EK 307: Electric Circuits

Fall 2017

Lecture 19

Nov 14, 2017

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Administrivia

- Homework 8 due
- Midterm #2: Thu Nov 16

Last time (Lecture 18):

1. **First-order op-amp circuits**
2. **Second-order circuits & resonaors**

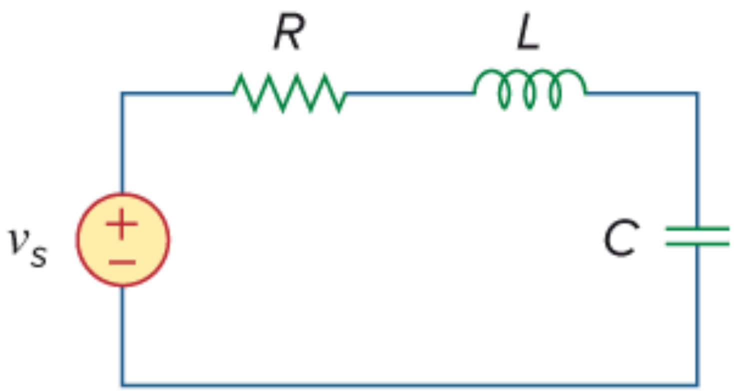
(Chapter 8)

Lecture 18: Chapter 8 – 2^{nd} order ccts

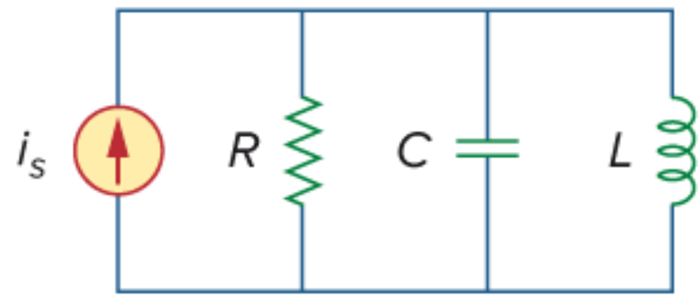
Second-order circuits

- 1^{st}-order circuits => 1^{st} order diff eq. => 1 energy storage element
- 2^{nd}-order circuits => 2^{nd} order diff eq. => 2 energy storage elements

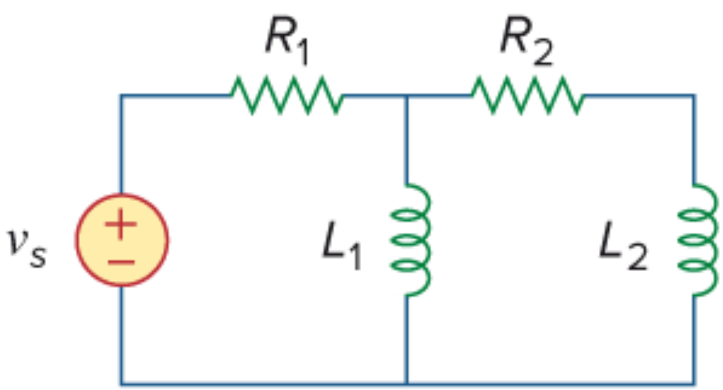
Examples

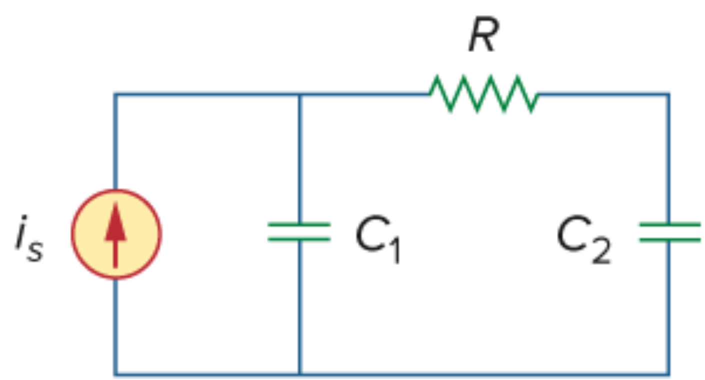


series RLC



parallel RLC





Initial conditions

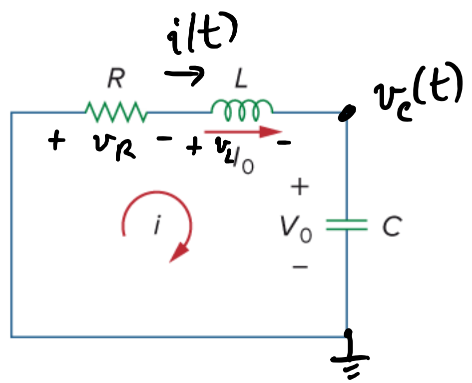
- We will need: $v(0)$, $i(0)$, $dv(0)/dt$, $di(0)/dt$, $i(\infty)$, and $v(\infty)$.

- Remember: use passive sign convention

- For capacitor: $v(0^-) = v(0^+)$
- For inductor: $i(0^-) = i(0^+)$

Source-free Series RLC Circuit

Give: $v(0)$, $i(0)$ at $t=0$. Find: $v(t)$, $i(t)$, $t>0$.



$$\boxed{\frac{d^2 i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{i}{LC}=0}$$

$$i = C\frac{dv_c}{dt} \rightarrow \boxed{\frac{dv_c(t)}{dt}=\frac{i(t)}{c}} \Rightarrow v_c(t)=\int^t \frac{1}{C}\, i(t')\,dt'$$

$$v_R(t)+v_L(t)+v_c(t)=0$$

$$iR + L\frac{di}{dt}+\frac{1}{C}{}^{t}\, i(t')\,dt'=0$$

$\frac{d}{dt}$

$$\frac{dv_R}{dt}+\frac{d}{dt}v_L+\frac{d}{dt}v_c=0$$

$$\boxed{R\frac{di}{dt}+L\frac{d^2}{dt^2}i+\frac{1}{C}i=0} \rightarrow \text{need } i(0)$$

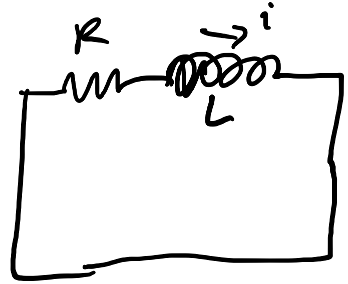
$$i'(0)=\frac{v_L(0)}{L} \Rightarrow v_c \cdots$$

$$\frac{d^2}{dt^2}i+\frac{R}{L}\frac{di}{dt}+\frac{1}{LC}i=0$$

$\frac{1}{\tau_{RL}}$

$s^{-2} \rightarrow \frac{1}{\sqrt{LC}} \rightarrow s^{-1}$

$$\boxed{L\frac{d}{dt}i + iR = 0}$$



$$\frac{d}{dt}i + \frac{R}{L}i = 0$$

$$\frac{1}{i}\frac{d}{dt}i = -\frac{R}{L}$$

$$\frac{d}{dt}\ln i = -\frac{R}{L}$$

$$\ln i = -\frac{R}{L}\int dt = -\frac{R}{L}t$$

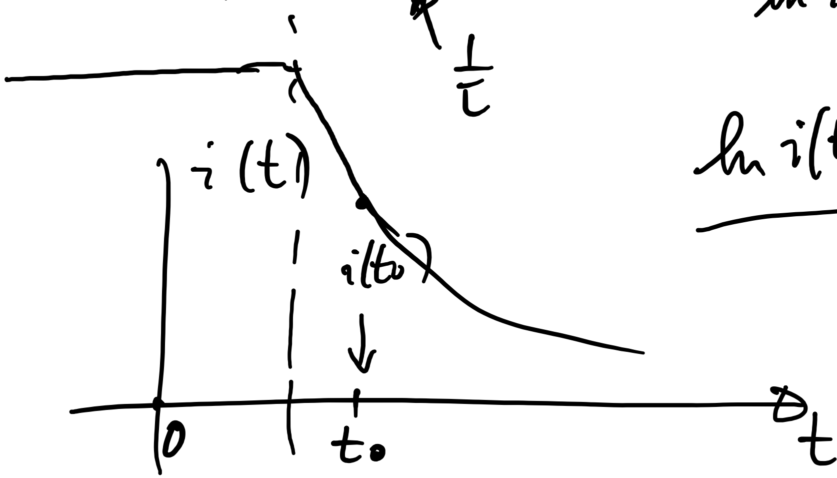
$$i(t) = e^{-\frac{R}{L}t}$$

$$\frac{1}{\tau}$$

$$\int_0^{\boxed{t}} \frac{d}{dt'}\ln i(t')\,dt' =$$

$$\ln i(t')\Big|_{t'=0}^{t'=t} = - \Big|_{t'=0}^{t'=t} = -\frac{R}{L}(t-0)$$

$$\ln i(t) - \ln i(0) = -\frac{R}{L}t \Rightarrow \frac{i(t)}{i(0)} = e^{-\frac{R}{L}t}$$



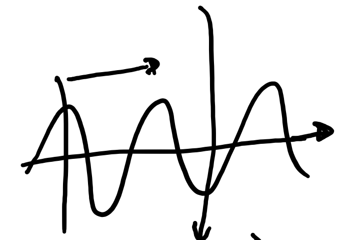
$$\int_{t_0}^{t}$$

$$\ln i(t) - \ln i(t_0) = -\frac{R}{L}(t-t_0)$$

$$\boxed{\frac{i(t)}{i(t_0)} = e^{-\frac{R}{L}(t-t_0)}}$$

Source-free Series RLC Circuit

$\cos(2\pi f_0 t)$ Hz



$$\frac{d^2}{dt^2} i + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$$

$$s^2 \cancel{Ae^{st}} + \frac{R}{L} s \cancel{Ae^{st}} + \frac{1}{LC}\cancel{Ae^{st}} = 0$$

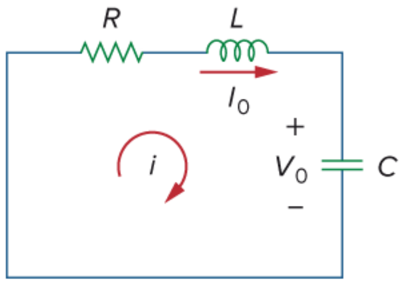
$$\boxed{s^2 + \frac{R}{L}s + \frac{1}{LC} = 0}$$

$s = -\frac{1}{\tau}$ $\quad e^{-\frac{1}{\tau}t}$

$$\cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

freq (rad/s)

$s = j\omega_0$ $\quad e^{st} \Rightarrow e^{(j\omega_0)t}$



$$\boxed{i = Ae^{st}} \rightarrow \quad \frac{di}{dt} = sAe^{st} \qquad \frac{d^2 i}{dt^2} = s^2 A e^{st}$$

$$\boxed{s^2 + \frac{R}{L}s + \frac{1}{LC} = 0}$$

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

s_1 and s_2 : *natural frequencies*, measured in nepers per second (Np/s).

