EK 307: Electric Circuits

Fall 2017

Lecture 7

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Administrivia

- HW 2 due today
- Should be reading Ch. 4

Lecture 6: Reminder

1. **Complete supernodes**
2. **Review circuits in context of lab: measuring voltage and current**
3. **Linear and nonlinear circuits: LED**
4. **Physics of the resistor (revisit Ohm's law)**
5. **Equivalent resistance**

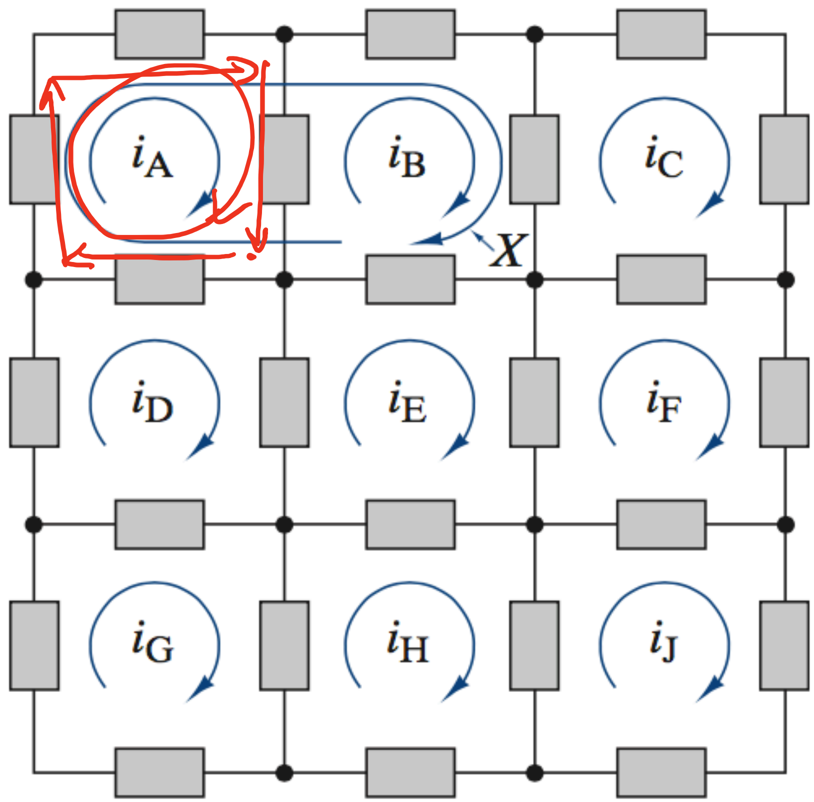
Lecture 7 (today): What you should know at end of this lecture

1. **Mesh current analysis (2 examples)**

2. **Linearity, superposition**
3. **Dependent sources, transistor model circuits**
4. **Thevenin and Norton theorems**
5. **Equivalent resistance (max power transfer)**

Mesh current analysis

Mesh current analysis



$$\bar{x} = \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \\ \vdots \\ i_G \end{bmatrix}$$

$$\bar{\bar{A}} \cdot \bar{x} = \bar{b}$$

$$\bar{x} = \bar{\bar{A}}^{-1} \cdot \bar{b}$$

Mesh current analysis

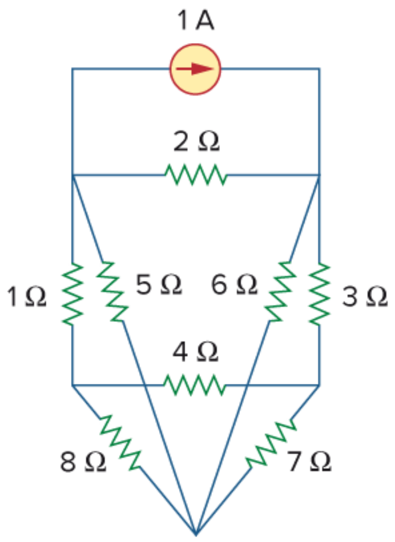
STEP 1 Identify a mesh current with every mesh and a voltage across every circuit element.

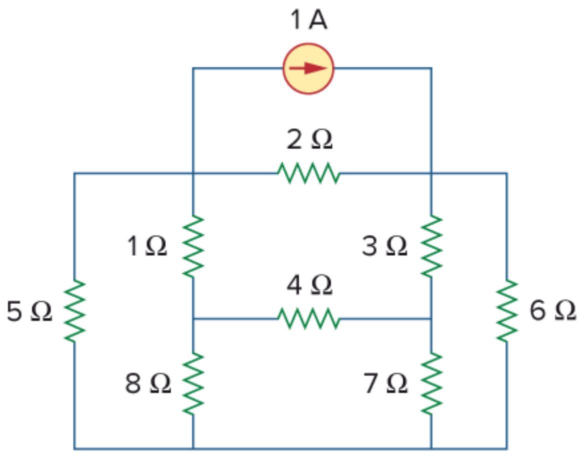
STEP 2 Write KVL connection constraints in terms of the element voltages around every mesh.

STEP 3 Use KCL and the i–v relationships of the elements to express the element voltages in terms of the mesh currents.

STEP 4 Substitute the element constraints from step 3 into the connection constraints from step 2 and arrange the resulting equations in standard form.

Planar meshes

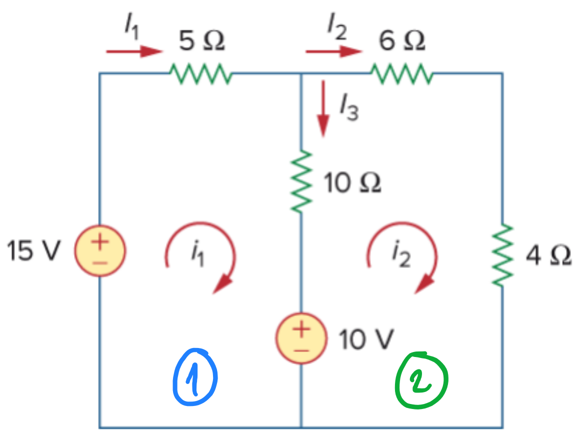




Example

(Ex 3.5 in book, 6th ed)

Find the branch currents I_1, I_2, and I_3 using mesh analysis.



