BOSTON UNIVERSITY

EK 307: Electric Circuits

Fall 2017

Lecture 6

Sep 21, 2017

Prof. Miloš Popović

Department of Electrical and Computer Engineering

Boston University

Administrivia

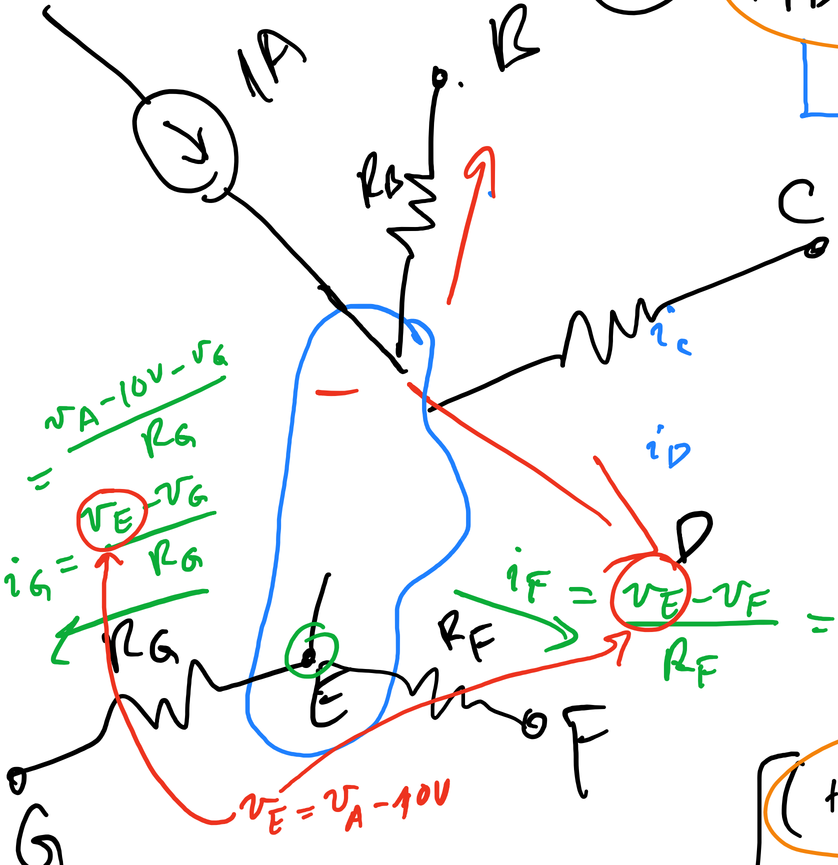
- You should have read Ch 1-3 and should be reading Ch 4!
- HW 1 is in, 2 is due next Tuesday, 3 is out Friday.
- First midterm exam is 3.5 weeks away.

Lecture 6: What you should know at end of this lecture

1. **Complete supernodes**
2. **Review circuits in context of lab: measuring voltage and current**
3. **Linear and nonlinear circuits: LED**
4. **Physics of the resistor (revisit Ohm's law)**
5. **Equivalent resistance**
6. **Mesh current analysis**
7. **Linearity, superposition**
8. **Dependent sources**
9. **Transistor model circuits**

Supernodes

(for practice see Fig. 3.7 in book and explanation, and practice prob 3.3 and around there)



$$i_G = \frac{v_E - v_G}{R_G} = \frac{v_A - 10V - v_G}{R_G}$$

$$i_F = \frac{v_E - v_F}{R_F} = \frac{v_A - 10V - v_F}{R_F}$$

$$v_E = v_A - 10V$$

A:

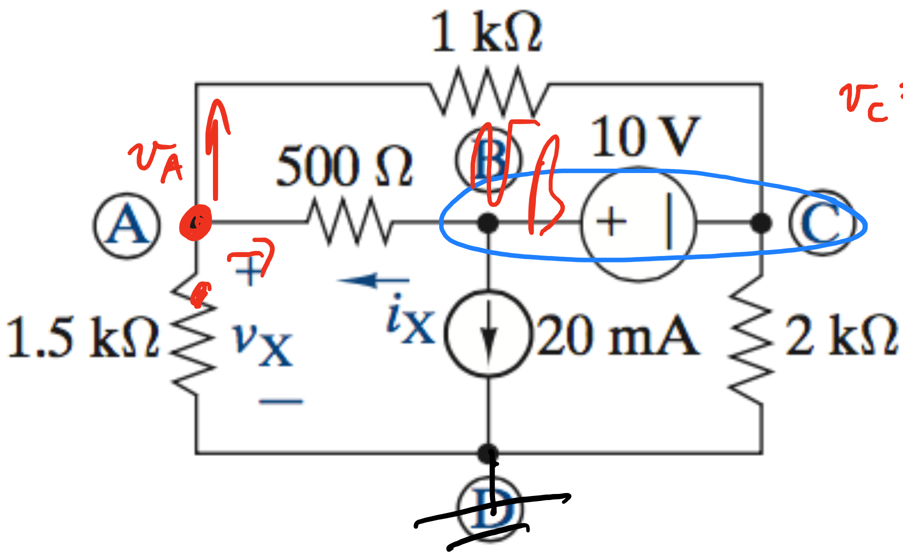
$$\left(\frac{1}{R_B}+\frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_F}+\frac{1}{R_G}\right) v_A \quad - \frac{1}{R_B} v_B \quad - \frac{1}{R_C} v_C \quad - \frac{1}{R_D} v_D$$

$$- \frac{1}{R_F} v_F - \frac{1}{R_G} v_G = \frac{10V}{R_F} + \frac{10V}{R_G} \quad +1A$$

$$\begin{bmatrix} (+ + + +) & -\frac{1}{R_B} & -\frac{1}{R_C} & \cdots & \cdot \\ & & & & \\ & & & & \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \\ v_D \\ v_F \\ v_G \end{bmatrix} = \begin{bmatrix} \frac{10V}{R_F} + \frac{10V}{R_G} + 1A \\ \\ \\ \end{bmatrix}$$

A:

Supernodes



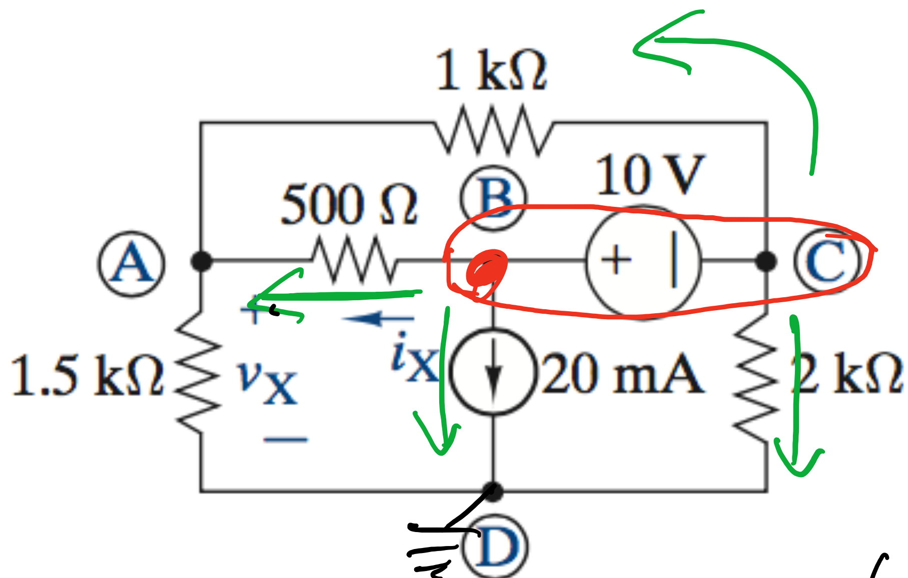
$v_C = v_B - 10V$

$$\begin{bmatrix} \ & \ & \ \\ \ & \ & \ \\ \ & \ & \ \end{bmatrix}\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$\text{A:} \left(\frac{1}{1.5k} + \frac{1}{0.5k} + \frac{1}{1k}\right) V_A + \left(\frac{-1}{0.5k}\right) V_B \quad \left(\frac{-1}{1k}\right)$$