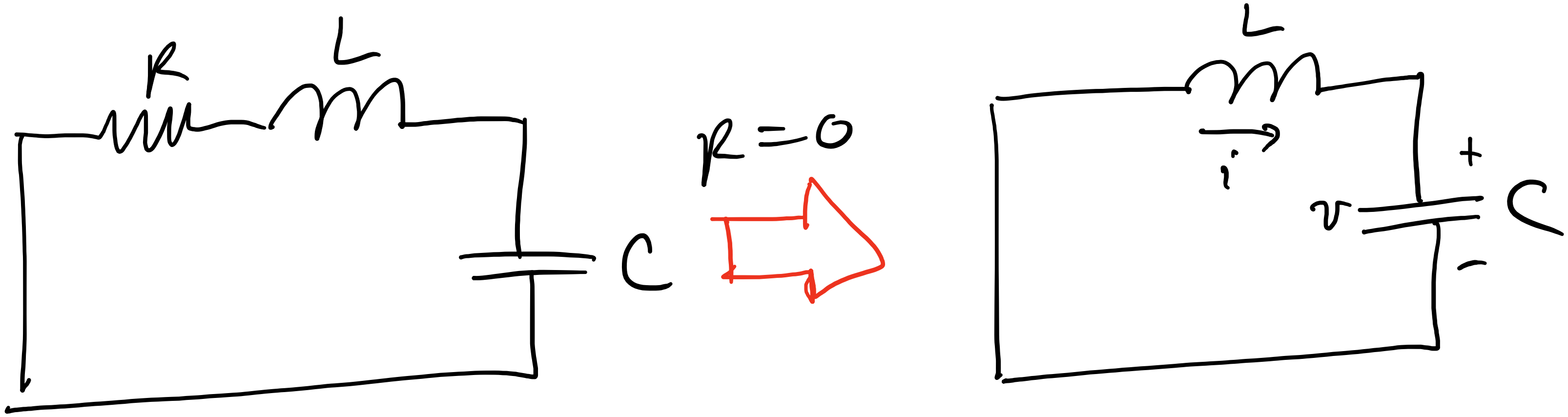
ω_0 : *resonant frequency* or strictly as the *undamped natural frequency*, expressed in radians per second (rad/s)

$$Ax^2 + Bx + C = 0$$

$$x = \frac{-B \pm \sqrt{}}{2A}$$

$$s = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\frac{1}{LC}}}{2}$$

$$\boxed{s = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}$$



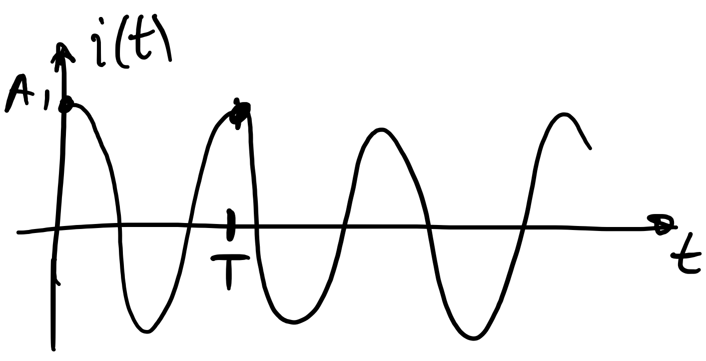
$$s = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s = 0 \pm \sqrt{0 - \frac{1}{LC}}$$

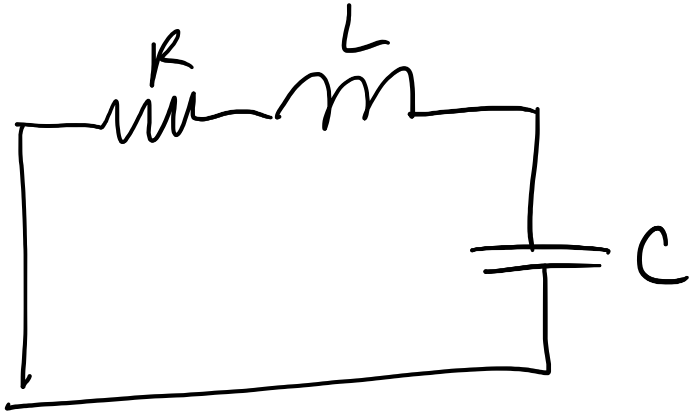
$$s_{1,2} = \pm j \frac{1}{\sqrt{LC}} = \pm j\omega_0$$

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} = A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t}$$

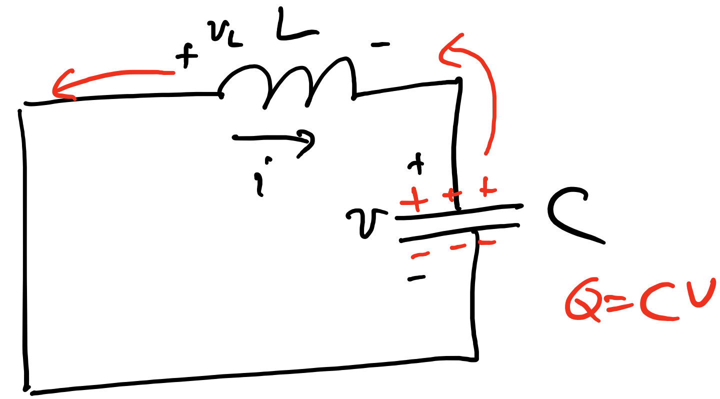
$$= A_1\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) = \underline{A_1 2\cos(\omega_0 t)}$$



$$T = \frac{2\pi}{\omega_0}$$



$R=0$

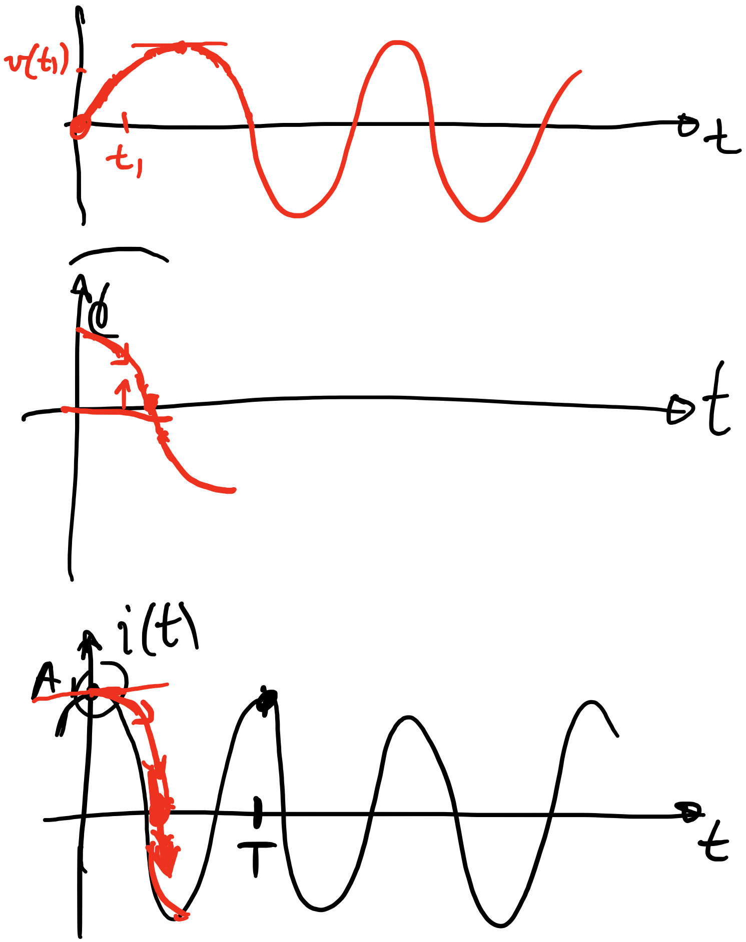


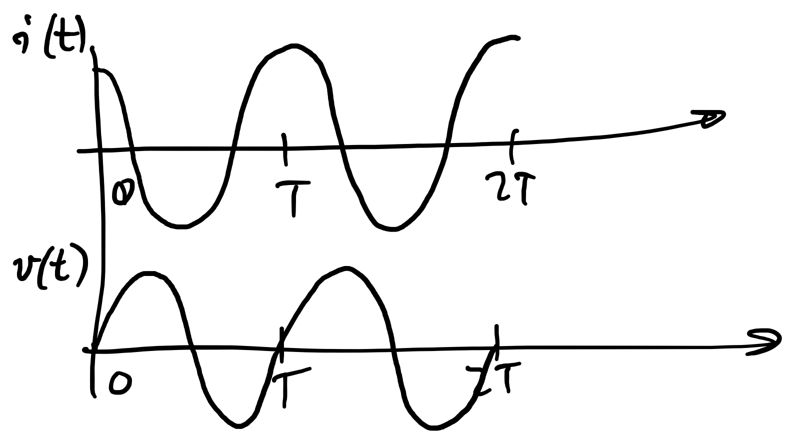
$$-v = +v_L = L\frac{di}{dt} \qquad i = C\frac{dv}{dt}$$

