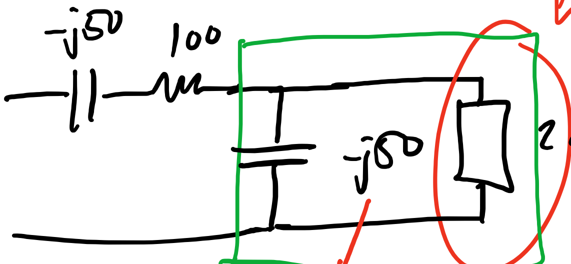


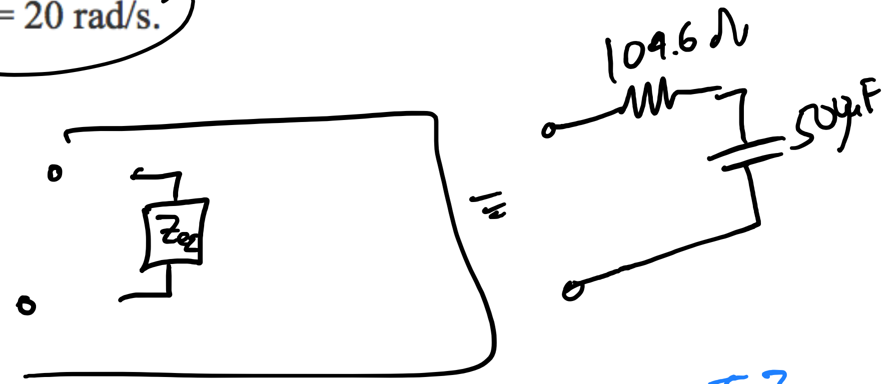
$j8 \cdot 20 = j160\,\Omega$

$-j50\,\Omega$

$Z_C = \dfrac{1}{j\,1\,\text{mF} \cdot 20\,\text{rad/s}} = \dfrac{1}{j\omega C}$

$\dfrac{1}{0.020} = -jS$





$Z_{eq} = -j50 + 100 + 9.60 - j55.3$

$= \boxed{109.6} - j\,\boxed{105.3}$

$= R - j\dfrac{1}{\omega C} + \cancel{j\omega L}$

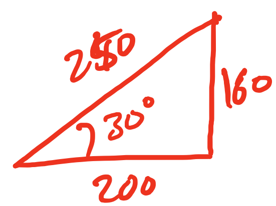
$R = 109.6\,\Omega$

$\dfrac{1}{\omega C} = 105.3\,\Omega$

$C = \dfrac{1}{20 \cdot 105.3}$

$= 0.0005\,F$

$C = 500\,\mu F$



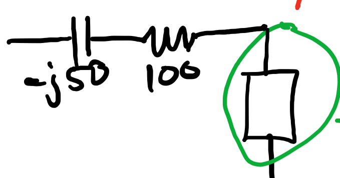
$Z \sim 250\,\angle 30° \to Z = 250\,e^{j\frac{30}{180}\pi}$

$Y = \dfrac{1}{Z} = \dfrac{1}{250\,e^{j30°}} = \dfrac{1}{250}e^{-j30°} = \dfrac{1}{250}\angle -30°$

$0.004\,\angle -30°$

$Y = 0.003049 - j0.002439$

$Y = j\dfrac{1}{50} = j0.020$



$Y = j0.020 + 0.003 - j0.002 = 0.003 + j0.018$

$Z = 1/Y = \cancel{52\angle 80°} = 9.60 - j55.3$

Practice Problem 9.10

Determine the input impedance of the circuit in Fig. 9.24 at $\omega = 20$ rad/s.

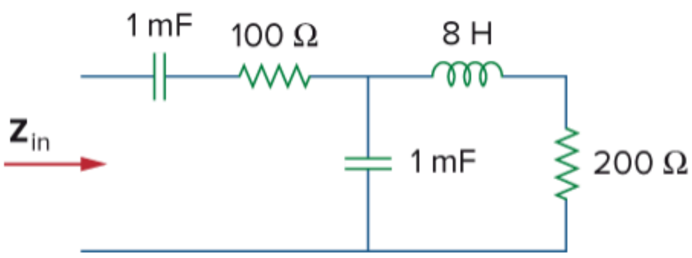


Figure 9.24 For Practice Prob. 9.10.