$$\text{B:} \quad 20mA + \frac{V_B - V_A}{\ }$$

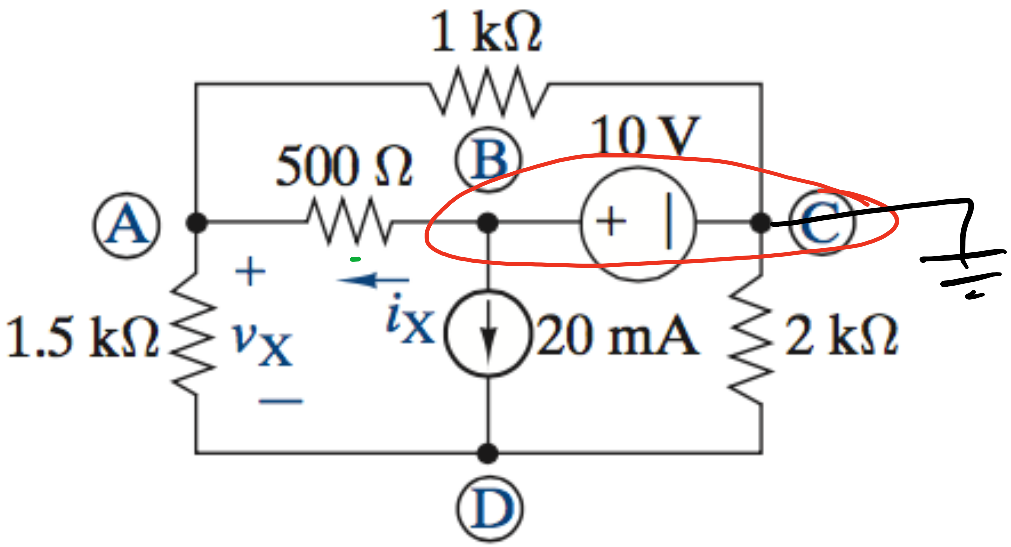
Supernodes



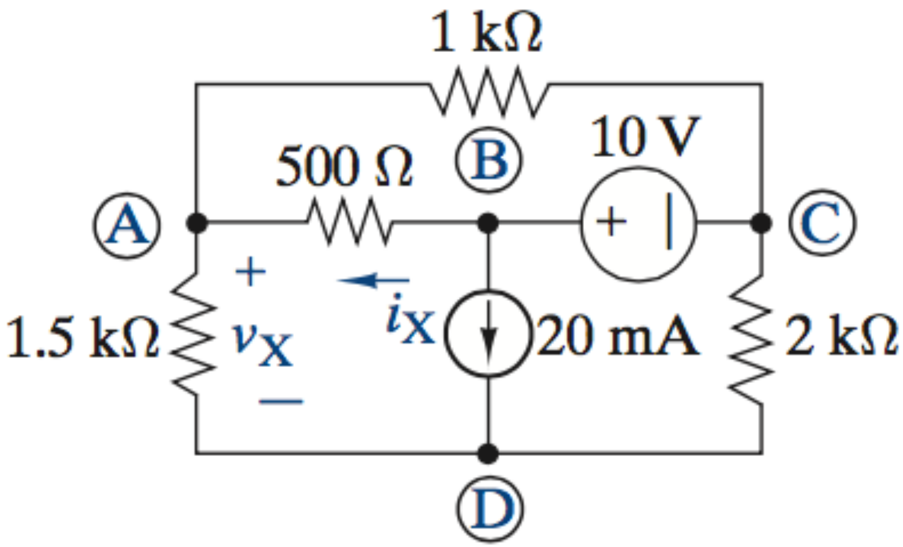
$$A: \frac{V_A - 0}{1.5k\Omega} + \frac{V_A - V_B}{500\Omega} + \frac{V_A - (V_B - 10V)}{1k\Omega} = 0$$

$$B: \frac{V_B - V_A}{500\Omega} + 20mA + \frac{(V_B - 10V) - V_A}{1k\Omega} + \frac{(V_B - 10V)}{2k\Omega} = 0$$