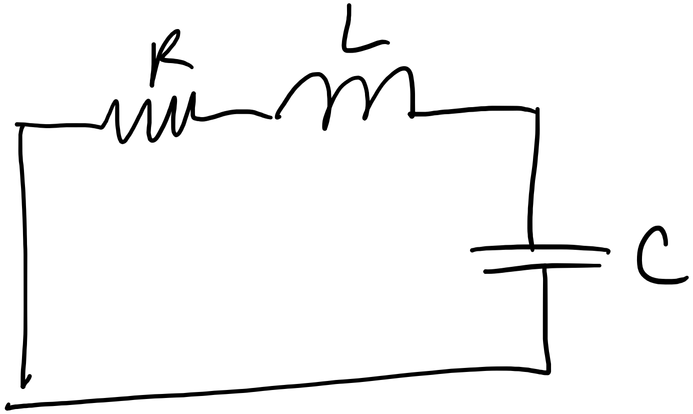
$$\frac{di}{dt} = \frac{-v}{L}$$

$$= 0 \text{ at } t=0$$

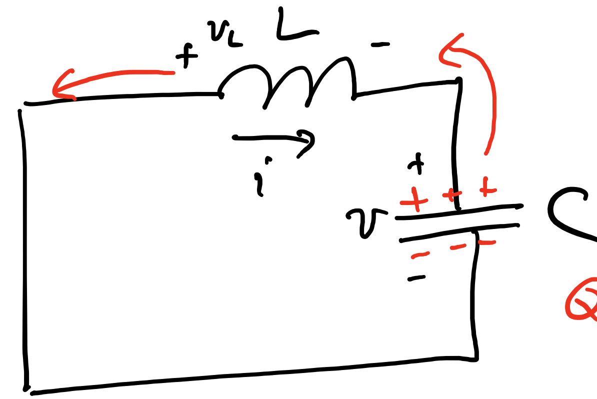
$$\left.\frac{di}{dt}\right|_{t=t_1} = \frac{-v(t_1)}{L} < 0$$