① $-15V + I_1 \cdot 5\Omega + (I_3 \cdot 10\Omega + 10V) = 0$

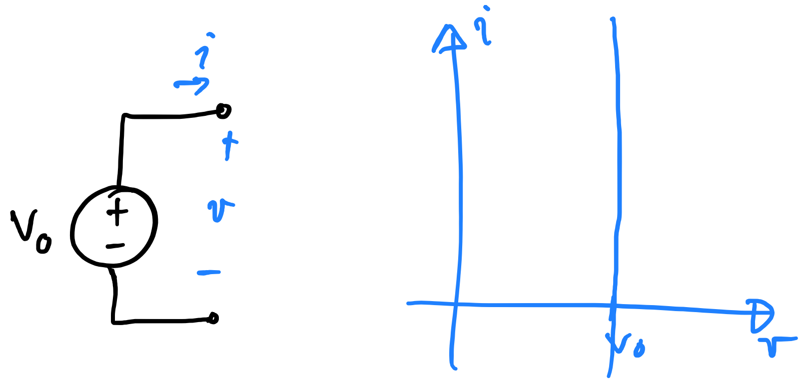
$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$

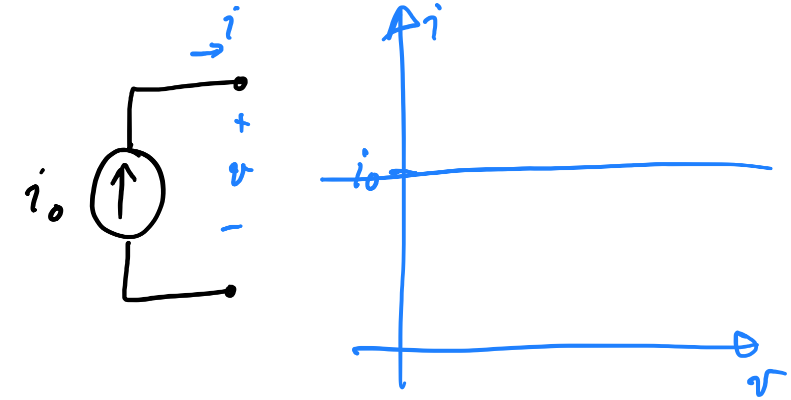
↳ $15 i_1 - 10 i_2 - 5 = 0$

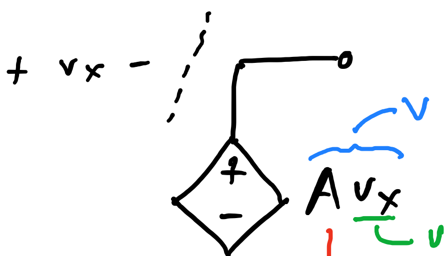
$-10 + (i_2 - i_1)10 + 6i_2 + 4i_2 = 0$

② ↳ $-10 i_1 + 20 i_2 - 10 = 0$

$$\begin{bmatrix} 15 & -10 \\ -10 & 20 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