Answer: $(109.53 - j105.29)\ \Omega$.

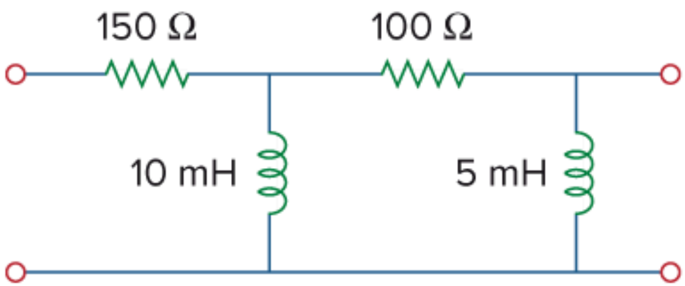
$$Z = 200 + j\omega 8$$

$$Y = \frac{1}{\boxed{Z}} = \frac{200 - j\omega 8}{\boxed{(200 + j\omega 8)}(200 - j\omega 8)} = \frac{200 - j\omega 8}{200^2 + (\omega 8)^2}$$

$$= \frac{200}{200^2 + (\omega 8)^2} - j\frac{\omega 8}{200^2 + (\omega 8)^2}$$

Example 9.14

For the *RL* circuit shown in Fig. 9.35(a), calculate the amount of phase shift produced at 2 kHz.



Solving AC Circuits

(Chapter 10)

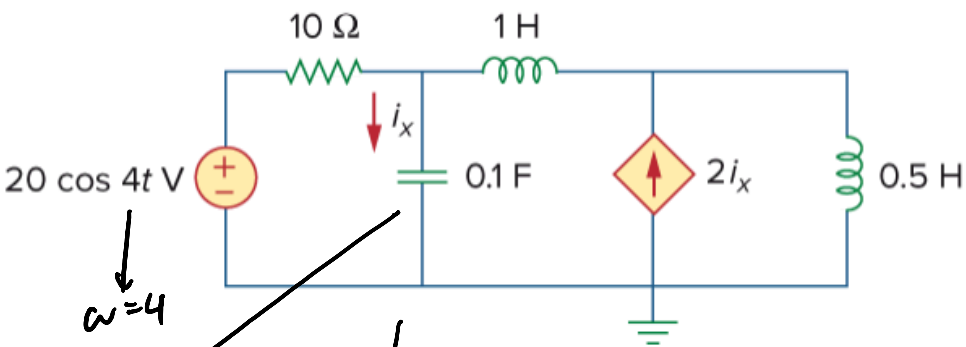
Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.

Node voltage method

Example 10.1

Find i_x in the circuit of **Fig. 10.1** using nodal analysis.



$\omega = 4$

$Z_C = \frac{1}{j\omega C}$

$= -j\frac{1}{4\cdot 0.1} = -j\frac{1}{0.4}$

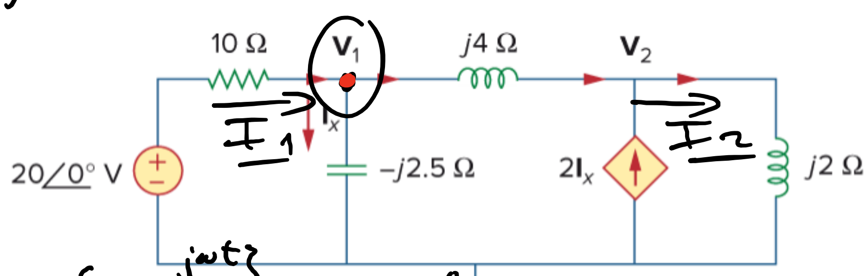
$= -j2.5\,\Omega$

Find impedances

(1): $$\overbrace{\frac{20\angle 0^\circ - V_1}{10}}^{I_1} = \overbrace{\frac{V_1 - V_2}{j4}}^{I_2} + \overbrace{\frac{V_1}{-j2.5}}^{I_x}$$

