Practice Problem 8.7

Having been in position a for a long time, the switch in **Fig. 8.21** is moved to position b at $t = 0$. Find $v(t)$ and $v_R(t)$ for $t > 0$.

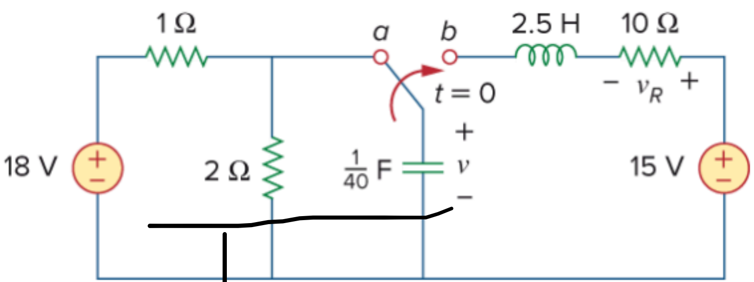
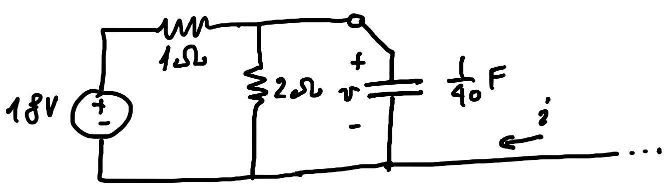


Figure 8.21 For **Practice Prob. 8.7.**

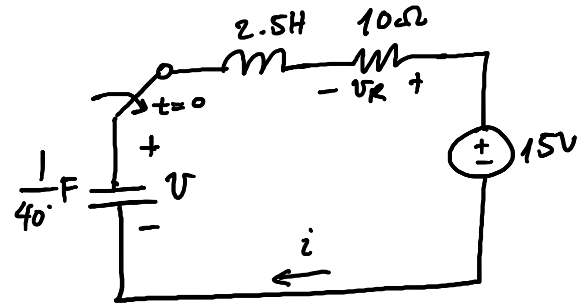
Answer: $15 - (1.7321 \sin 3.464t + 3 \cos 3.464t)e^{-2t}$ V, $3.464e^{-2t} \sin 3.464t$ V.

① $v(0^-), i(0^-)$.



$$\boxed{\begin{aligned} v(0^-) &= \frac{2}{3}\,18V = 12V \\ i(0^-) &= 0 \end{aligned}}$$

② Find response of cct.



$v(0) = 12V$

KVL: $v - L\frac{di}{dt} + v_R - 15V = 0$

$$\boxed{\begin{aligned} &v - L\frac{di}{dt} - iR - 15V = 0 \\ &-i = C\frac{dv}{dt} \end{aligned}}$$

We can replace either i or v. In class we replaced v with i, and the resulting math was complicated. Here, we replace i with v:

$v + L\frac{d}{\ } \quad + RC\frac{d}{dt}v - 15V = 0$

$LCv'' + RCv' + v = 15V$

$$\boxed{v'' + \frac{R}{L}v' + \frac{1}{LC}v = \frac{15V}{LC}}$$

Has solutions

$v(t) = v_{transient}(t) + v_{steadystate}(t)$

$= v_t(t) \qquad + 15V$

Practice Problem 8.7

Having been in position a for a long time, the switch in **Fig. 8.21** is moved to position b at $t = 0$. Find $v(t)$ and $v_R(t)$ for $t > 0$.

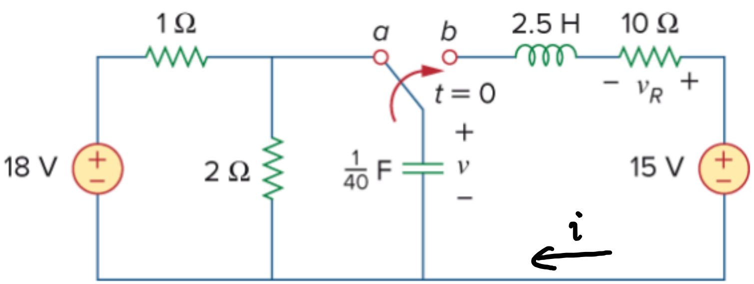


Figure 8.21 For **Practice Prob. 8.7**.

Answer: $15 - (1.7321 \sin 3.464t + 3 \cos 3.464t)e^{-2t}$ V, $3.464e^{-2t} \sin 3.464t$ V.

(3) Now, $\alpha = \frac{R}{2L} = \frac{10\Omega}{2 \cdot 2.5H} = 2$ Np/s, $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.5 \cdot \frac{1}{40}}} = \sqrt{16} = 4$ rad/s

$$\boxed{\alpha < \omega_0 \Rightarrow \text{underdamped.}}$$

Now, this means a solution with roots :

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2} \equiv -\alpha \pm j\omega_d$$

Note that this has a frequency $\boxed{\omega_d = \sqrt{\omega_0^2 - \alpha^2}}$

∴ The solution is

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$= A_1 e^{-\alpha t} e^{j\omega_d t} + A_2 e^{-\alpha t} e^{-j\omega_d t}$$

$$v_t(t) = e^{-\alpha t}\left[A_1(\cos\omega_d t + j\sin\omega_d t) + A_2(\cos\omega_d t - j\sin\omega_d t)\right]$$
$$= e^{-\alpha t}\left[B_1 \cos\omega_d t + B_2 \sin\omega_d t\right]$$

where

$$\omega_d \equiv \sqrt{\omega_0^2 - \alpha^2}$$
$$= \sqrt{4^2 - 2^2}$$
$$\boxed{\omega_d = \sqrt{12} \approx 3.464 \text{ rad/s}}$$

Practice Problem 8.7

Having been in position a for a long time, the switch in **Fig. 8.21** is moved to position b at $t = 0$. Find $v(t)$ and $v_R(t)$ for $t > 0$.