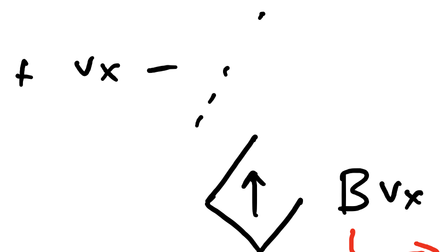




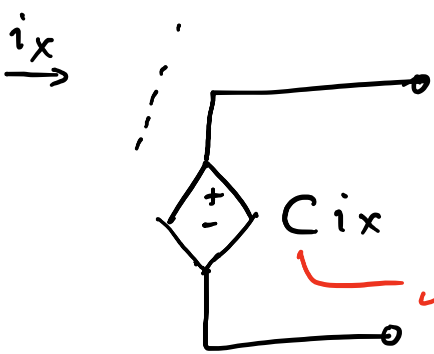


VCVS

↳ unitless, voltage gain

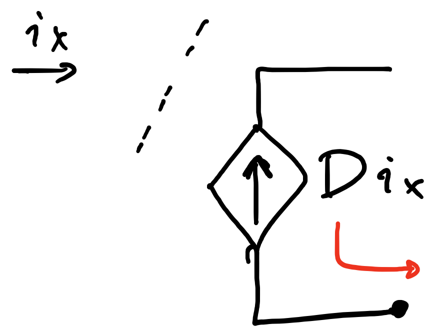


↳ $A/V = 1/\Omega$, transconductance

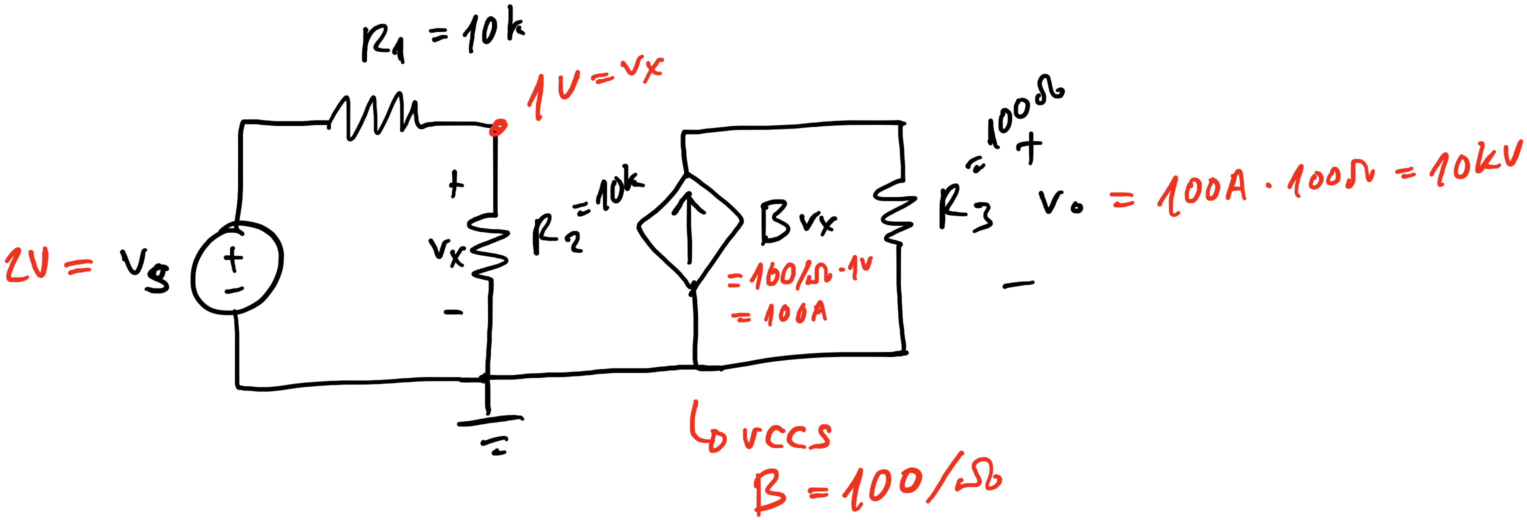


CCVS

↳ Ω, transresistance



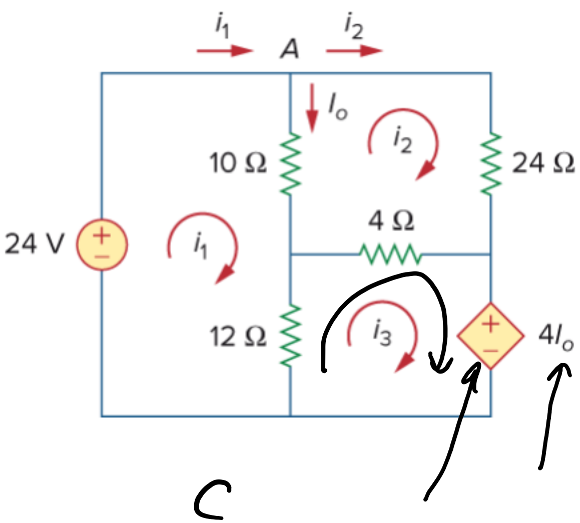
↳ unitless, current gain



$R_1 = 10k$
$1V = v_x$
$2V = v_g$
v_x
$R_2 = 10k$
Bv_x
$= 100/\Omega \cdot 1V$
$= 100A$
$R_3 = 100\Omega$
$v_o = 100A \cdot 100\Omega = 10kV$
↳ VCCS
$B = 100/\Omega$

Example

Use mesh analysis to find the current I_o in the circuit



① $-24 + 10(i_1 - i_2)$

② $24 i_2 + 4(i_2 - i_3) + 10$

③ $12(i_3 - i_1) + 4(i_3$

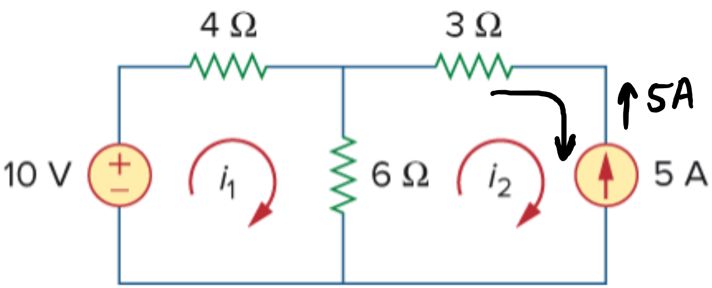
$= 0$

$I_o = i_1 - i_2$

$12(i_3 - i_1) + 4(i_3 - i_2) + 4(i_1 - i_2) = 0$

Example

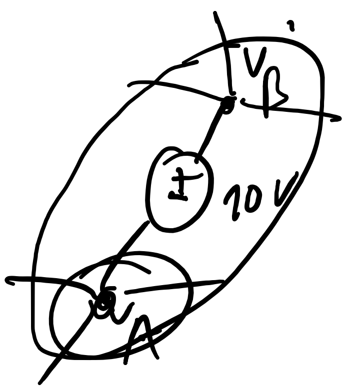
Mesh analysis with a current source…?

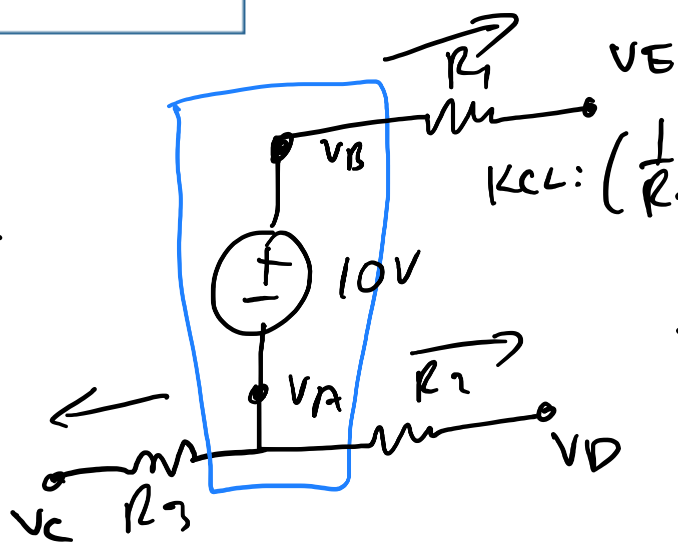


$$i_2 = -5A$$

$$① \quad -10 + 4i_1 + 6(i_1 - i_2) = 0$$

$$② \quad 6(i_2 - i_1) + 3i_2 + \ ? \ = 0$$



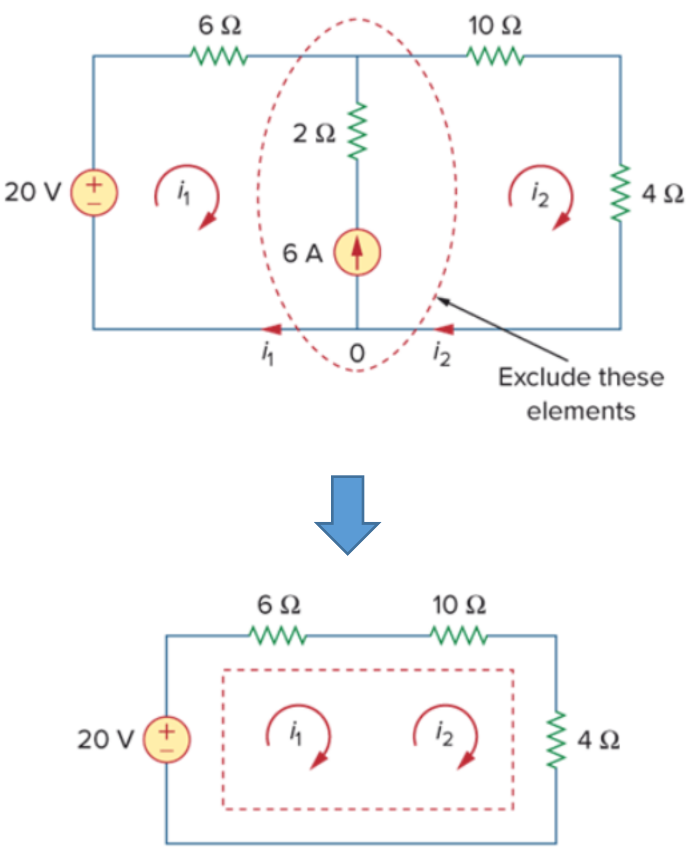


KCL: $\left(\frac{1}{R_2}+\frac{1}{R_3}\right) v_A - \frac{1}{R_2} v_C - \frac{1}{R_3} v_D +$

$+ \frac{1}{R_1}(v_A + 10V) - \frac{1}{R_1} v_E = 0$

Example

Mesh analysis with a current source between two mesh panels.