(2): $$\frac{V_1 - V_2}{j4} + 2\frac{V_1}{-j2.5} = \frac{V_2}{j2}$$

$$I_x = \frac{V_1}{-j2.5}$$



$$\frac{20\angle 0^\circ - \boxed{V_1}}{10} = \frac{\boxed{V_1} - \boxed{V_2}}{j4} + \frac{\boxed{V_1}}{.5}$$

$$\frac{\boxed{V_1} - \boxed{V_2}}{j4} + 2\frac{\boxed{V_1}}{-j2.5} = \frac{\boxed{V_2}}{j2}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

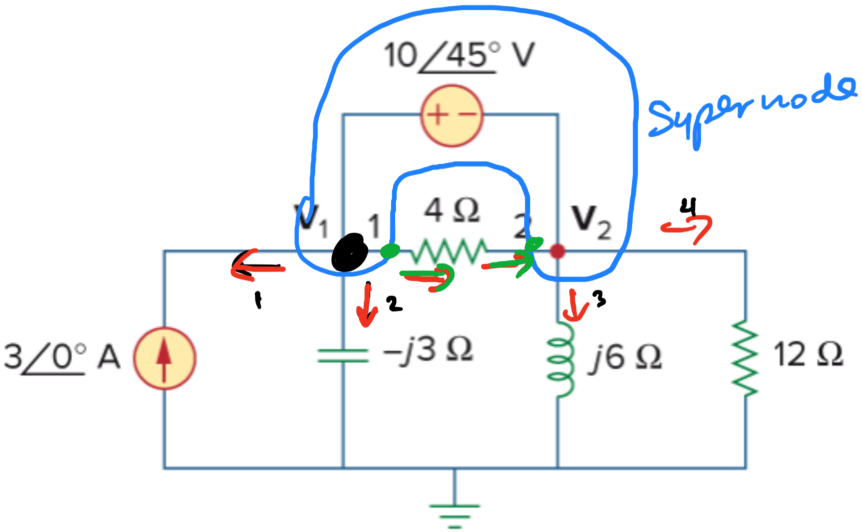
$i_x(t) = Re\{I_x e^{j\omega t}\}$

$= Re\{10 e^{-j\pi/4} e^{j\omega t}\}$ say $= 10\angle -45^\circ$

$i_x(t) = 10\cos(\omega t - 45^\circ) \leftarrow I_x = \frac{V_1}{-j2.5}$

Example 10.2

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of **Fig. 10.4**.



$$\underline{V_2} = \underline{V_1} - 10\angle 45^\circ$$

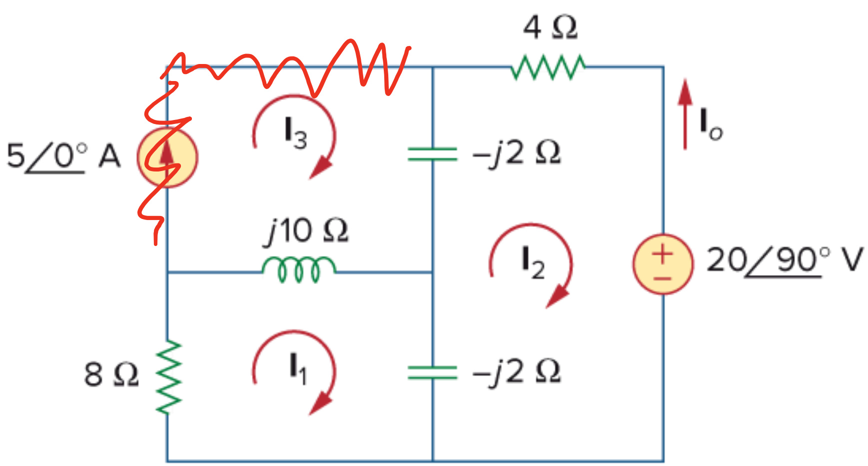
$$\underset{1}{-3\angle 0^\circ} + \underset{2}{\frac{V_1}{-j3}} + \underset{3}{\frac{V_1 - 10\angle 45^\circ}{j6}} + \underset{4}{\frac{V_1 - 10\angle 45^\circ}{12}} = 0$$

$$\underline{V_1}$$

$$\underline{V_2} = \underline{V_1} - 10\angle 45^\circ$$

Example 10.3

Determine current $\mathbf{I}_o$ in the circuit of **Fig. 10.7** using mesh analysis.