Supernodes



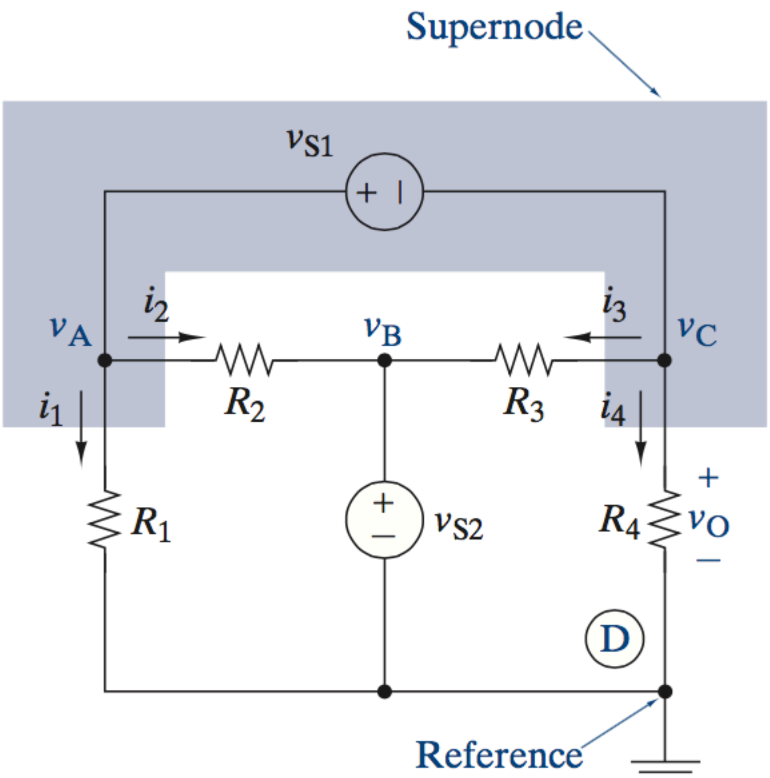
Supernodes



Node A: $$\frac{V_A - 10}{0.5k} + \frac{V_A - V_D}{1.5k} + \frac{V_A}{1k} = 0$$

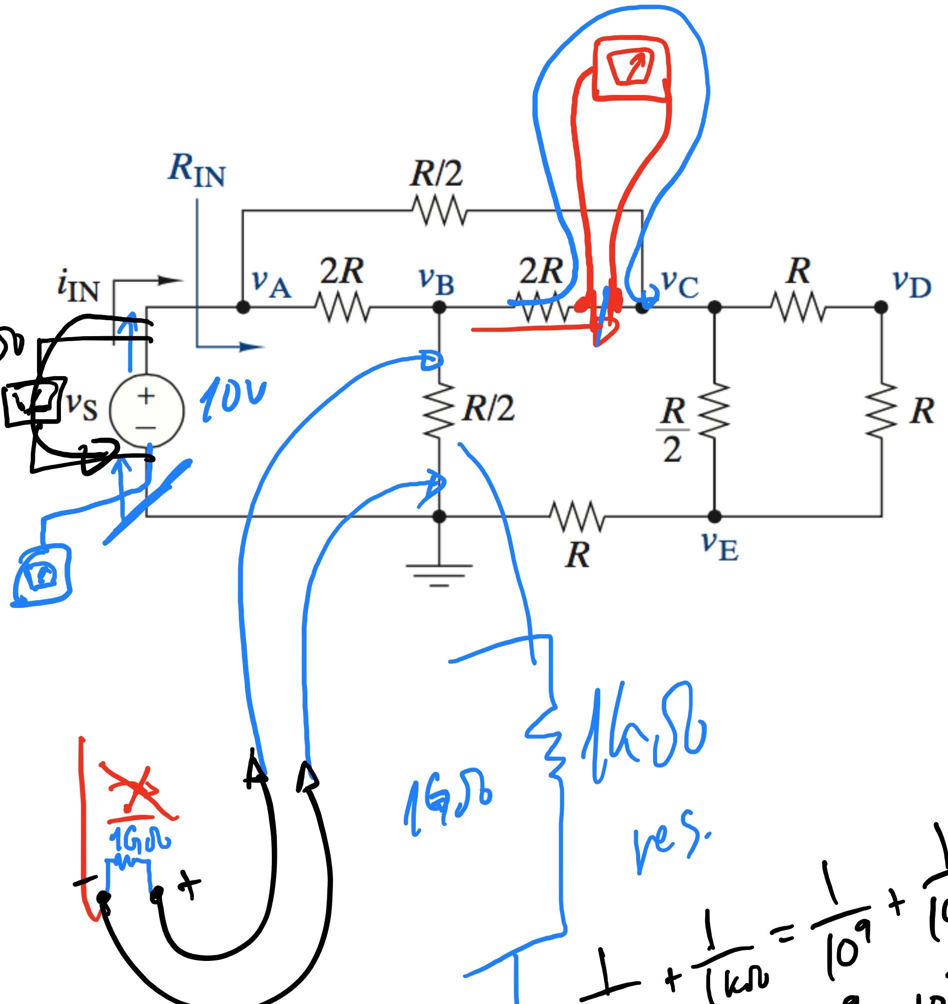
Node D: $$\frac{V_D - V_A}{1.5k} - 0.020 + \frac{V_D}{2k} = 0$$

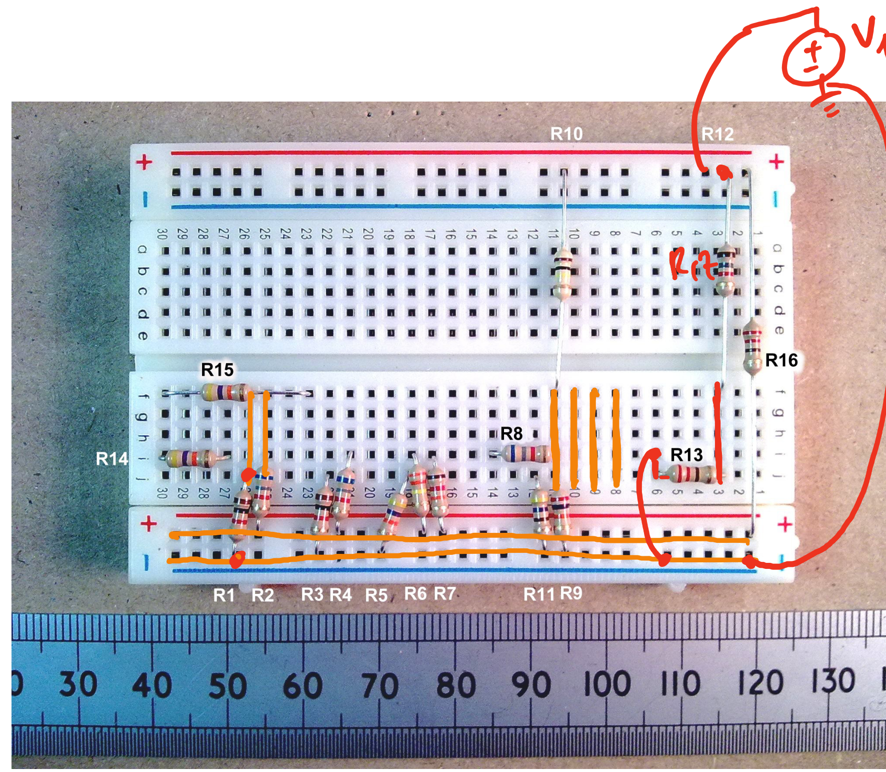
Another supernode problem



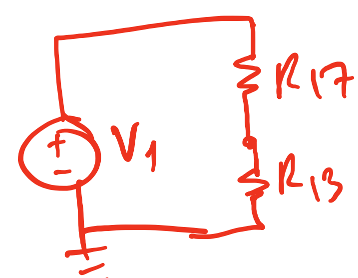
Review circuits in context of lab: measuring voltage and current