$$i_1 + 6A = i_2 \quad \rightarrow \quad i_1 = i_2 - 6A$$

KVL: $-20 + 6i_1 + 10\underbrace{i_2}_{(i_1+6)} + 4\underbrace{i_2}_{(i_1+6)} = 0$

$$(6+10+4)i_1 - 20 + 60 + 24 = 0$$

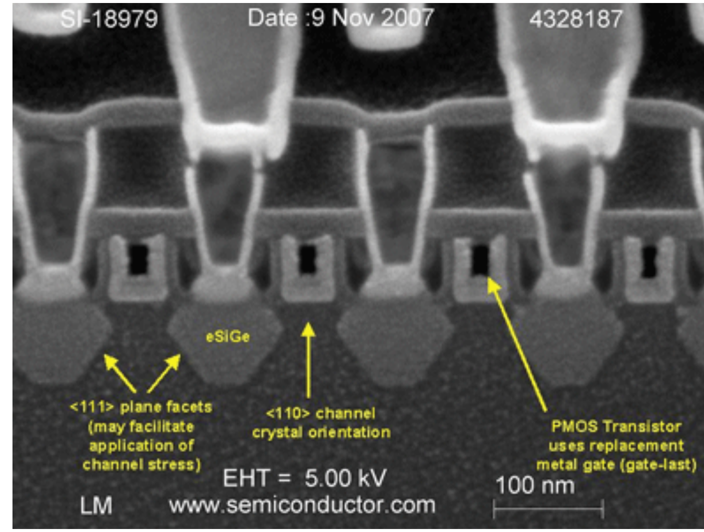
$$20i_1 + 44 = 0$$

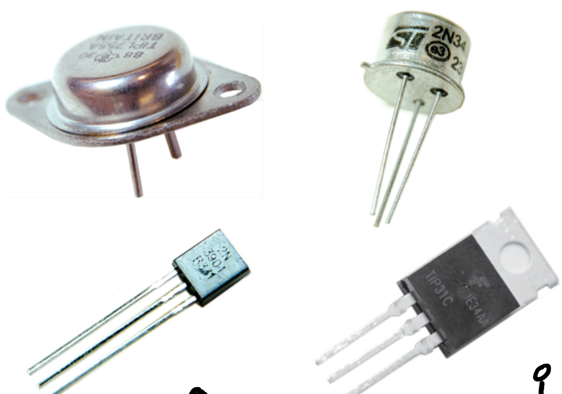
$$\boxed{i_1 = \frac{-44}{20} = -2.2A}$$

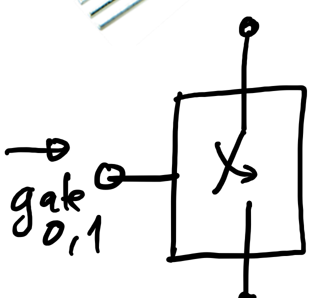
Dependent sources and DC Transistor Circuits

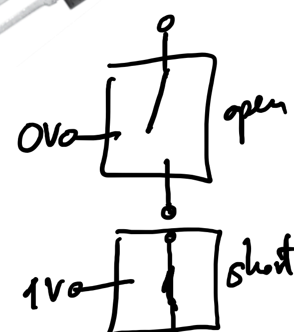
Transistors

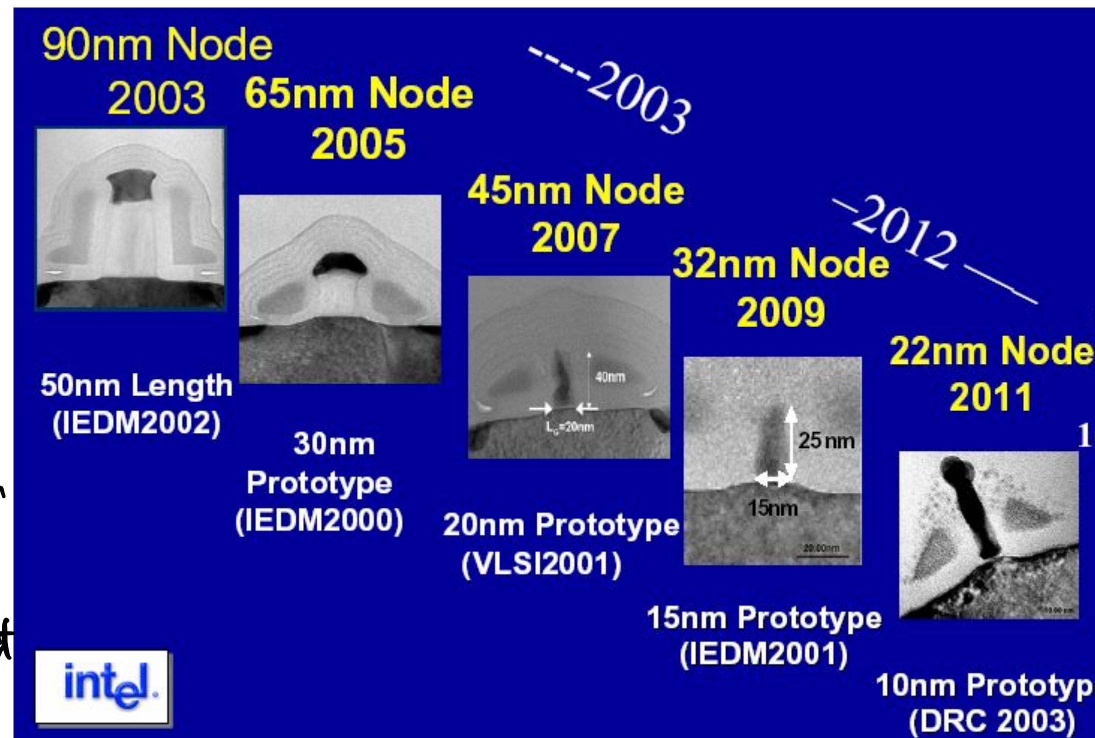
- Bipolar junction transistor (BJT)
- Field effect transistor (FET)