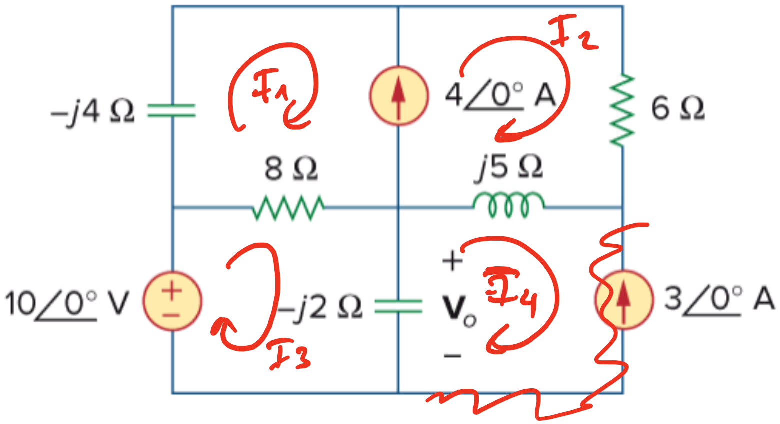
2 eqns bc $\underline{I_3} = 5\angle 0^\circ$ A, $i_3(t) = 5\cos(\omega t)$ A

$\Rightarrow [\;+\;]\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\hookrightarrow \underline{I_o} = -\underline{I_2}$

Example 10.4

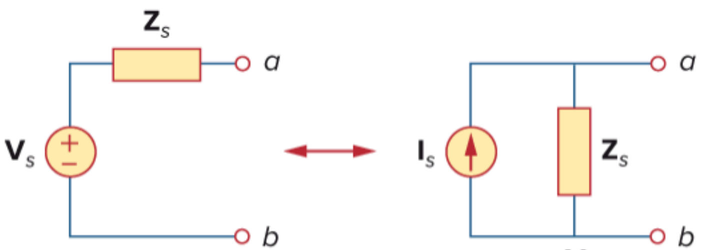
Solve for $\mathbf{V}_o$ in the circuit of Fig. 10.9 using mesh analysis.



$I_4 = 3\angle 0°$ A

Source transformations

$$\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}$$

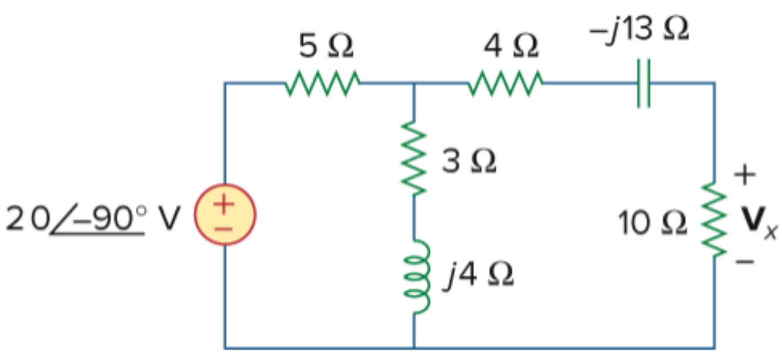


$\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s$

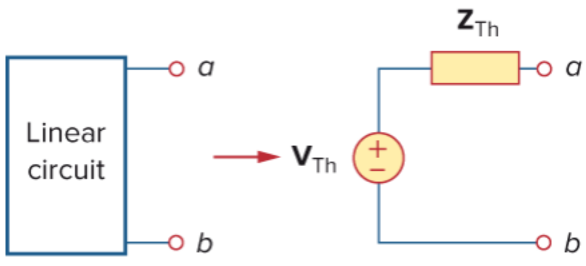
$\mathbf{I}_s = \dfrac{\mathbf{V}_s}{\mathbf{Z}_s}$

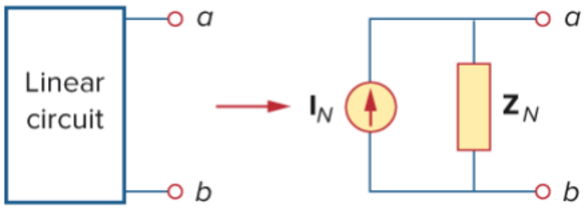
Example 10.7

Calculate $\mathbf{V}_x$ in the circuit of **Fig. 10.17** using the method of source transformation.