Measuring voltage and current in the lab

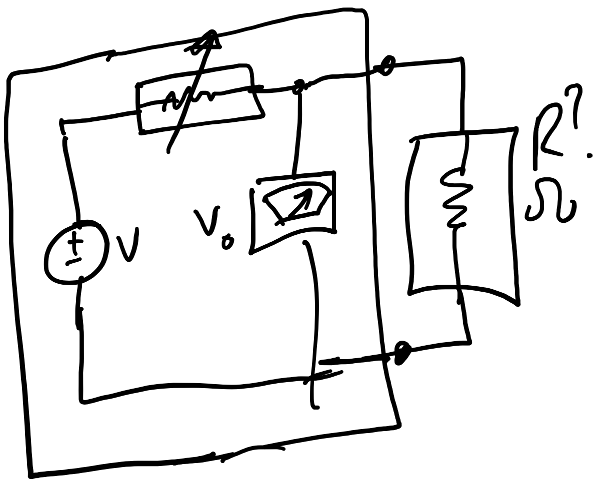


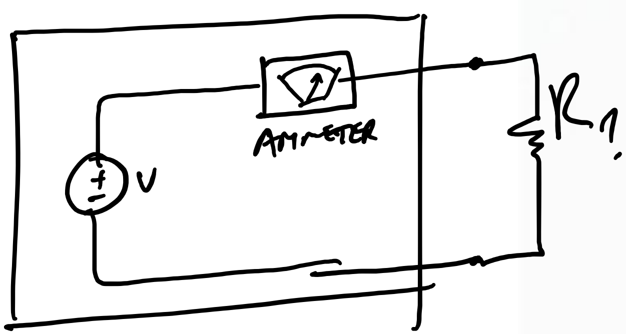


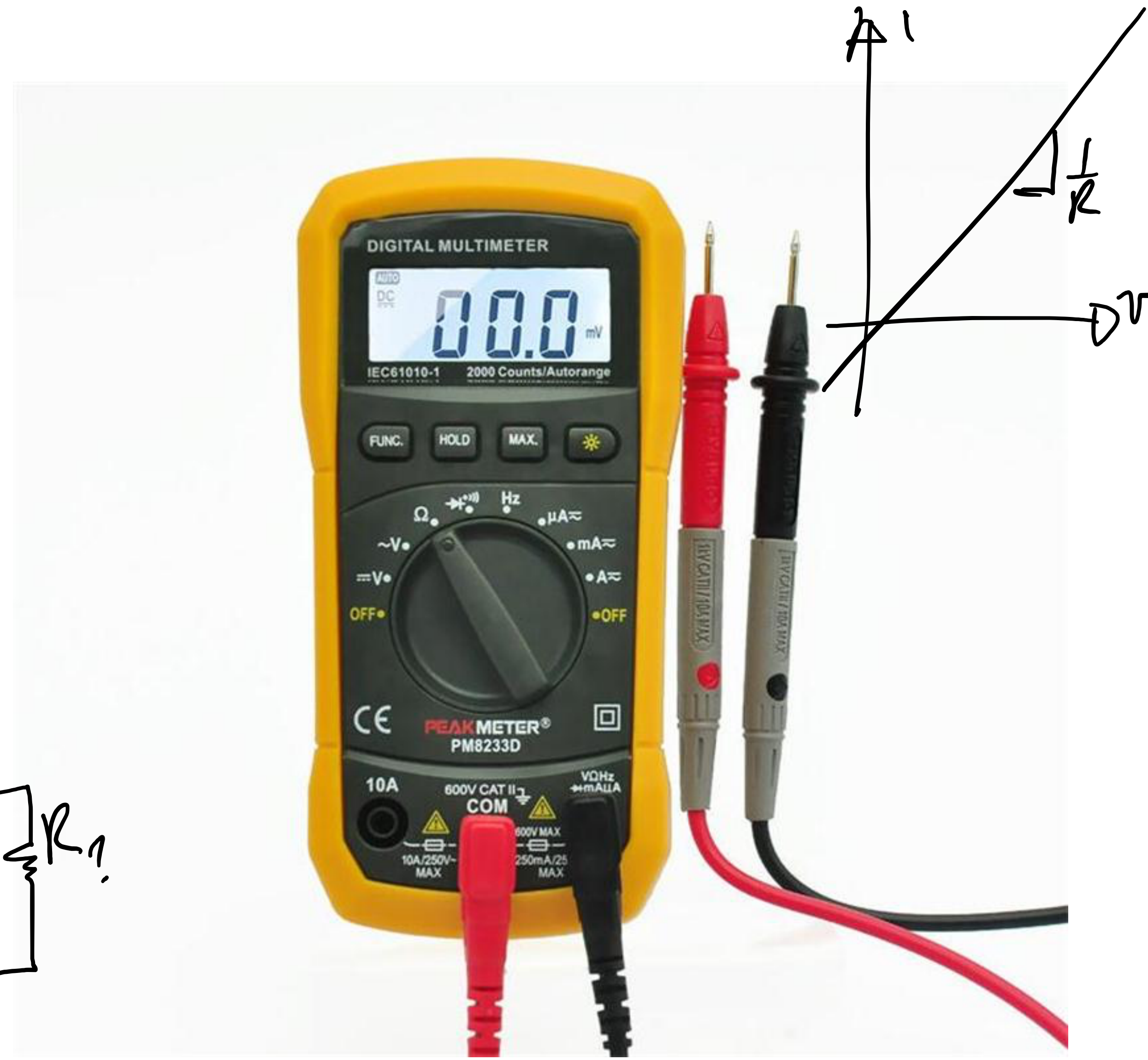
$$eq = \frac{1}{1G\Omega} + \frac{1}{1k\Omega} = \frac{1}{10^9} + \frac{1}{10^3}$$
$$= 10^{-9} + 10^{-3} = 10^{-3}\,\frac{1}{\Omega}$$
$$R_{eq} = 1k\Omega$$



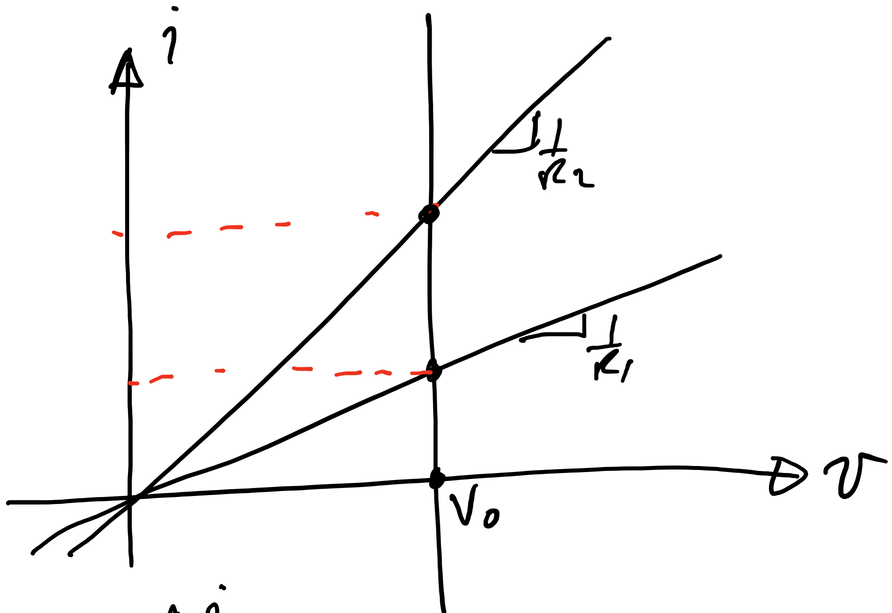
Measuring voltage and current in the lab

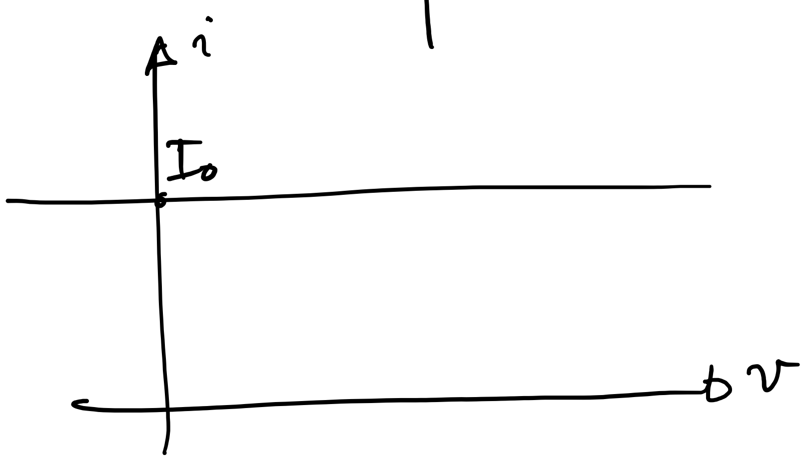


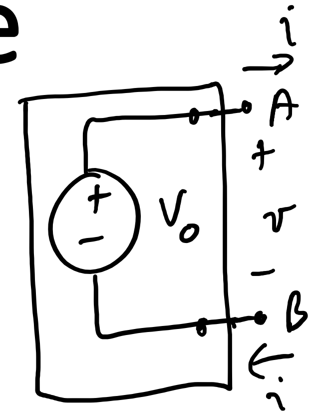


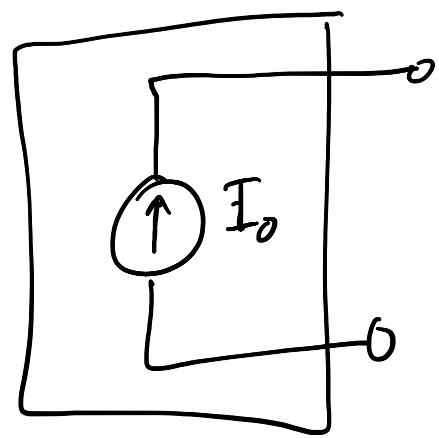


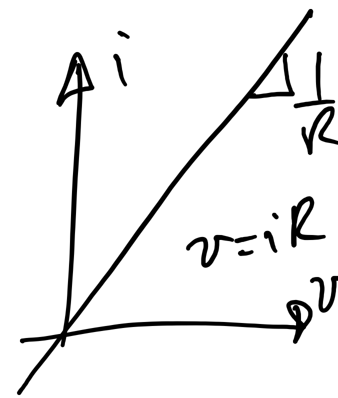
Linear and nonlinear circuits: the Light Emitting Diode