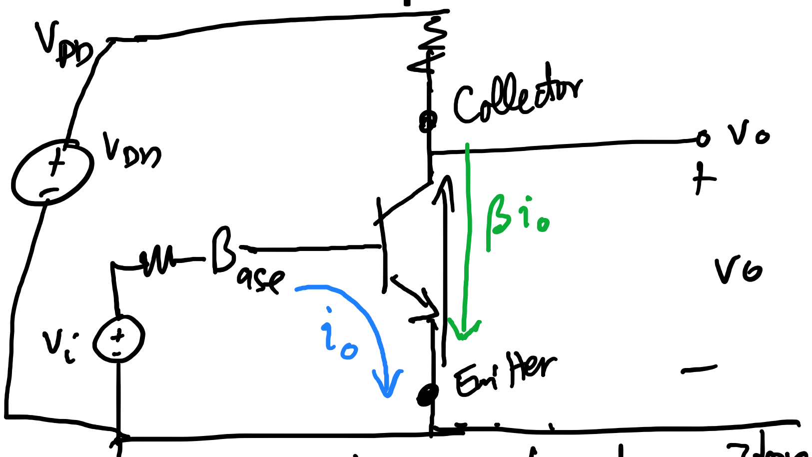




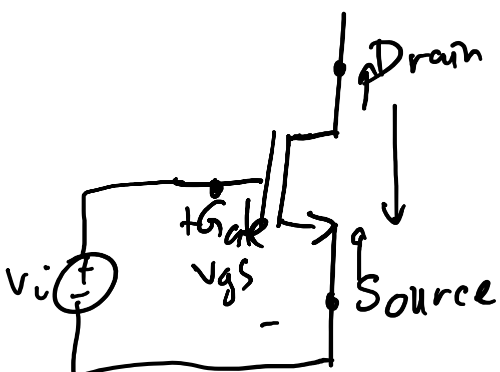




Examples

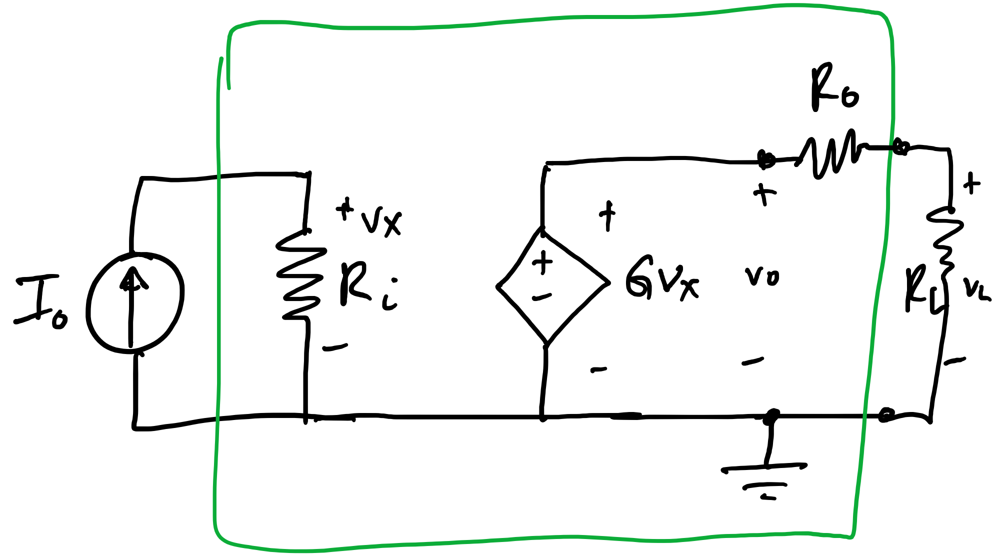


Bipolar junction transistor



MOS Field Effect transistor

Typical cct with dependent source
(we will revisit similar ones when modeling transistors)