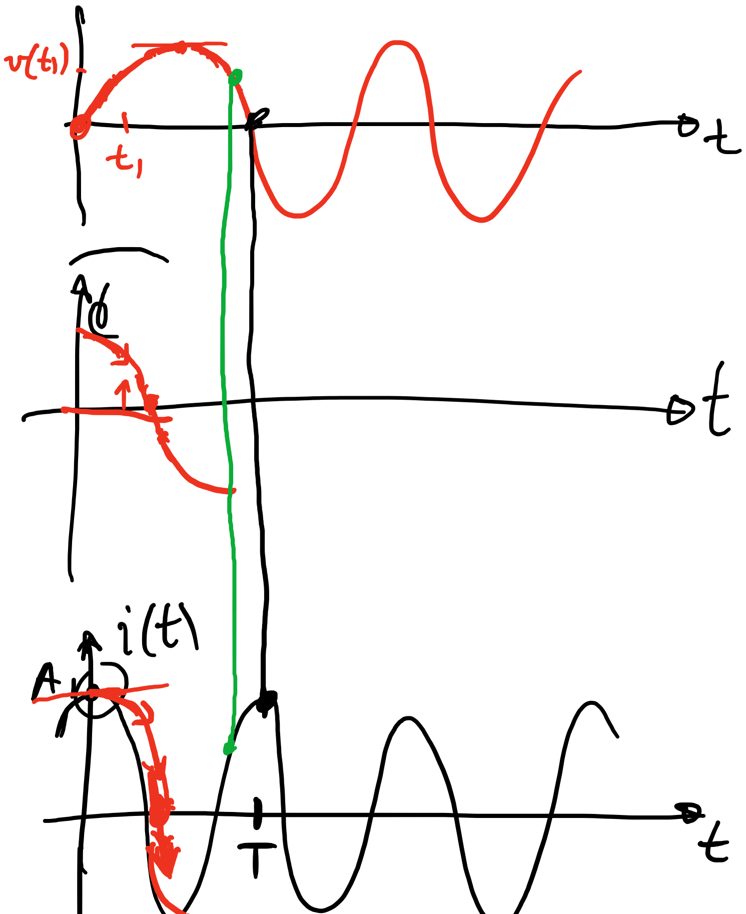


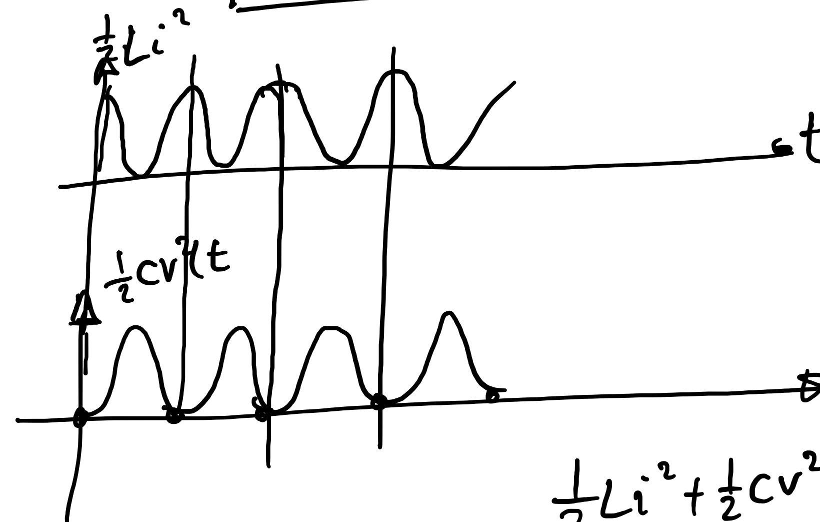


$R = 0$

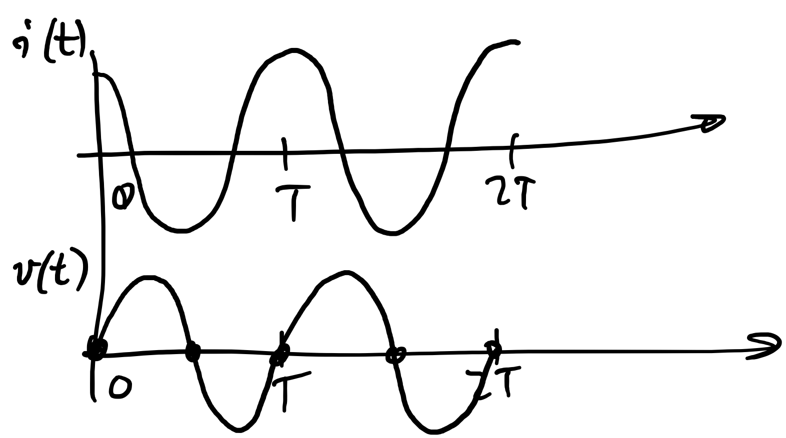


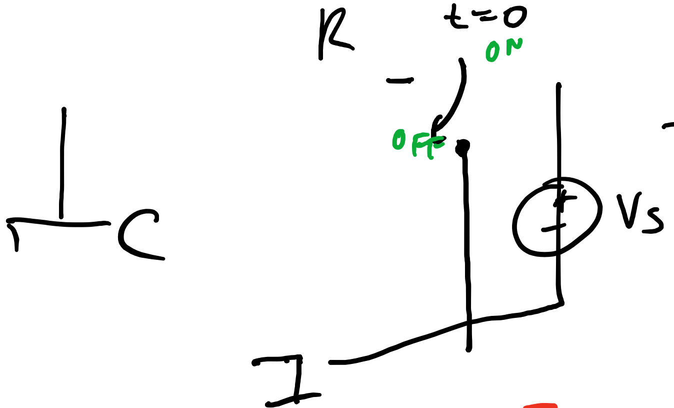
$Q = CV$

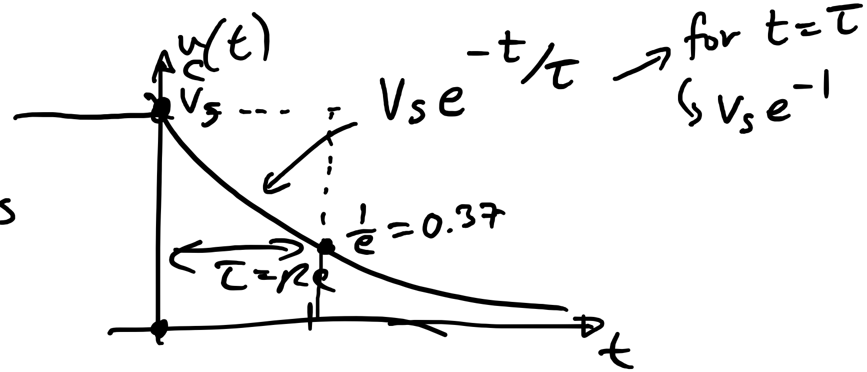


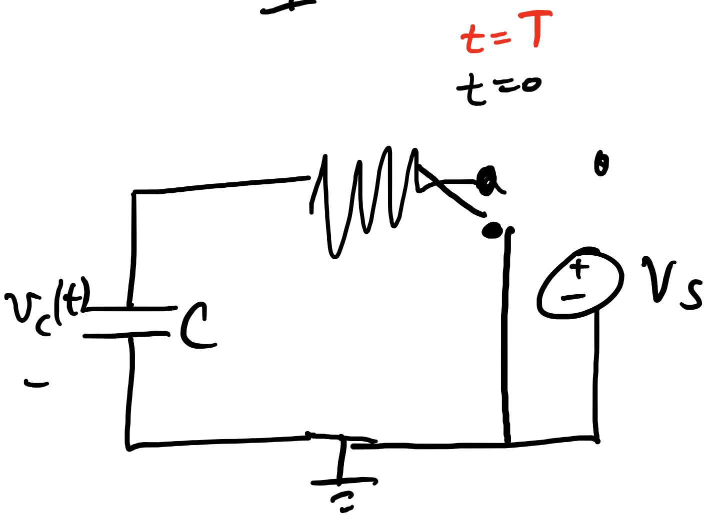


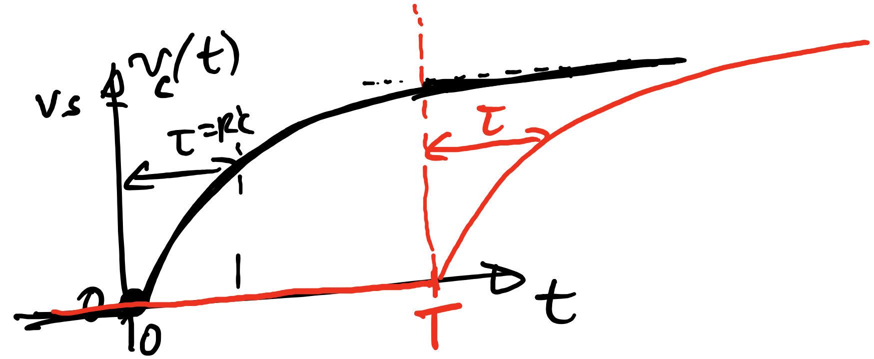
$$\frac{1}{2}Li^2 + \frac{1}{2}cv^2 = \text{const.}$$

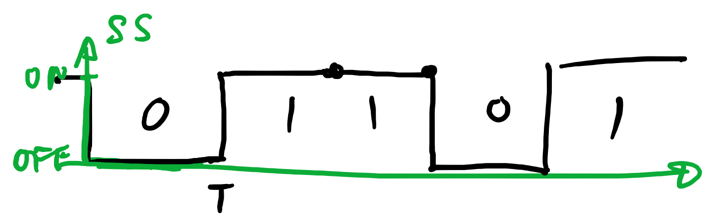






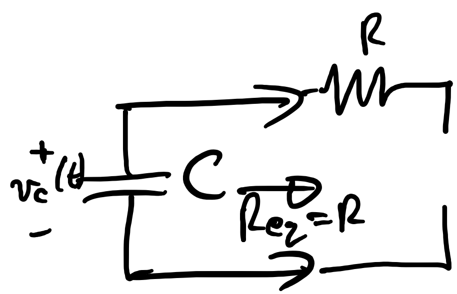




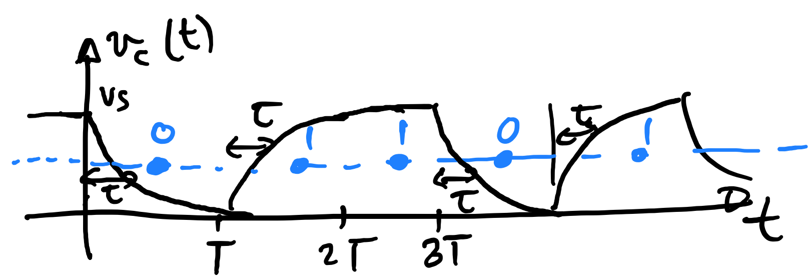


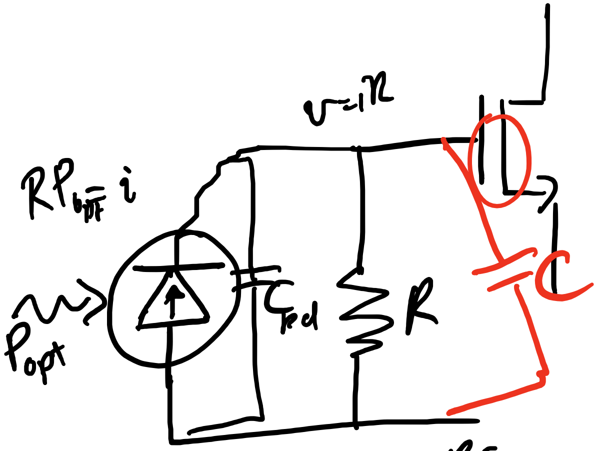
$\frac{1}{T}$ bps

1ns → 1Gbps.



$\tau = RC$





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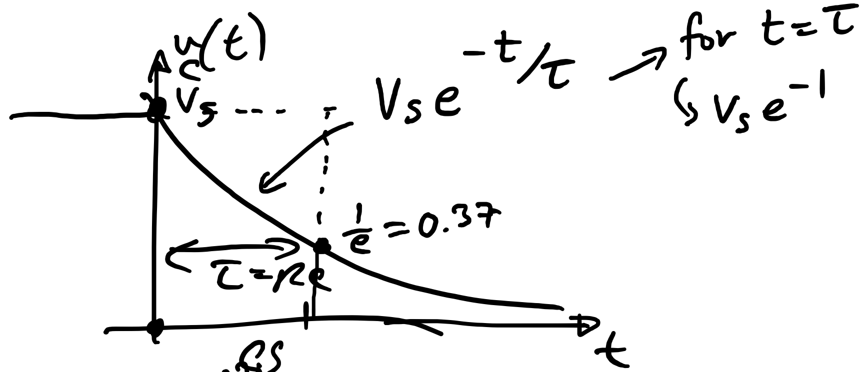
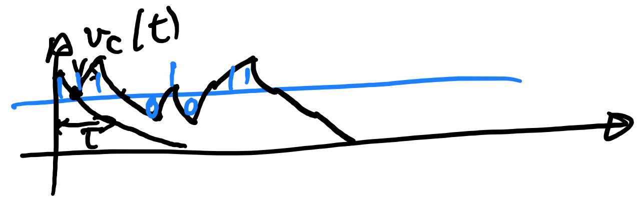
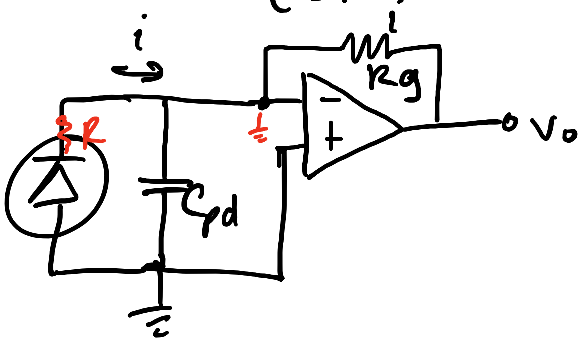
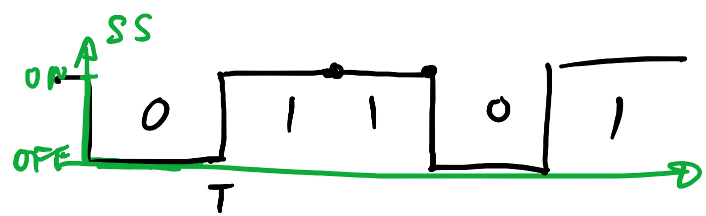


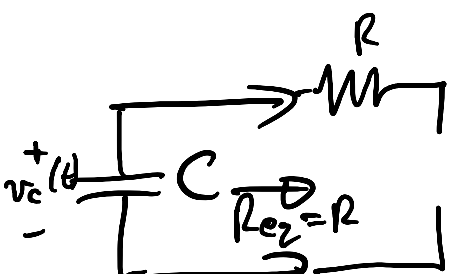




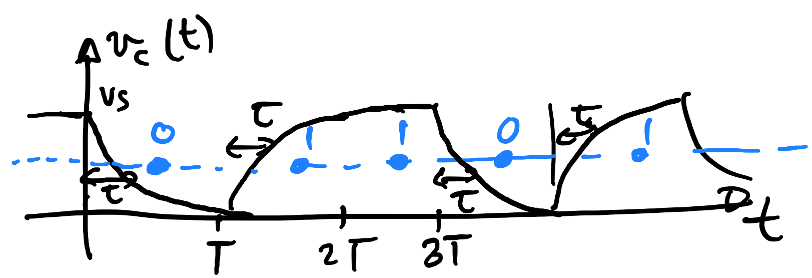


$\frac{1}{T}$ bps

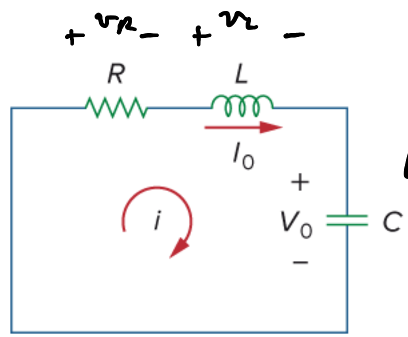
1ns → 1Gbps.