Thevenin and Norton equivalents

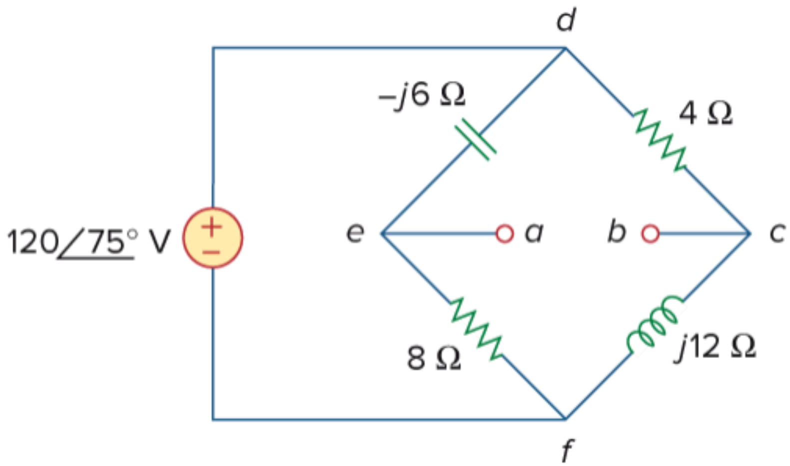




$$\mathbf{V}_{\text{Th}} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{\text{Th}} = \mathbf{Z}_N$$

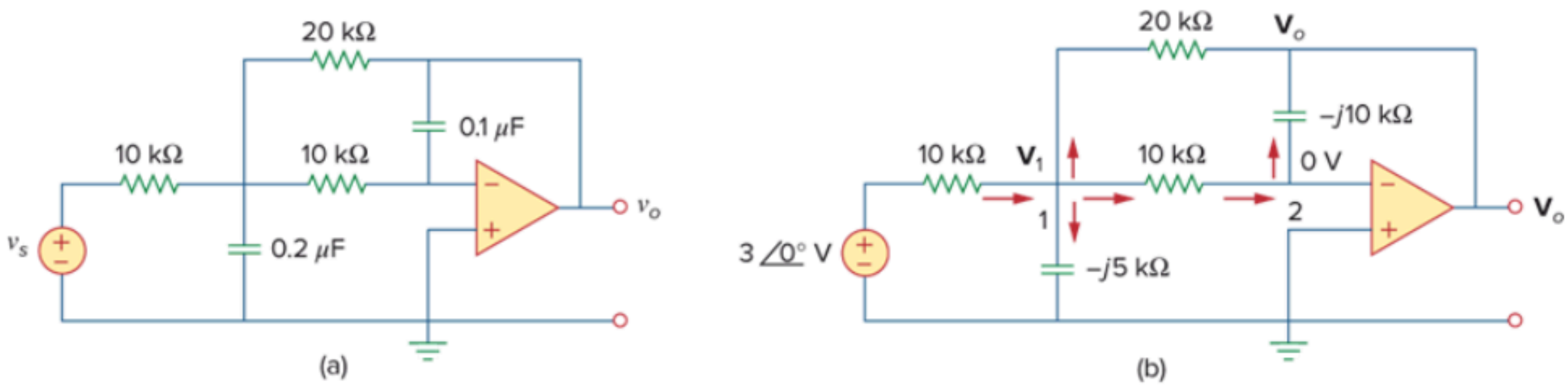
Example 10.8

Obtain the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 10.22.