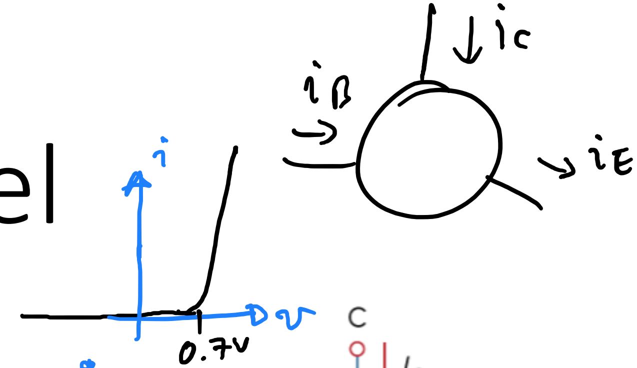


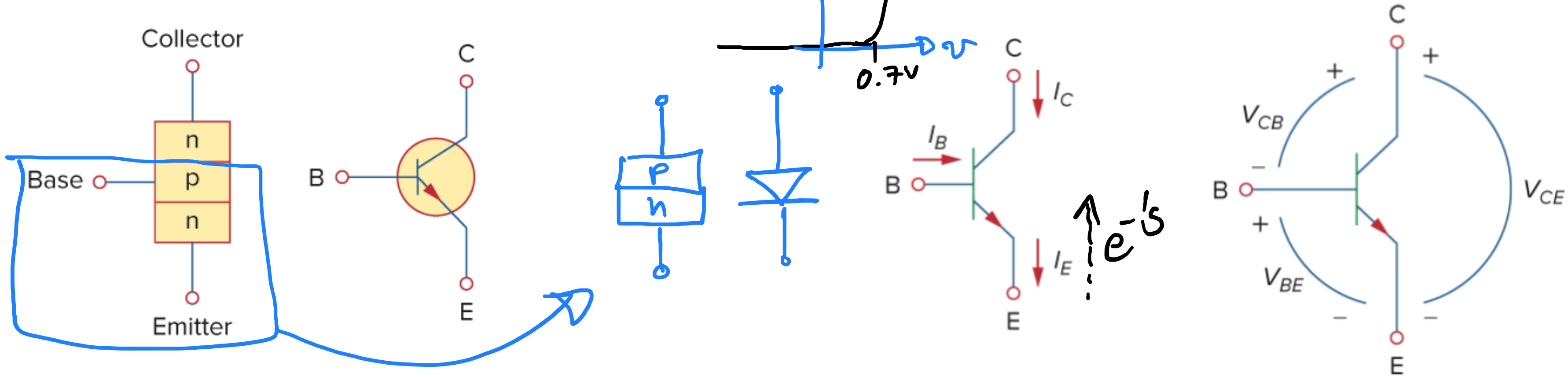
$v_x = I_o R_i$

$v_o = G v_x = G I_o R_i$

$v_L = \dfrac{R_L}{R_o + R_L} v_o$

Transistor Model



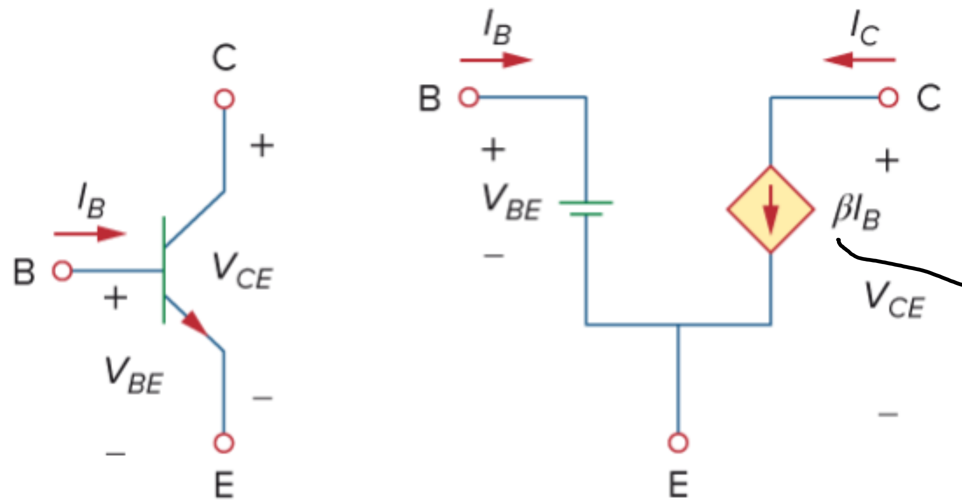


KCL: $I_E = I_B + I_C$

- Modes:
  - Active => $\boxed{I_C = \beta I_B}$ current gain
  - Cutoff
  - Saturation

- Active mode: v_{BE} ~ 0.7V, $I_C = \beta\ I_B$, β is current gain

Transistor Model

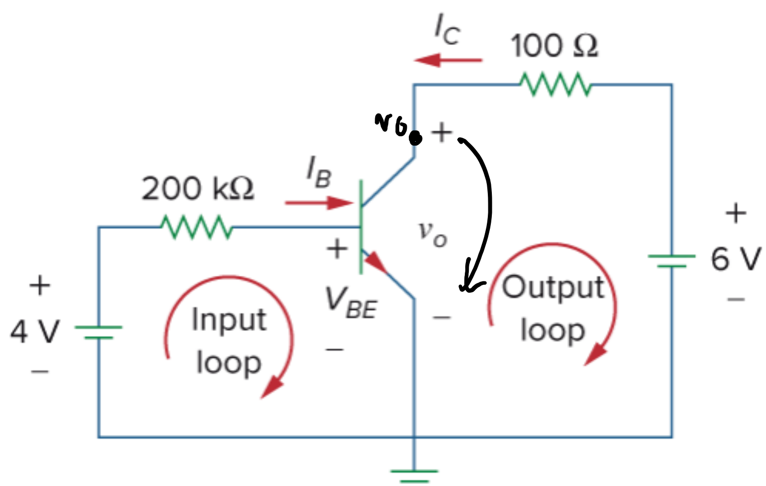


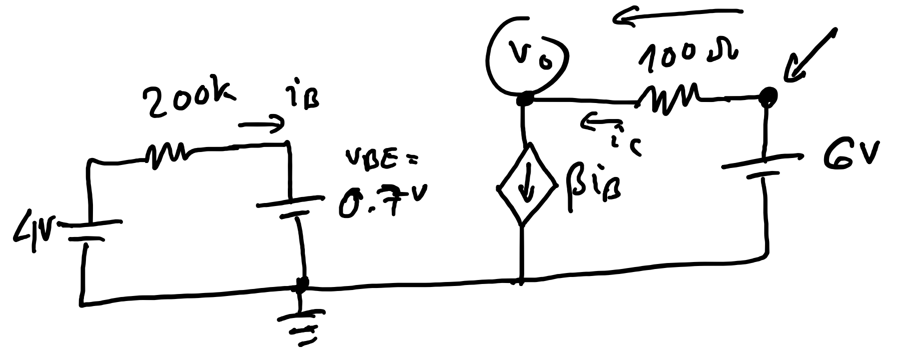
(CCCS)

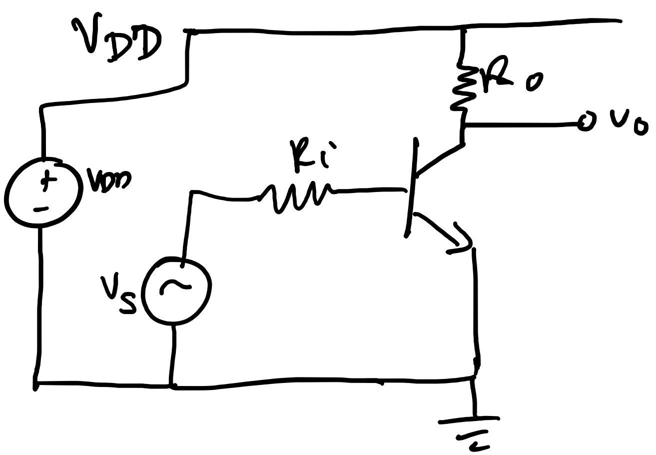
current gain

Transistor Example

Find I_B, I_C and v_o. Assume active mode and $\beta = 50$.







$$i_B = \frac{4V - 0.7V}{200k} = \frac{3.3V}{200k} = \frac{16.7}{1000} mA$$

$$= 16.7 \mu A$$

$$i_c = \beta i_B = \frac{1670}{2} \mu A = 835 \mu A$$

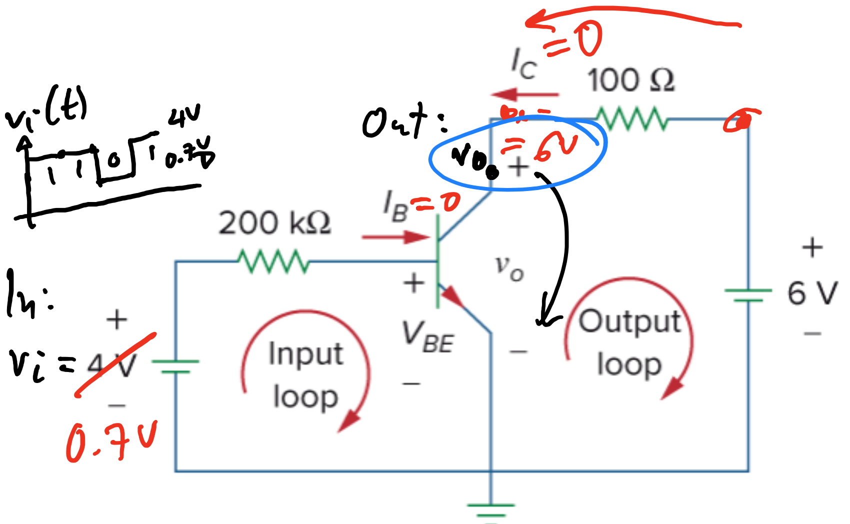
$$v_o = 6V - 100 i_c = 6V - 100\Omega \cdot 835 \mu A$$