$\tau = RC$



Source-free Series RLC Circuit



$$iR + L\frac{di}{dt} + \int \frac{1}{C} i\,dt = 0$$

$$Li'' + Ri' + \frac{1}{C}i = 0$$

$$LCi'' + RCi' + i = 0$$

$$i'' + \frac{R}{L}i' + \frac{1}{LC}i = 0$$

$$As^2 + Bs + C = 0$$

$$1s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \leftarrow s^2\cancel{i} + \frac{R}{L}s\cancel{i} + \frac{1}{LC}\cancel{i} = 0$$

$$\boxed{s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}}$$

where $s = \frac{-B}{2A} \pm \sqrt{\frac{B^2}{2A} - \frac{4AC}{A}}$

$$\boxed{\alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}}$$

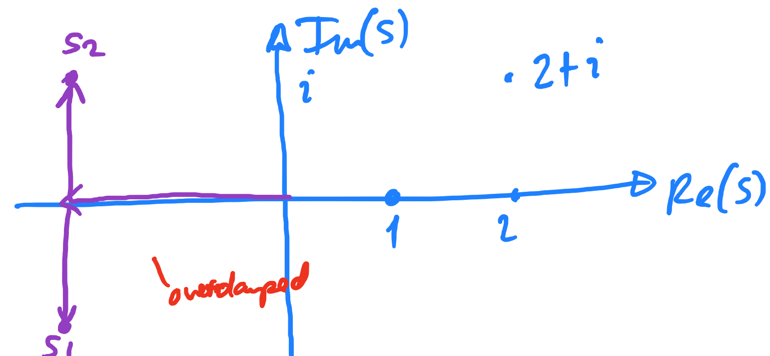
resonant freq.

$$i = \underline{A}\,\underline{e^{st}} \qquad \frac{d}{dt}i = \underline{A}s\underline{e^{st}} = si$$

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$i_1 = A_1e^{s_1t}, \qquad i_2 = A_2e^{s_2t}$$

$$i(t) = A_1e^{s_1t} + A_2e^{s_2t}$$



We can infer 3 types of soln:

1) If $\alpha > \omega_0$, we have the *overdamped* case.
2) If $\alpha = \omega_0$, we have the *critically damped* case.
3) If $\alpha < \omega_0$, we have the *underdamped* case.

Source-free Series RLC Circuit

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1) If $\alpha > \omega_0$, we have the *overdamped* case.
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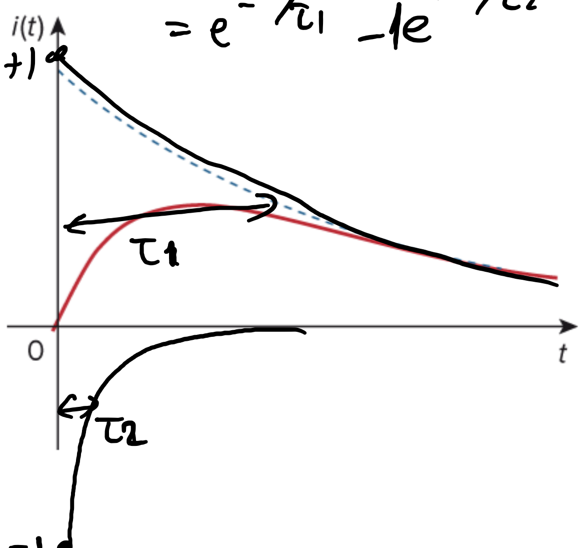
3) If $\alpha < \omega_0$, we have the *underdamped* case.

$s = -\alpha \pm \Delta\alpha$

$i(t) = e^{-\alpha_1 t} - e^{-\alpha_2 t}$

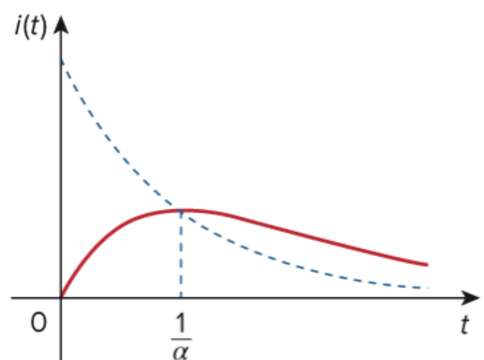
$= e^{-t/\tau_1} - 1e^{-t/\tau_2}$



Overdamped

$s = -\alpha \pm 0$

$i(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t}$



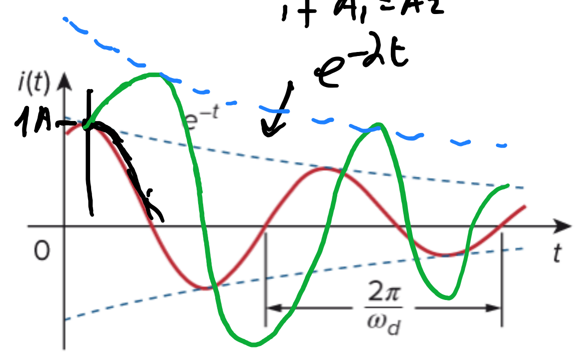
Critically damped

$s = -\alpha \pm j\omega_0$

$i(t) = A_1 e^{(-\alpha + j\omega_0)t} + A_2 e^{(-\alpha - j\omega_0)t}$

$= A_1 2\cos(\omega_0 t)\, e^{-\alpha t}$

if $A_1 = A_2$

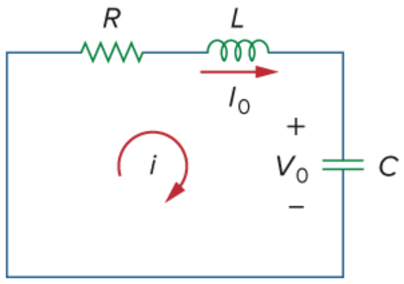


$i(0) = 1A,\ v(0) = 0$

$i(0) = 1A,\ v(0) = 1V$

Underdamped

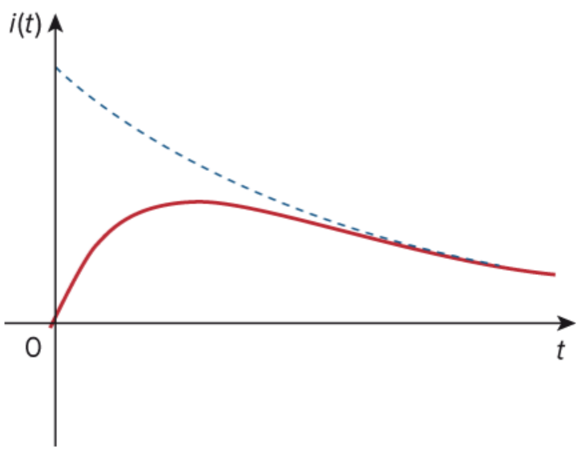
Overdamped case



$\alpha > \omega_0 \Rightarrow C > 4L/R^2$

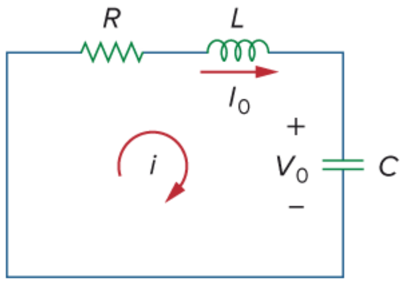
$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$



Overdamped case

$\alpha > \omega_0 \Rightarrow C > 4L/R^2$



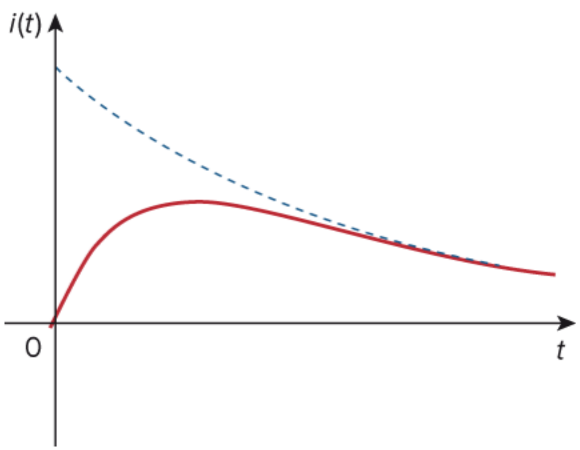
$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\boxed{\alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}}$$

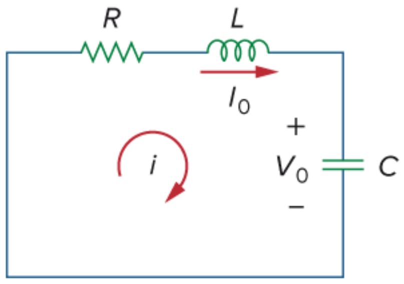
$$\boxed{s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\boxed{i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$$



Critically damped case



$\boxed{\alpha = \omega_0}$ ⇒ $C = 4L/R^2$

$$s_1 = s_2 = -\alpha = -\frac{R}{2L}$$

$$i(t) = A_1 e^{-\alpha t} + A_2 e^{-\alpha t} = A_3 e^{-\alpha t}$$

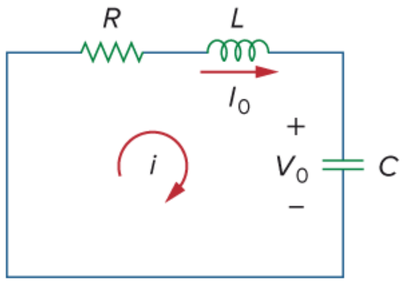
$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Assumption of exponential solution wrong for critical damping case!

$$\frac{d^2 i}{dt^2} + \underbrace{\frac{R}{L}}_{2\alpha}\frac{di}{dt} + \underbrace{\frac{i}{LC}}_{\omega_0^2 \to \alpha^2} = 0 \Rightarrow \frac{d^2 i}{dt^2} + 2\alpha\frac{di}{dt} + \alpha^2 i = 0$$