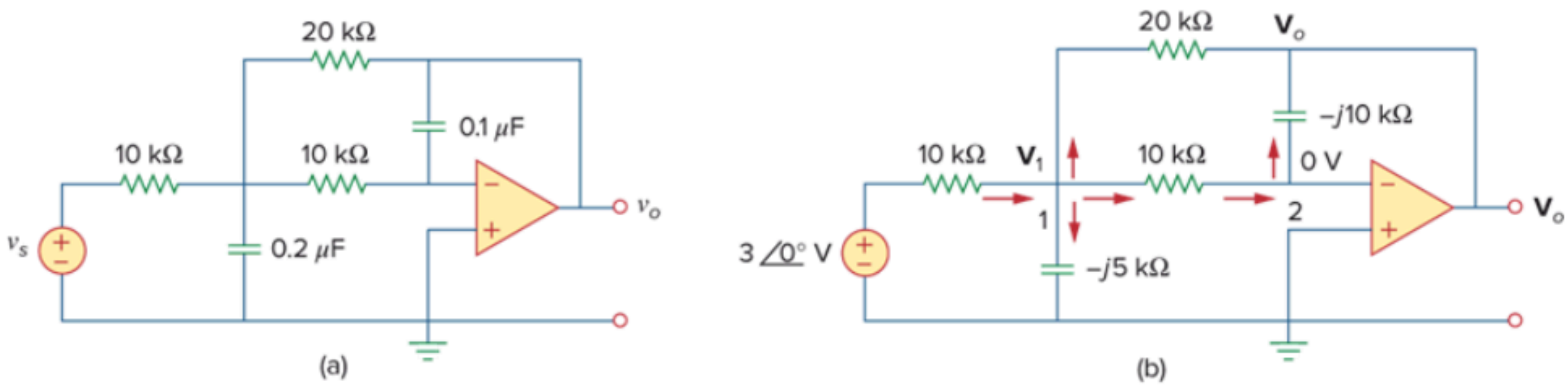
Example 10.11

Determine $v_o(t)$ for the op amp circuit in **Fig. 10.31(a)** if $v_s = 3 \cos 1000t$ V.



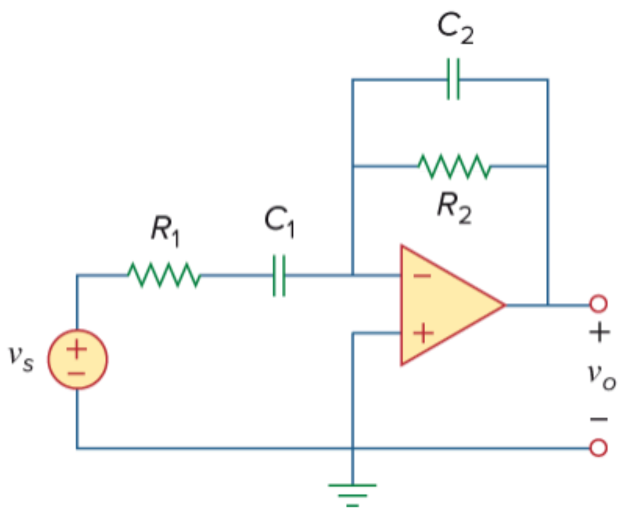
Example 10.11

Determine $v_o(t)$ for the op amp circuit in **Fig. 10.31(a)** if $v_s = 3 \cos 1000t$ V.



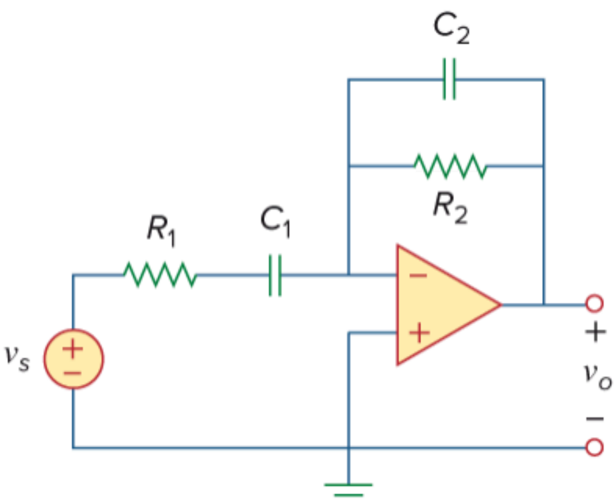
Example 10.12

Compute the closed-loop gain and phase shift for the circuit in **Fig. 10.33**. Assume that $R_1 = R_2 = 10\ \text{k}\Omega$, $C_1 = 2\ \mu\text{F}$, $C_2 = 1\ \mu\text{F}$, and $\omega = 200$ rad/s.