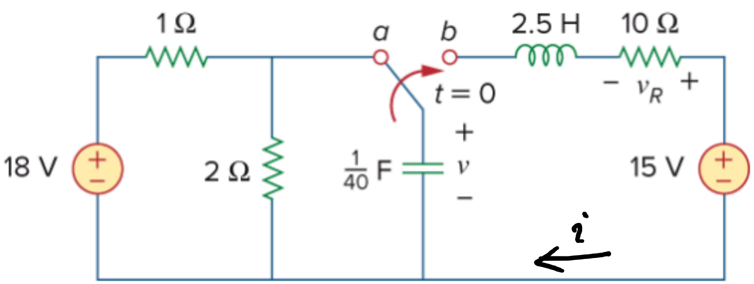


Figure 8.21 For **Practice Prob. 8.7**.

Answer: $15 - (1.7321 \sin 3.464t + 3 \cos 3.464t)e^{-2t}$ V, $3.464e^{-2t} \sin 3.464t$ V.

④ Now, match initial conditions at $t=0$ $(v(0^-) = 12V,\ i(0^-) = 0)$.

$$v_t(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

$$\alpha = 2, \quad \omega_d = \sqrt{12}$$

$$v(t) = v_t(t) + v_{ss} = v_t(t) + 15$$

THEN: $\boxed{v(0) = B_1 + 15V = 12V} \rightarrow \boxed{B_1 = -3V}$

$$-i = C\frac{dv}{dt} = C\left[-\alpha e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t) + e^{-\alpha t}(-B_1 \omega_d \sin \omega_d t + B_2 \omega_d \cos \omega_d t)\right]$$

$$\boxed{i(0) = 0 = -C[-\alpha B_1 + B_2 \omega_d]}$$

$$\therefore \boxed{B_2 = \frac{\alpha B_1}{\omega_d} = \frac{2 \times -3}{\sqrt{12}} = \frac{-6}{\sqrt{12}} = \frac{-3}{\sqrt{3}} = -\sqrt{3} \approx -1.732}$$

⑤ FINAL SOLUTION:

$$v(t) = v_t(t) + v_{ss}$$
$$= v_t(t) + 15V$$
$$= e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t) + 15V$$
$$= e^{-2t}(-3 \cos \omega_d t - 1.732 \sin \omega_d t) + 15V$$

$$\boxed{v(t) = 15 - e^{-2t}(3 \cos \omega_d t + 1.732 \sin \omega_d t)}$$

(same as answer key) $\boxed{\omega_d = \sqrt{12} = 3.464}$

Now for v_R:

$$v_R(t) = -iR, \quad -i = C\frac{dv}{dt}$$

$$\therefore v_R(t) = RCe^{-\alpha t}\left[\cancel{-\alpha B_1 \cos \omega_d t} - \alpha B_2 \sin \omega_d t + \cancel{B_2 \omega_d \cos \omega_d t} - B_1 \omega_d \sin \omega_d t\right]$$

$$= -RCe^{-\alpha t}(\alpha B_2 + \omega_d B_1) \sin \omega_d t$$

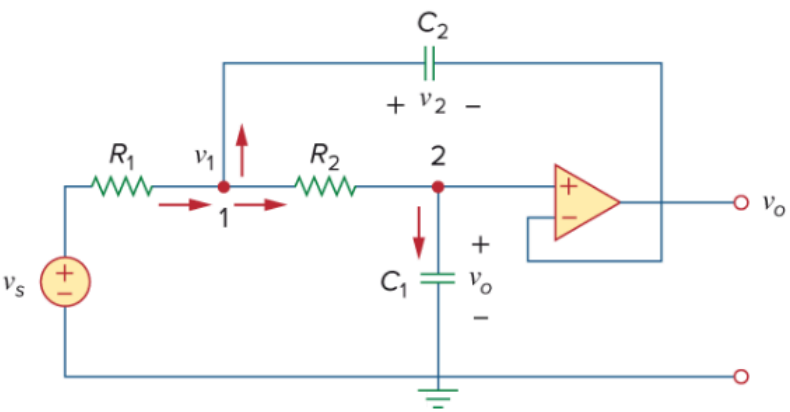
$$RC(\alpha B_2 + \omega_d B_1)$$
$$= 10 \cdot \frac{1}{40}[2 \cdot (-\sqrt{3}) + \sqrt{12}(-3)]$$
$$= \frac{-1}{4}[2\sqrt{3} + 3 \cdot 2\sqrt{3}]$$
$$= -2\sqrt{3}$$
$$= -3.464$$

$$\therefore \boxed{v_R(t) = 3.464 e^{-\alpha t} \sin \omega_d t}$$

DONE!

Example 8.11

In the op amp circuit of Fig. 8.33, find $v_o(t)$ for $t > 0$ when $v_s = 10u(t)$ mV. Let $R_1 = R_2 = 10$ kΩ, $C_1 = 20\ \mu$F, and $C_2 = 100\ \mu$F.



Please study example 8.11 in Section 8.8 (6th edition of book). pages 342–344.