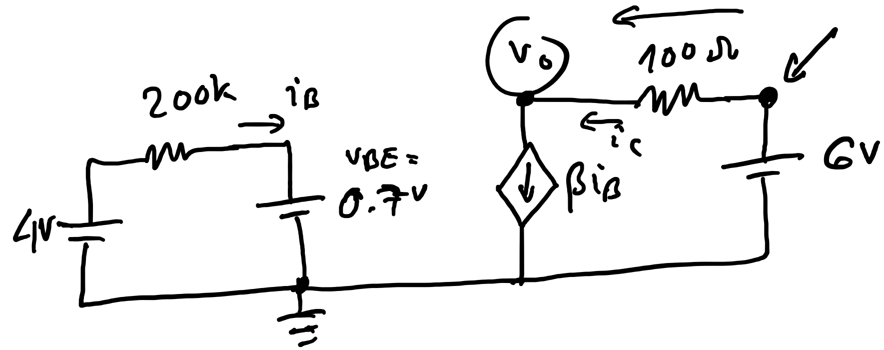
$$= 6V - 83.5 mV = 6V - 0.0835 V$$

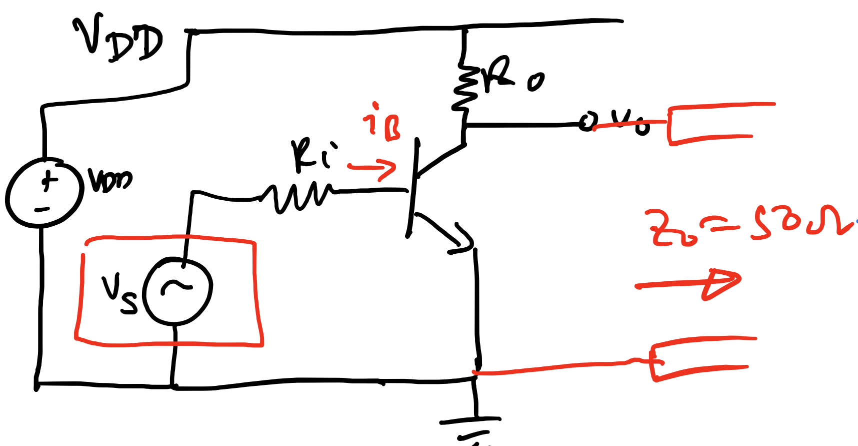
$$= 5.9165 V$$

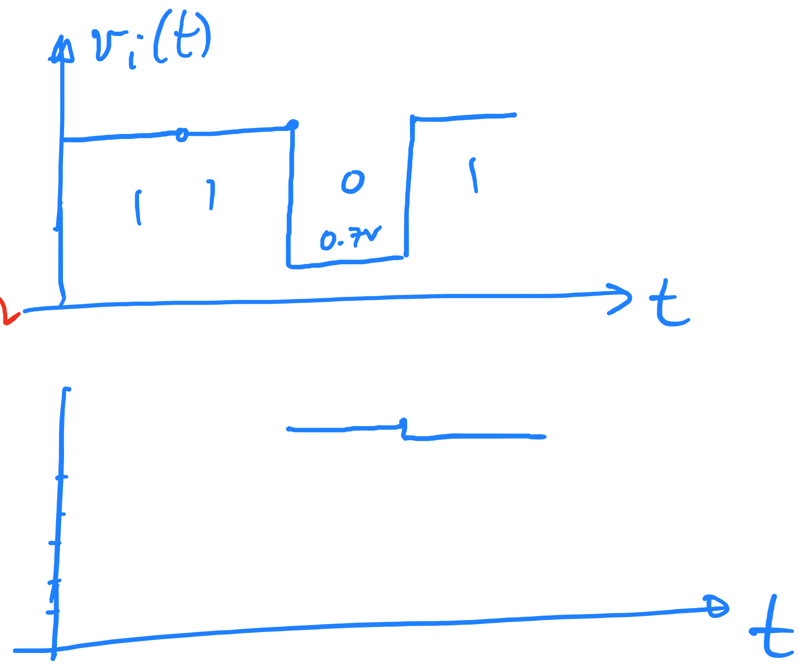
Transistor Example

Find I_B, I_C and v_o. Assume active mode and $\beta = 50$.