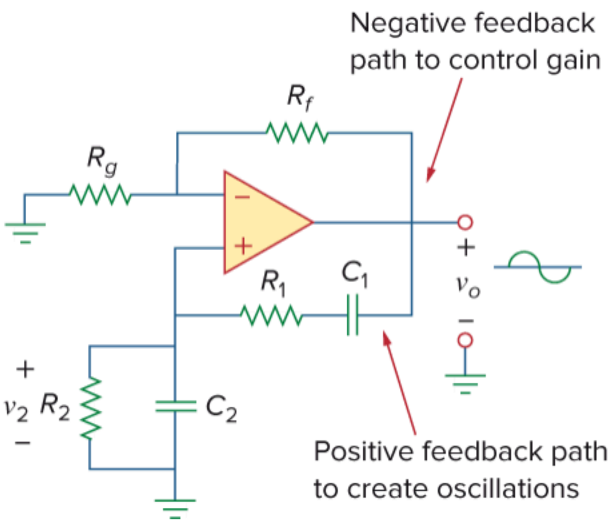
Example 10.12

Compute the closed-loop gain and phase shift for the circuit in Fig. 10.33. Assume that $R_1 = R_2 = 10\ \text{k}\Omega$, $C_1 = 2\ \mu\text{F}$, $C_2 = 1\ \mu\text{F}$, and $\omega = 200$ rad/s.



Oscillators

An **oscillator** is a circuit that produces an ac waveform as output when powered by a dc input.

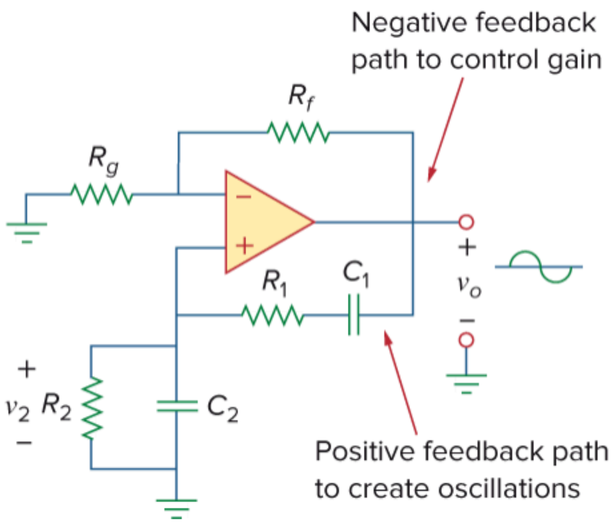


In order for sine wave oscillators to sustain oscillations, they must meet the *Barkhausen criteria*:

1. The overall gain of the oscillator must be unity or greater. Therefore, losses must be compensated for by an amplifying device.
2. The overall phase shift (from input to output and back to the input) must be zero.

Oscillators

An **oscillator** is a circuit that produces an ac waveform as output when powered by a dc input.

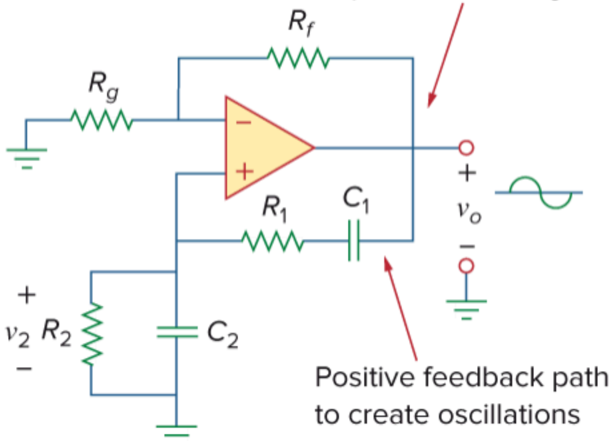


$$\mathbf{Z}_s = R_1 + \frac{1}{j\omega C_1} = R_1 - \frac{j}{\omega C_1}$$

$$\mathbf{Z}_p = R_2 \| \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2}$$