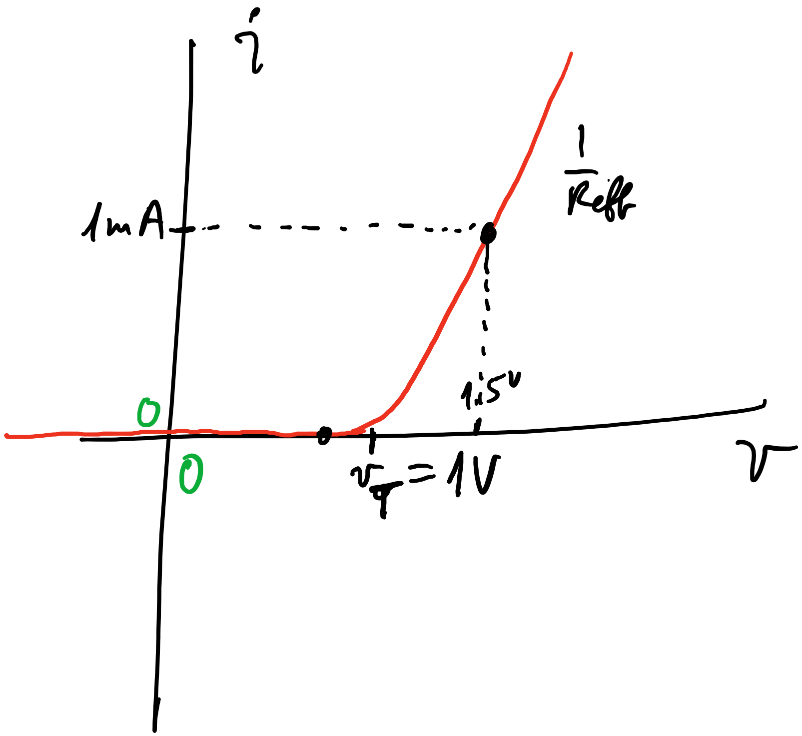


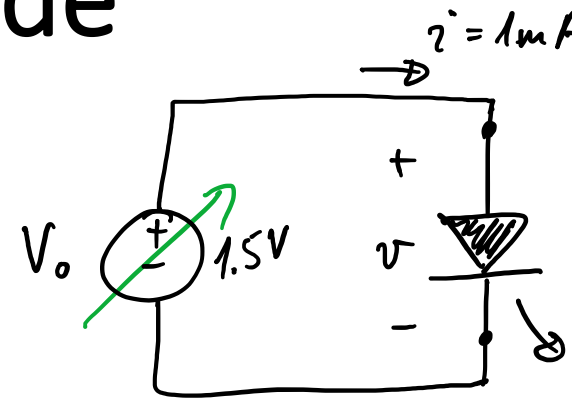


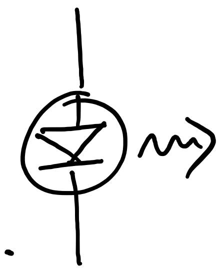




Linear and nonlinear circuits: the Light Emitting Diode



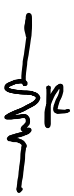




$p = vi$

$= 1.5V \cdot 1mA$

$= 1.5mW.$