Critically damped case

$\alpha = \omega_0 \Rightarrow C = 4L/R^2$



$$\frac{d^2i}{dt^2} + 2\alpha\frac{di}{dt} + \alpha^2 i = 0$$

$$(A+B)^2 = A^2 + 2AB + B^2$$

$$\left(\frac{d}{dt}+\alpha\right)^2 i = \left(\frac{d}{dt}^2 + \frac{d}{dt}\alpha + \alpha\frac{d}{dt} + \alpha^2\right) i$$

$$= \left(\frac{d^2}{dt^2} + 2\alpha\frac{d}{dt} + \alpha^2\right) i$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

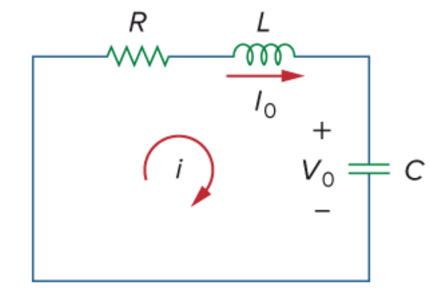
$$\left(\frac{d}{dt}+\alpha\right)\left(\frac{d}{dt}+\alpha\right)i = \frac{d}{dt}\left(\frac{di}{dt} + \alpha i\right) + \alpha\left(\frac{di}{dt} + \alpha i\right) = 0$$

$$f(t) = f = \frac{di}{dt} + \alpha i$$

$$\boxed{\frac{df}{dt} + \alpha f = 0} \rightarrow \frac{df}{dt} = -\alpha f$$

$$f(t) = Ae^{-\alpha t} = \frac{di}{dt} + \alpha i$$

Critically damped case



$\alpha = \omega_0 \Rightarrow C = 4L/R^2$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

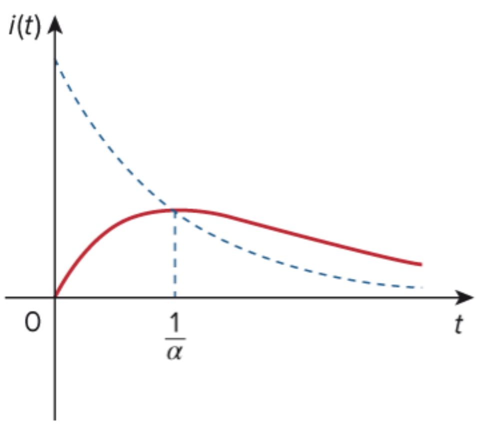
$$\frac{df}{dt} + \alpha f = 0$$

$$-\quad \frac{di}{dt} + \alpha i = \boxed{A_1 e^{-\alpha t}} \rightarrow e^{\alpha t}\left(\frac{di}{dt} + \alpha i\right) = A_1$$

$$e^{\alpha t}\frac{di}{dt} + \alpha i e^{\alpha t} = A_1$$

$$\frac{d}{dt}\left(e^{\alpha t} i\right) = A_1$$

$$\frac{d}{dt}(\underbrace{e^{\alpha t} i}_{g}) = A_1$$

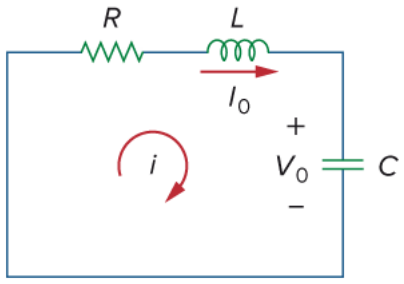


$$e^{\alpha t} i = A_1 t + A_2$$

$$i = (A_1 t + A_2)e^{-\alpha t}$$

Critically damped case

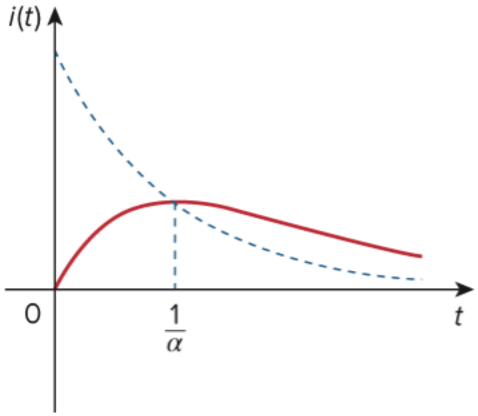
$\alpha = \omega_0 \Rightarrow C = 4L/R^2$



$$\boxed{\alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}}$$

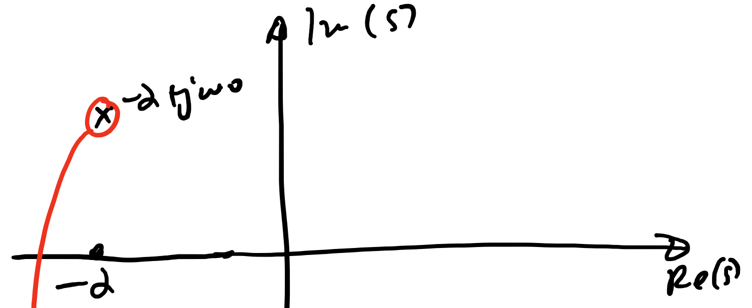
$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\boxed{s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}}$$

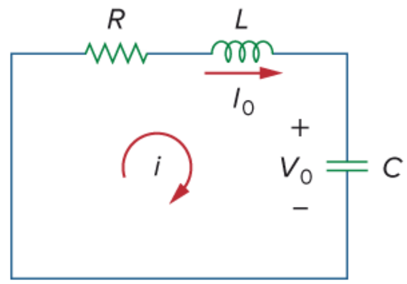


$$\boxed{i(t) = (A_2 + A_1 t)e^{-\alpha t}}$$

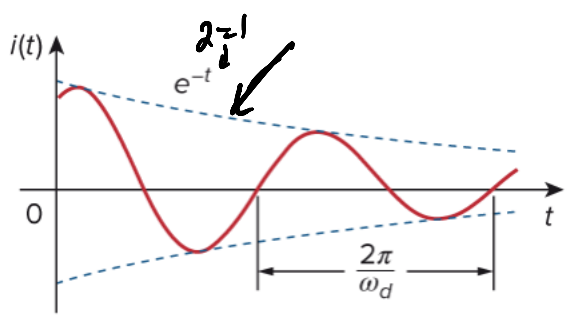
Under damped Case



$\alpha < \omega_0$ ⇒ $C < 4L/R^2$



$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$



X $-\alpha - j\omega_0$

$$i(t) = A_1 e^{-(\alpha - j\omega_0)t} + A_2 e^{-(\alpha + j\omega_0)t}$$

$$= e^{-\alpha t}(A_1 e^{+j\omega_0 t} + A_2 e^{-j\omega_0 t})$$

$$e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$$

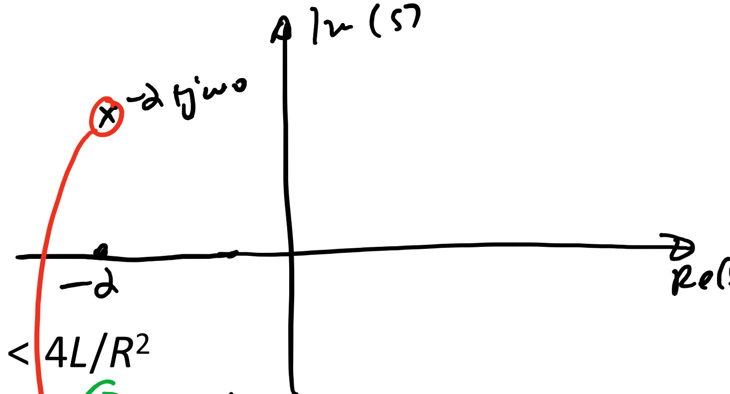
$$i(t) = e^{-\alpha t}\left[A_1(\cos\omega_0 t + j\sin\omega_0 t) + A_2(\cos\omega_0 t - j\sin\omega_0 t)\right]$$

$$= e^{-\alpha t}\left[\underbrace{(A_1 + A_2)}_{B_1}\cos\omega_0 t + j\underbrace{(A_1 - A_2)}_{B_2}\sin\omega_0 t\right]$$

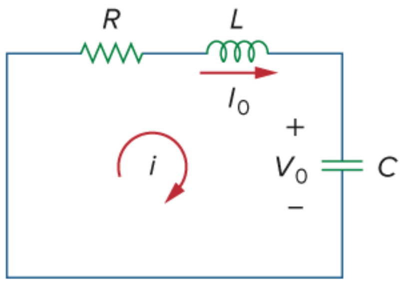
$$A_1 = A_2^*$$

$$\boxed{i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)}$$

Under damped Case



$\alpha < \omega_0$ ⇒ $C < 4L/R^2$



$x = -\alpha - j\omega_0$

$$i(t) = A_1 e^{-(\alpha - j\omega_0)t} + A_2 e^{-(\alpha + j\omega_0)t}$$

$$= e^{-\alpha t}(A_1 e^{+j\omega_0 t} + A_2 e^{-j\omega_0 t})$$

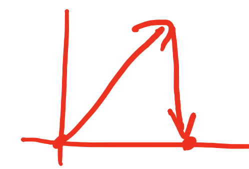
$$i(t) = e^{-\alpha t}\left(A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t}\right)$$

real real real

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

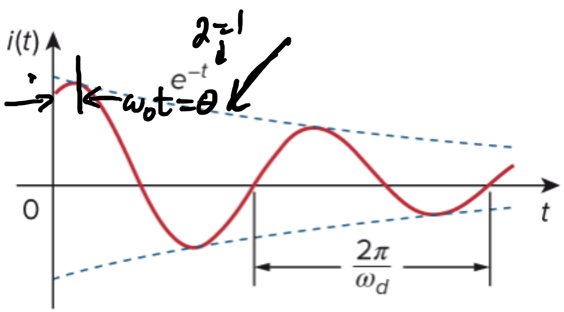
$A_1 = Ae^{j\theta}$

$A_2 = Ae^{-j\theta}$



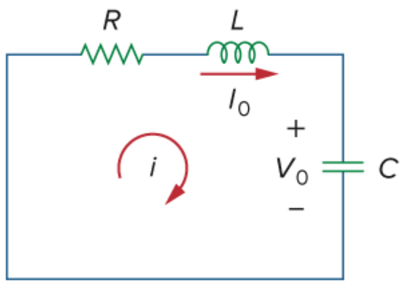
$$= e^{-\alpha t}\left(A e^{j\theta} e^{j\omega_0 t} + A e^{-j\theta} e^{-j\omega_0 t}\right)$$

$$= e^{-\alpha t}\left(A e^{j(\omega_0 t + \theta)} + A e^{-j(\omega_0 t + \theta)}\right)$$



$$\boxed{i(t) = e^{-\alpha t} A \cos(\omega_0 t + \theta)}$$

Summary



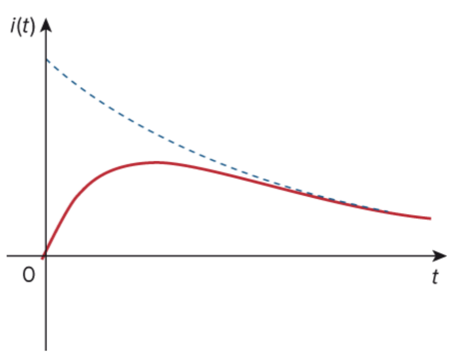
$$\alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