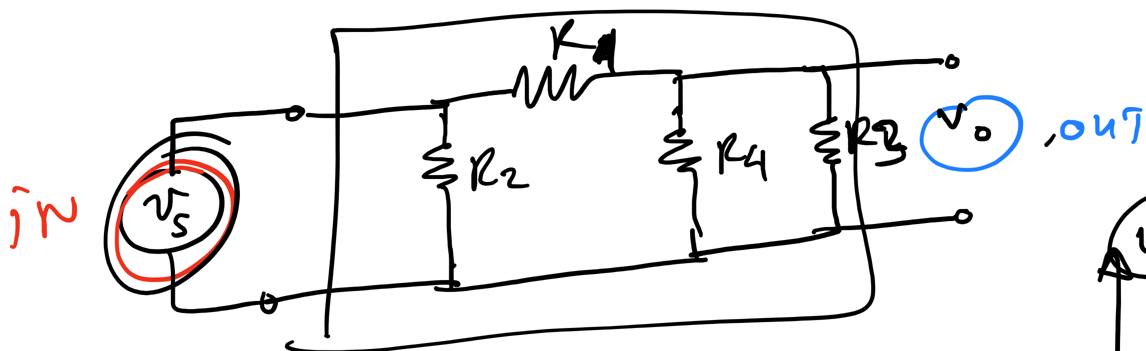


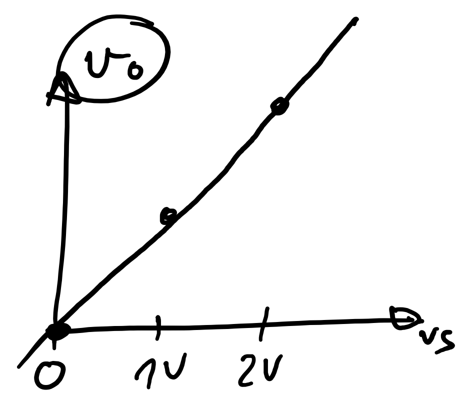
$P_i = v_s\, i_B$

Linearity, superposition, source transformation

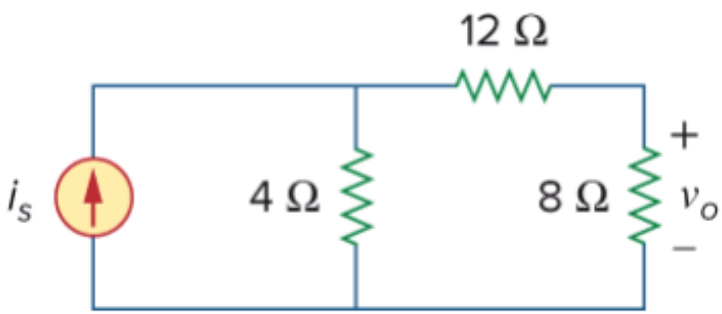
Linearity

- Any output is proportional to any input

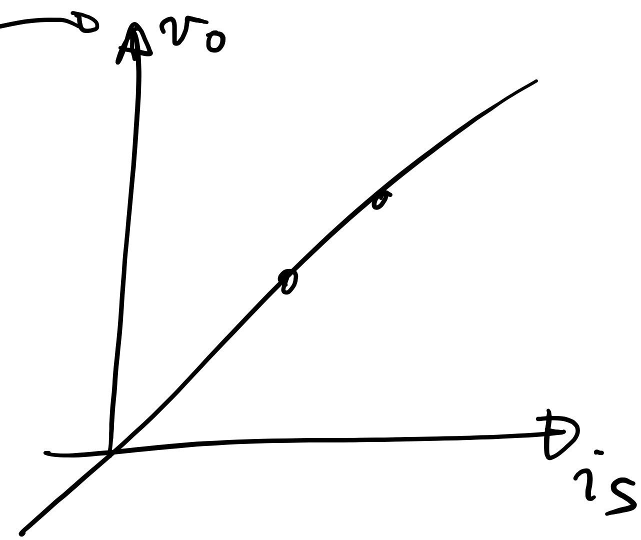




Linearity: example

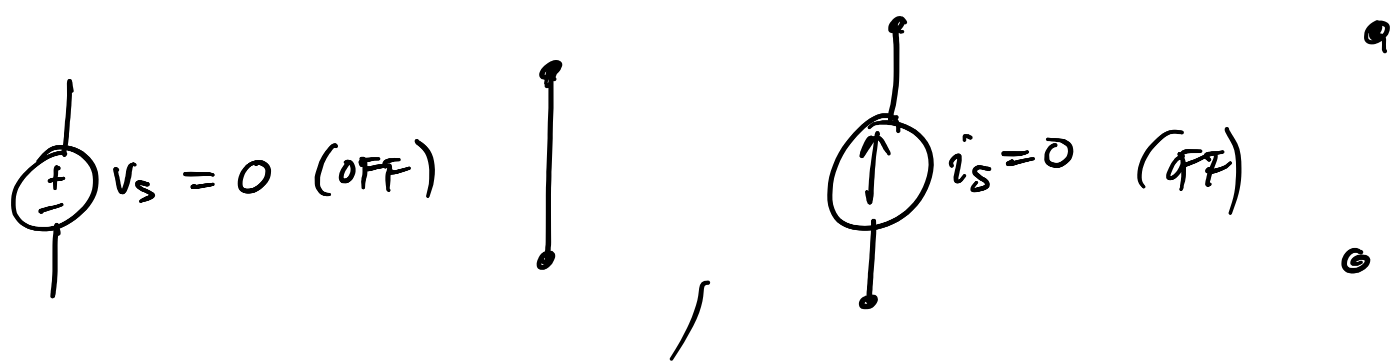


Plot v_o vs. i_s. Try i_s = 30A, 45A



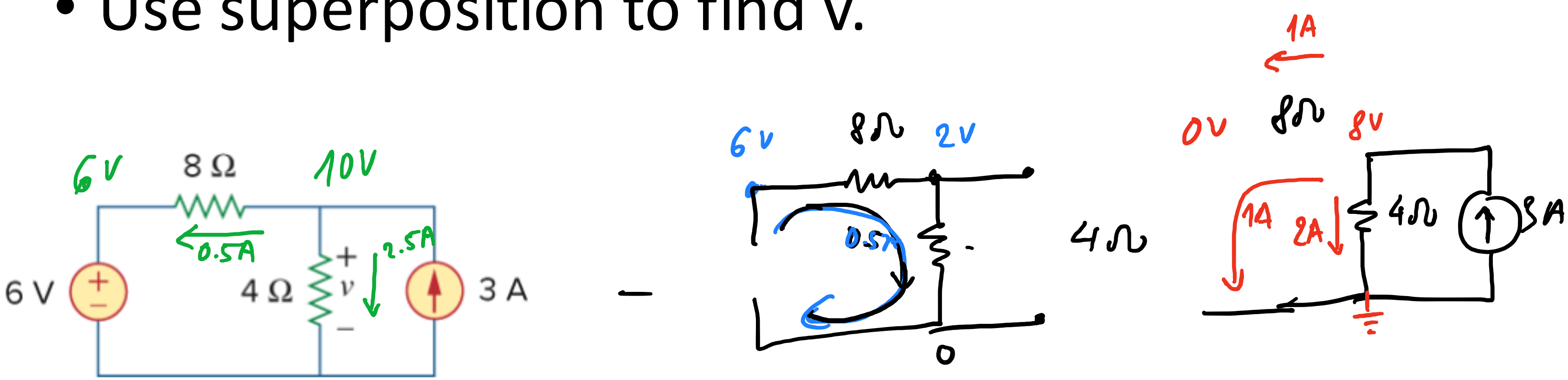
Superposition

- For multiple inputs (e.g. sources):
  - Any output is the sum of outputs due to each input turned on separately
  - “Turned off input” means V=0 for voltage sources, I=0 for current sources.
  - Dependent sources stay.



Superposition: example 1

- Use superposition to find v.



Superposition: example 2

Use superposition to find v_x in the circuit of Fig. 4.11.

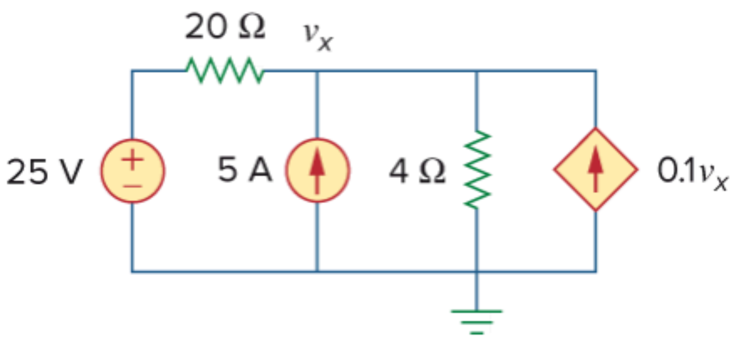


Figure 4.11 For Practice Prob. 4.4.