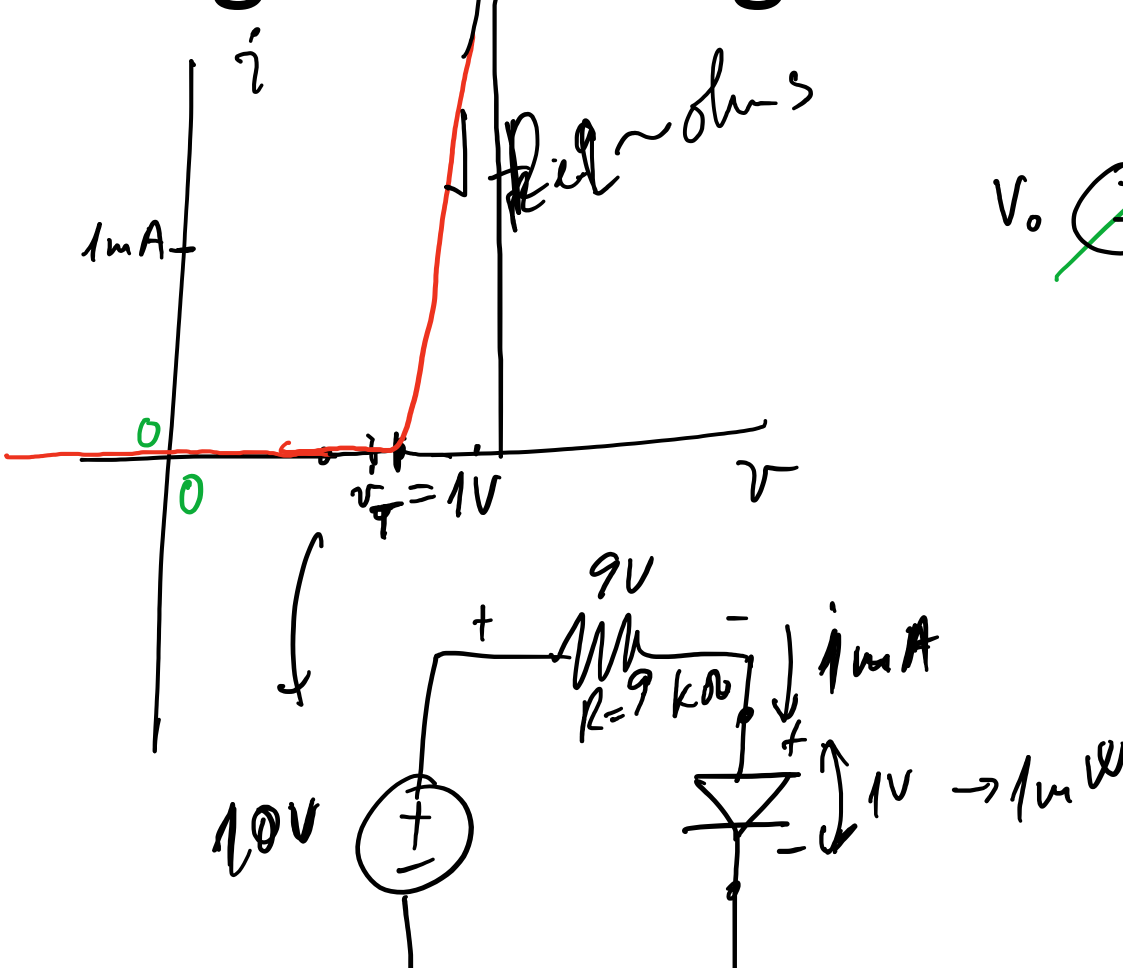


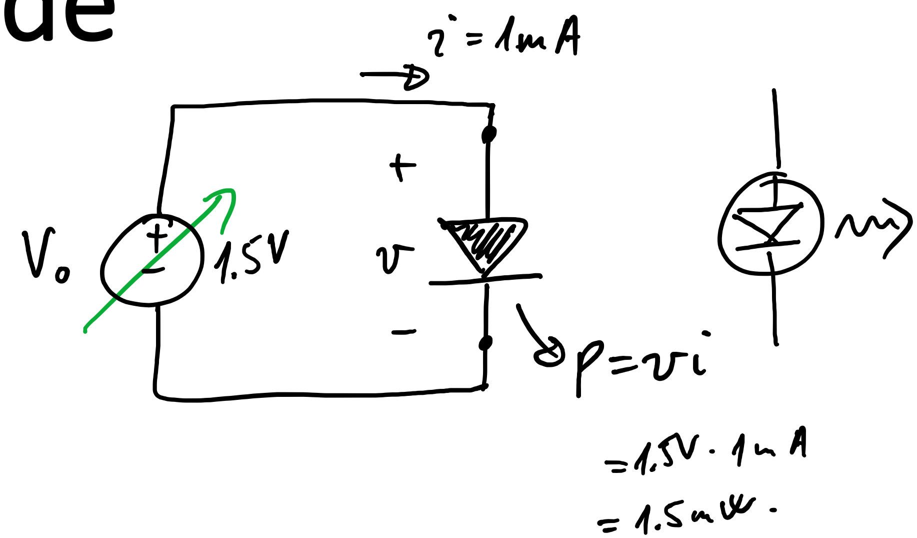
$i = \frac{1}{R} v$

$\frac{di}{dv} = \frac{1}{R}$

$$\boxed{i_D = I_S(e^{v_D/V_T} - 1)}$$

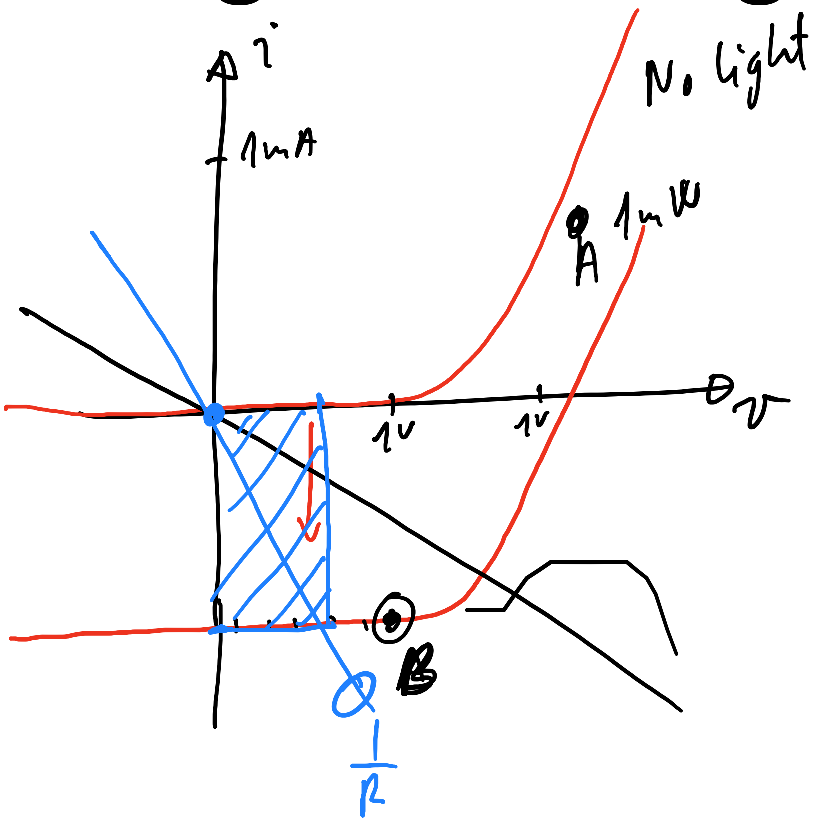
Linear and nonlinear circuits: the Light Emitting Diode