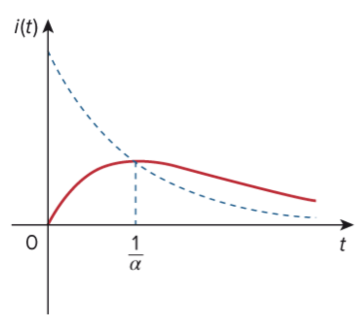
Overdamped

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$



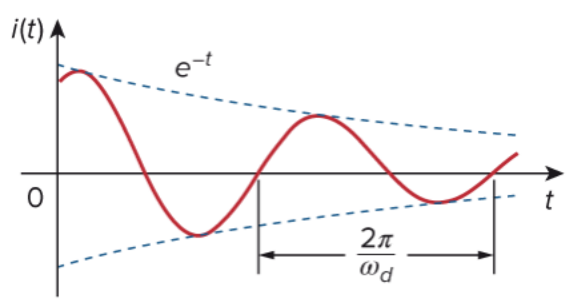
Critically damped

$$i(t) = (A_2 + A_1 t)e^{-\alpha t}$$



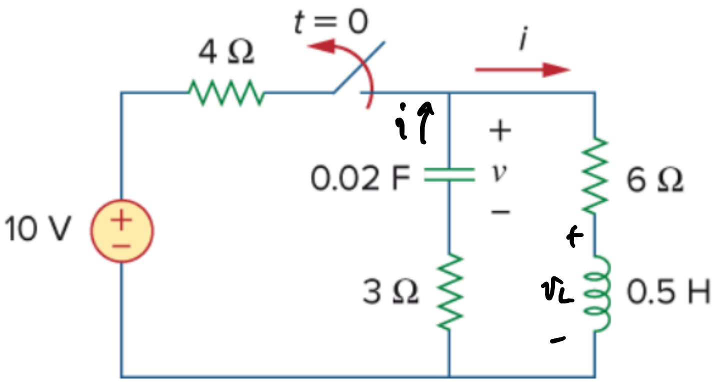
Underdamped

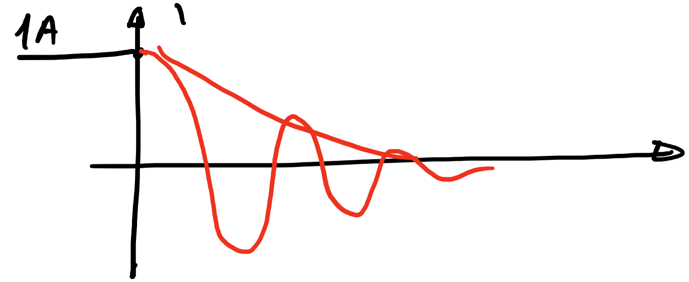
$$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

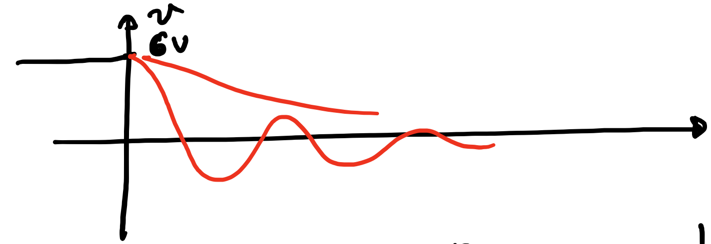


Example 8.4

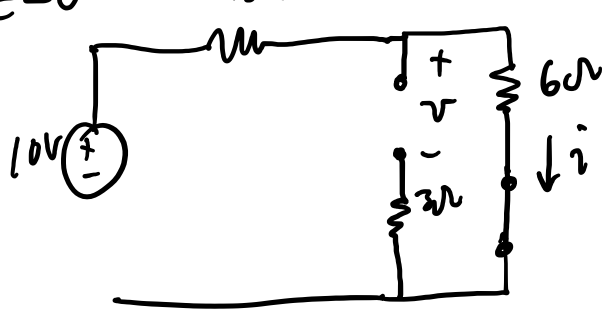
Find $i(t)$ in the circuit of **Fig. 8.10**. Assume that the circuit has reached steady state at $t = 0^-$.







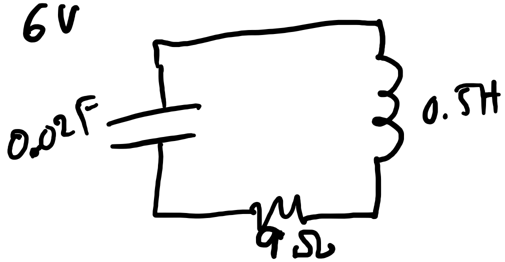
$t<0$:



$i(0^+) = i = 1A$

$v(0^+) = v = 6V$

$t>0$:



$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \rightarrow \alpha = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$i'' + \frac{R}{L}i' + \frac{1}{LC} = 0$$

$$\alpha = \frac{9\Omega}{2\times 0.5H} = 9\ Np/s \qquad \omega_0 = \frac{1}{\sqrt{0.01}} = 10\ rad/s$$

$$\boxed{\alpha < \omega_0}$$

underdamped

$$i(t) = e^{-\alpha t}\left[B_1\cos(\omega_0 t) + B_2\sin(\omega_0 t)\right]$$

$i(0) = 1A = B_1$

$$v_L(0) = v(0) - i6 - i3 = v(0) - 9\,i(0)$$

$$= 6V - 9\cdot 1A = -3V$$

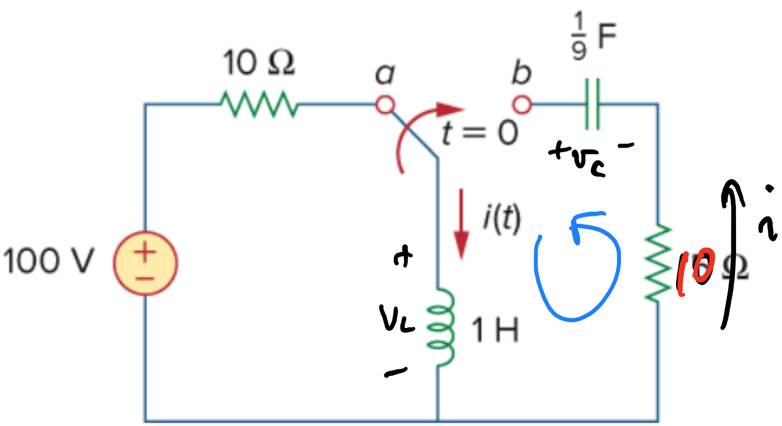
$$v_L(t) = L\frac{di}{dt} = L\left[-\alpha e^{-\alpha t}(B_1\cos\omega_0 t + B_2\cancel{\sin}\,\omega_0 t) + e^{-\alpha t}(-B_1\omega_0\cancel{\sin}\,\omega_0 t + B_2\omega_0\cos\omega_0 t)\right]$$

$$v_L(0) = L\left[-\alpha(B_1) + (B_2\omega_0)\right]$$

$$= L(B_2\omega_0 - B_1\alpha) = 3V$$

Practice Problem 8.4

The circuit in **Fig. 8.12** has reached steady state at $t = 0^-$. If the make-before-break switch moves to position b at $t = 0$, calculate $i(t)$ for $t > 0$.