Oscillators

An **oscillator** is a circuit that produces an ac waveform as output when powered by a dc input.



$$\mathbf{Z}_s = R_1 + \frac{1}{j\omega C_1} = R_1 - \frac{j}{\omega C_1}$$

$$\mathbf{Z}_p = R_2 \| \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2}$$

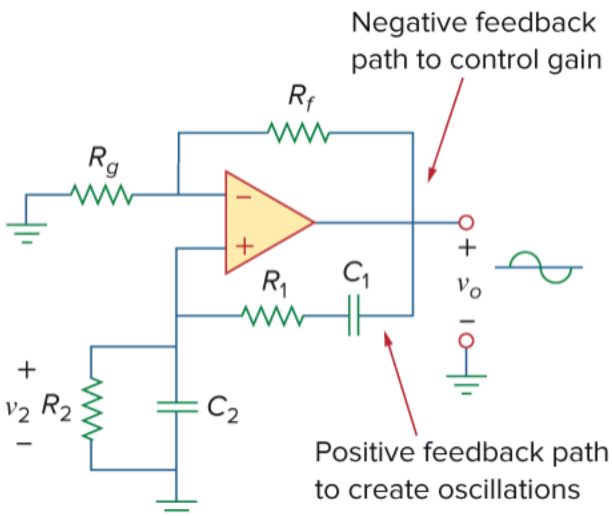
$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{R_2}{R_2 + \left(R_1 - \frac{j}{\omega C_1}\right)(1 + j\omega R_2 C_2)}$$

$$= \frac{\omega R_2 C_1}{\omega(R_2 C_1 + R_1 C_1 + R_2 C_2) + j(\omega^2 R_1 C_1 R_2 C_2 - 1)}$$

Oscillators

An **oscillator** is a circuit that produces an ac waveform as output when powered by a dc input.



$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{R_2}{R_2 + \left(R_1 - \dfrac{j}{\omega C_1}\right)(1 + j\omega R_2 C_2)}$$

$$= \frac{\omega R_2 C_1}{\omega(R_2C_1 + R_1C_1 + R_2C_2) + j(\omega^2 R_1C_1R_2C_2 - 1)}$$

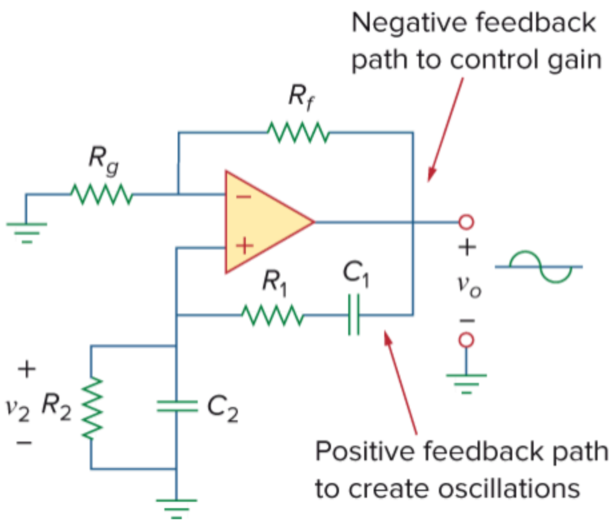
$$\omega_o^2 R_1C_1R_2C_2 - 1 = 0$$

$$\omega_o = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$\omega_o = \frac{1}{RC} = 2\pi f_o$$

Oscillators

An **oscillator** is a circuit that produces an ac waveform as output when powered by a dc input.



$$\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\omega_o = \frac{1}{RC} = 2\pi f_o$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{R_f}{R_g} = 3$$

$$R_f = 2R_g$$