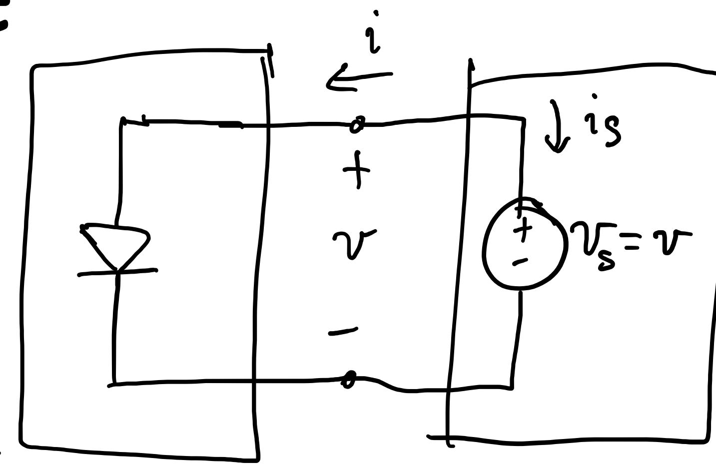




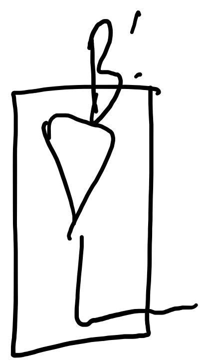
$$\boxed{i_D = I_S(e^{v_D/V_T} - 1)}$$

Linear and nonlinear circuits: the Light Emitting Diode





$p = v_s i_s$
$= v(-i)$
$= (1V)(-(-1mA))$
$= 1mW$

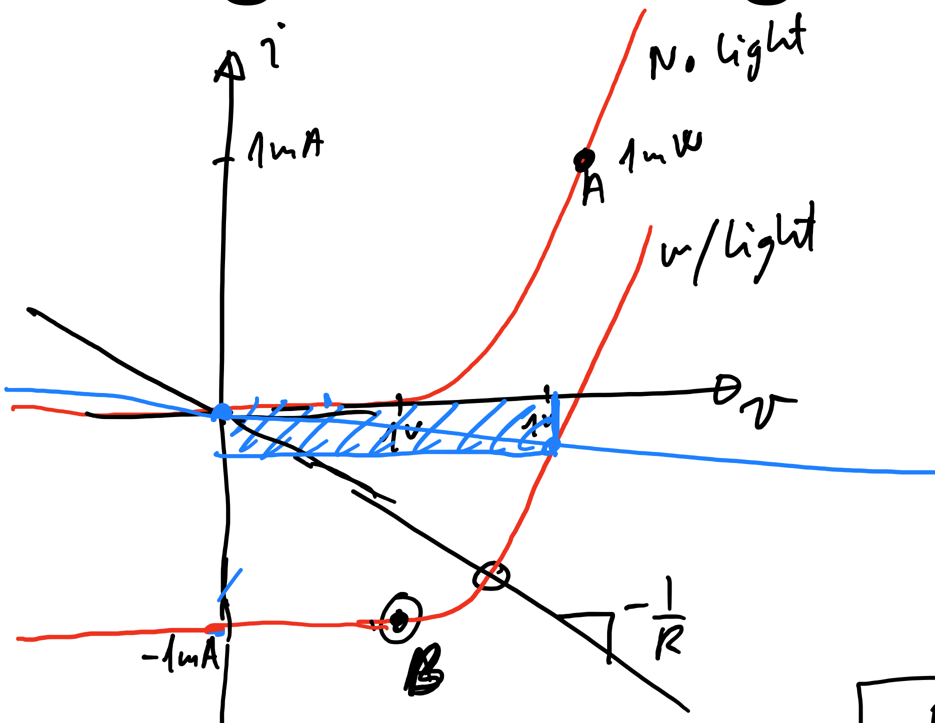


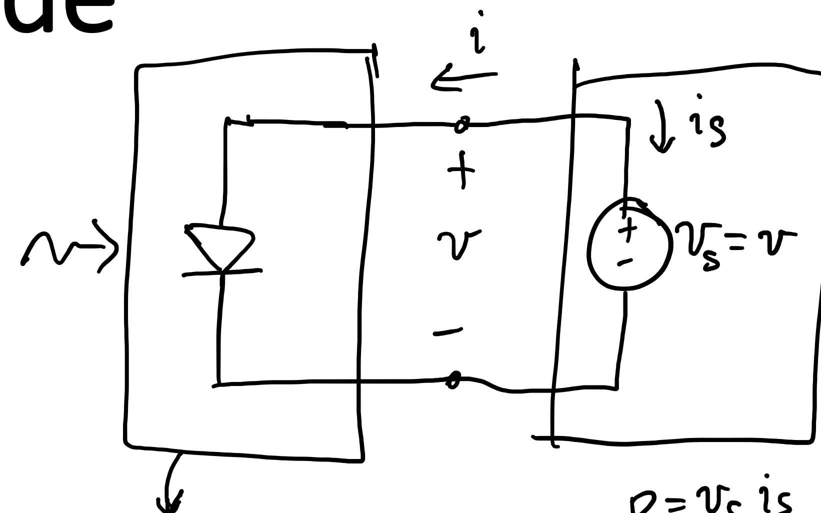
$p = vi$

$= (1V)(-1mA) = -1mW$

$$i_D = I_S(e^{v_D/V_T} - 1)$$

Linear and nonlinear circuits: the Light Emitting Diode





B: $p = vi$

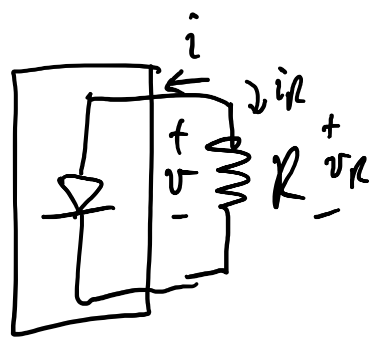
$= (1V)(-1mA) = -1mW$

$p = v_s i_s$

$= v(-i)$

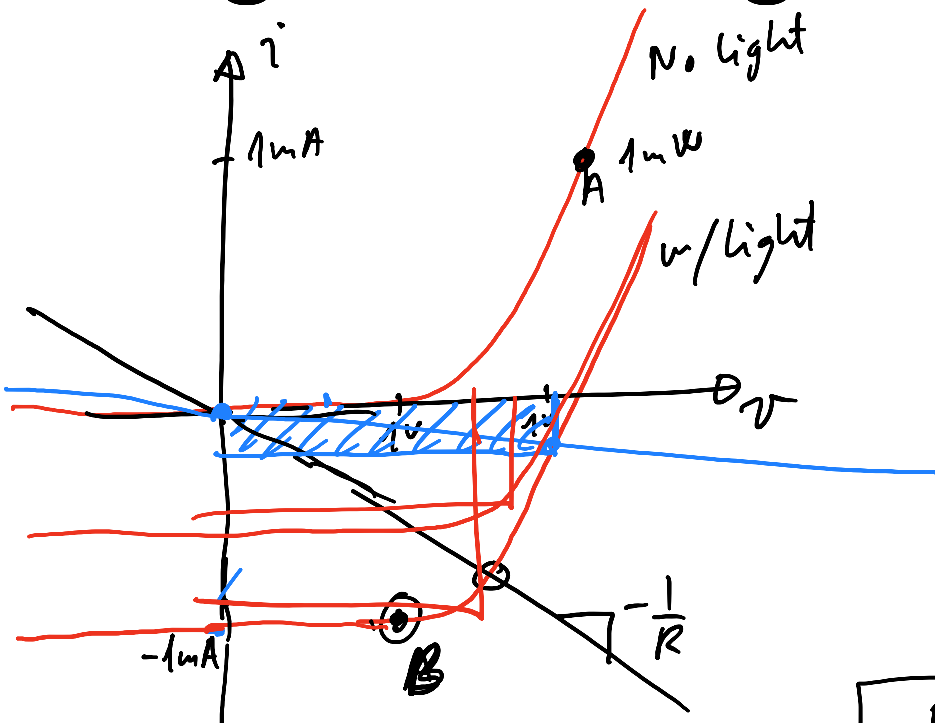
$= (1V)(-(-1mA))$

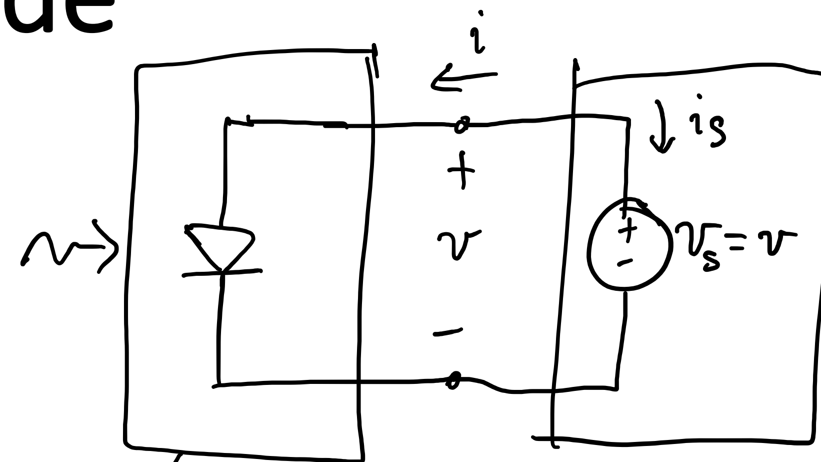
$= 1mW$



$$i_D = I_S(e^{v_D/V_T} - 1)$$

Linear and nonlinear circuits: the Light Emitting Diode





B: $p = vi$