$i(0^-) = 10A$

$i(0^+) = 10A$

$v_c(0^-) = 0 = v_c(0^+)$

$\alpha = \frac{R}{2L} = \frac{10}{2} = \cancel{2.5}\ 5$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{9}}} = 3\ rad/s$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$= -5 \pm \sqrt{5^2 - 3^2}$

$i(t) = A_1 e + A_2 e$

$i(0) = 10A$

$= A_1 + A_2$

$v_L = L\frac{di}{dt} = L\left[(-9)A_1 e^{-9t} - A_2 e^{-t}\right]$

At $t = 0^+$:

$v_L(0^+) + i\,10 + (-v_c) = 0$

$v_L(0^+) = -i(0^+)\,10 + v_c(0^+)$

$= -100 + 0$

$\rightarrow$ At $t = 0^+ \Rightarrow -100$

$1H(-9A_1 - (A_1 + A_2)) = -100$ ($A_1 + A_2 = 10$)

$-8A_1 = -90$

$A_1 = \frac{90}{8} \sim 11$

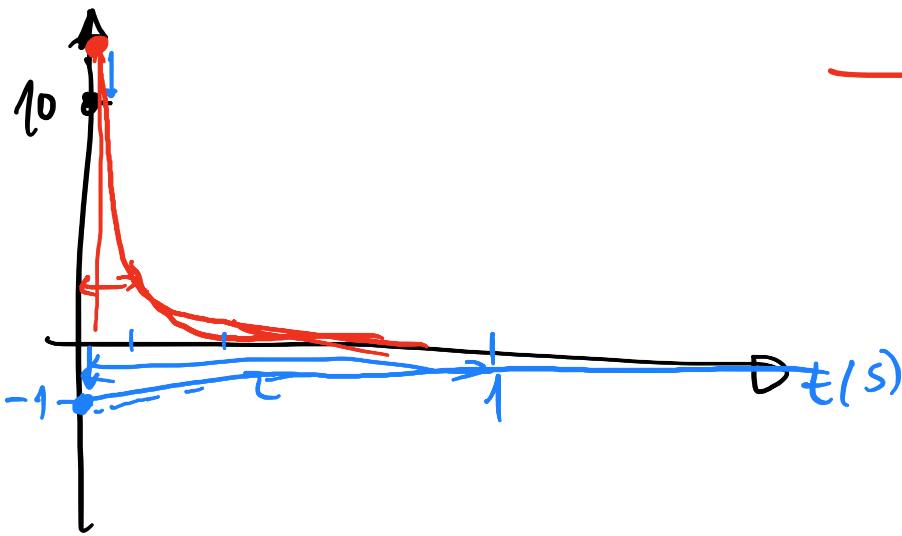
$A_2 = 10 - 90/8 \sim -1$

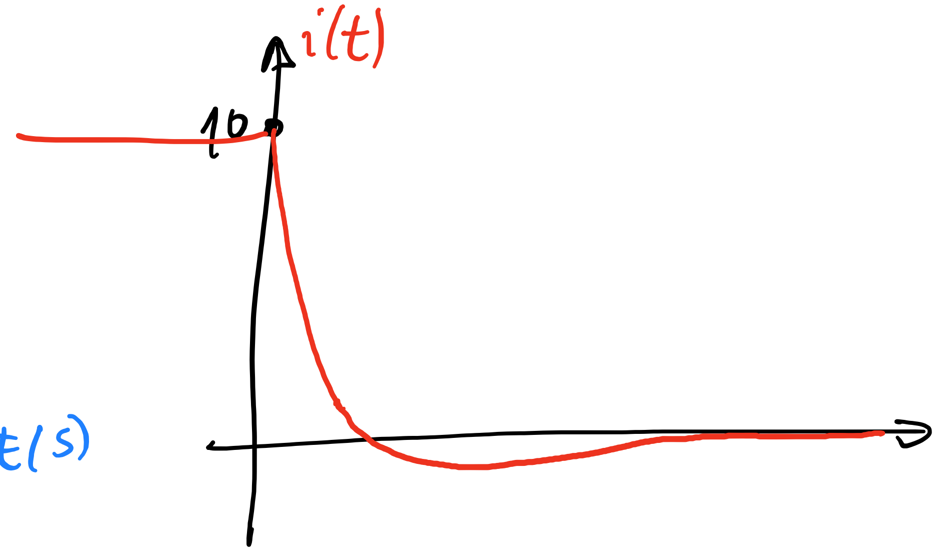
$$i(t) = 11e^{-9t} - 1e^{-t}$$

$$\tau_{1,2} = \frac{1}{9}, 1 \qquad e^{-t/\tau}$$

$$\downarrow$$

$$e^{-t/\frac{1}{9}\,s} \qquad e^{-t/1\,s}$$





Step Response

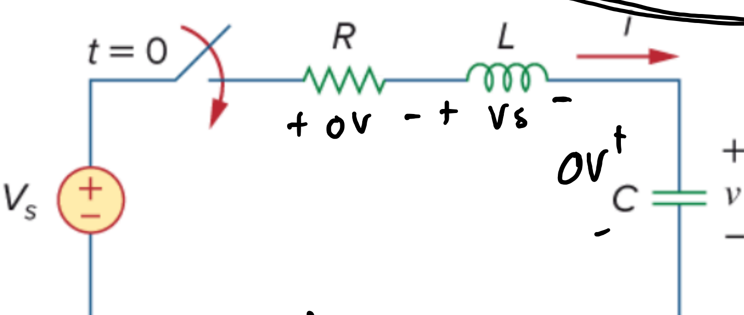
$v(0^-) = 0,\ i(0^-) = 0$

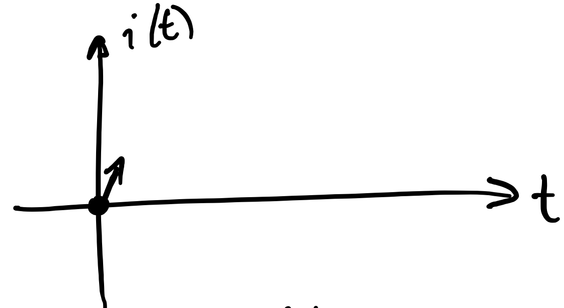
At $t=0^+$: $\boxed{\underline{v(0^+)} = 0,\ \underline{i(0^+)} = 0}$

KVL → $L\frac{di}{dt} + Ri + v = V_s$

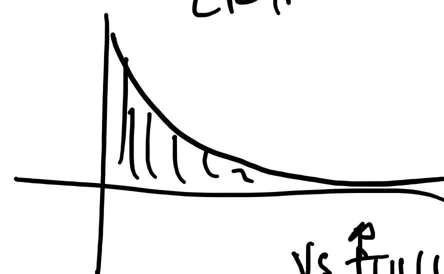
$\frac{di}{dt} = \frac{V_s}{L}$

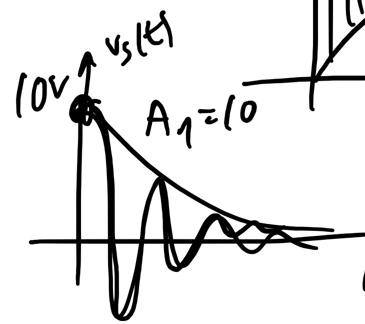
$i = C\frac{dv}{dt}$





LR, RC



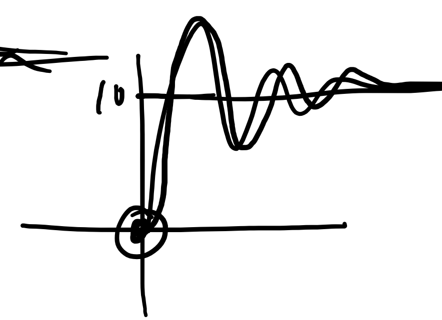


$LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v = V_s$

$$\boxed{\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC}}$$

$s^2 + \frac{R}{L}s + \frac{1}{LC} = \frac{V_s}{LC}$

$$\boxed{v(\infty) = V_s}$$



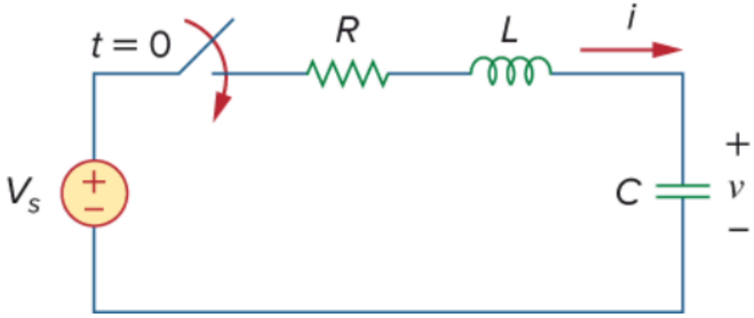
$V_s = 10$

$A_1 = -10$

Step Response

$$L\frac{di}{dt} + Ri + v = V_s$$

$$v(t) = v_t(t) + v_{ss}(t)$$



$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \text{(Overdamped)}$$

$$v_t(t) = (A_1 + A_2 t)e^{-\alpha t} \qquad \text{(Critically damped)}$$

$$v_t(t) = (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \qquad \text{(Underdamped)}$$

$$\frac{d^2 v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC}$$

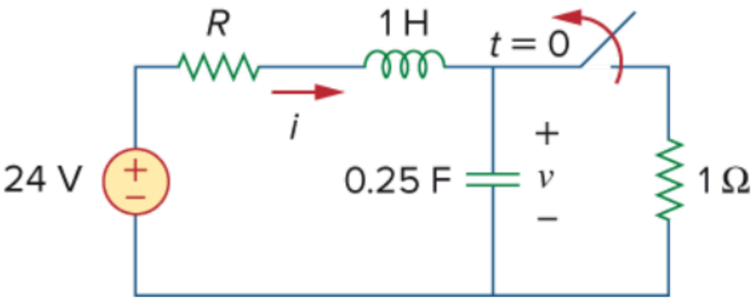
$$v(t) = V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \text{(Overdamped)}$$

$$v(t) = V_s + (A_1 + A_2 t)e^{-\alpha t} \qquad \text{(Critically damped)}$$

$$v(t) = V_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \qquad \text{(Underdamped)}$$

Example 8.7

For the circuit in **Fig. 8.19**, find $v(t)$ and $i(t)$ for $t > 0$. Consider these cases: $R = 5\ \Omega$, $R = 4\ \Omega$, and $R = 1\ \Omega$.



Example 8.11

In the op amp circuit of Fig. 8.33, find $v_o(t)$ for $t > 0$ when $v_s = 10u(t)$ mV. Let $R_1 = R_2 = 10$ kΩ, $C_1 = 20$ μF, and $C_2 = 100$ μF.