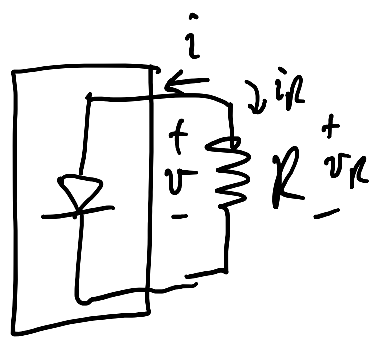
$= (1V)(-1mA) = -1mW$

$p = v_S i_S$

$= v(-i)$

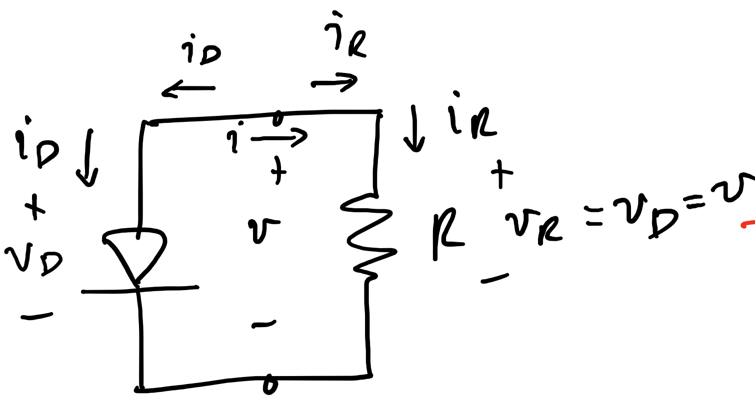
$= (1V)(-(-1mA))$

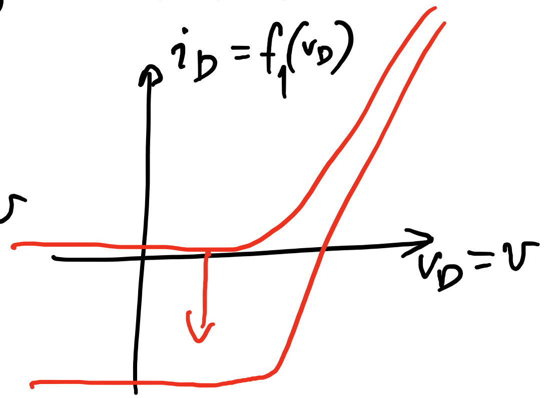
$= 1mW$

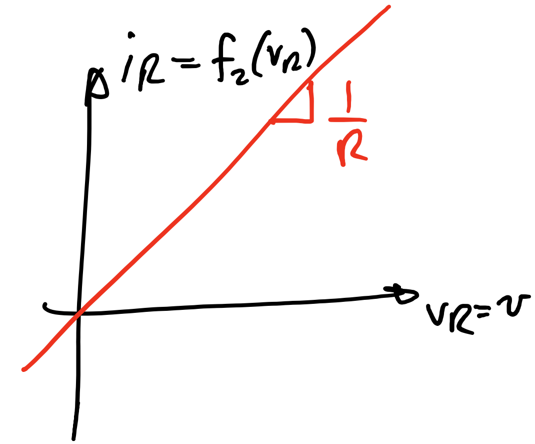


$$i_D = I_S(e^{v_D/V_T} - 1)$$

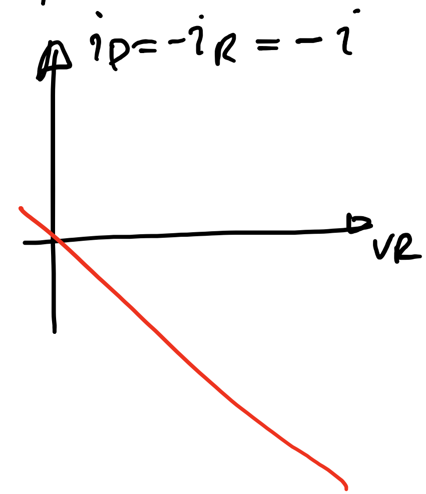
Linear and nonlinear circuits: the Light Emitting Diode

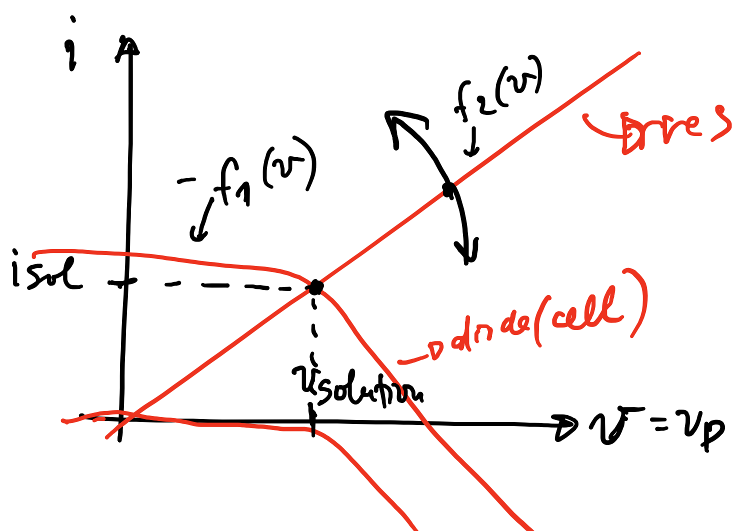






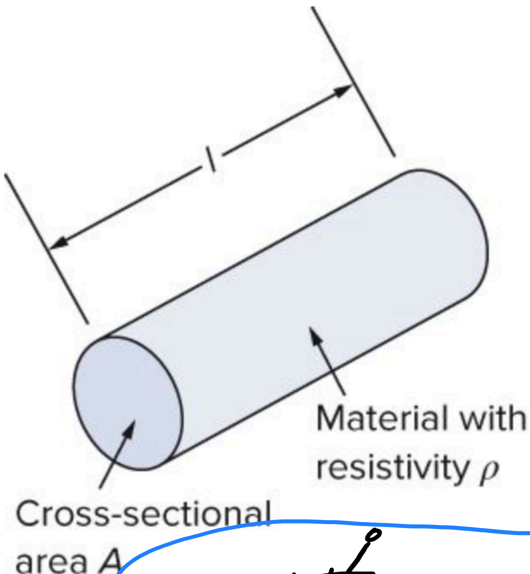
$$\boxed{i_R = -i_D = i} \rightarrow f_2(v_D) = -f_1(v_D)$$

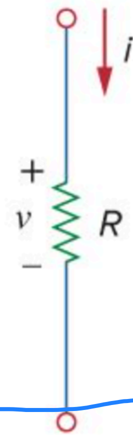


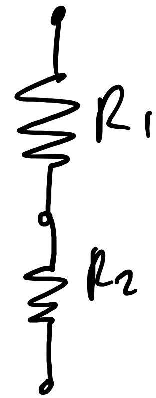


$$i_D = I_S(e^{v_D/V_T} - 1)$$

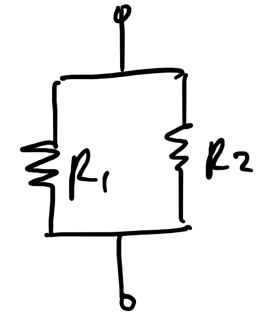
Ohm's law



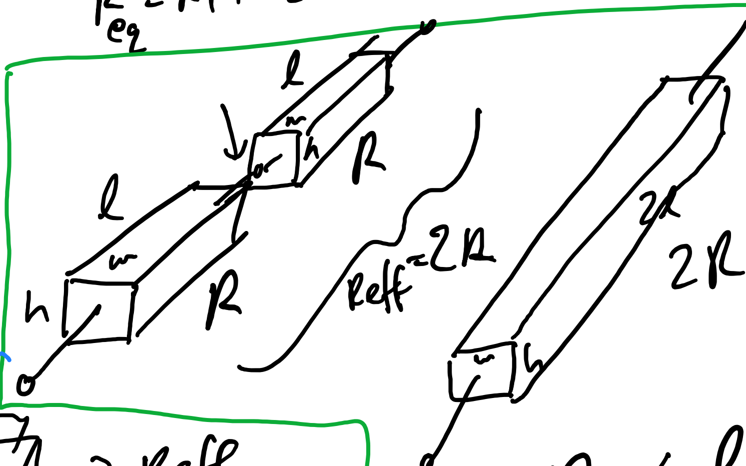




$R_{eq} = R_1 + R_2$



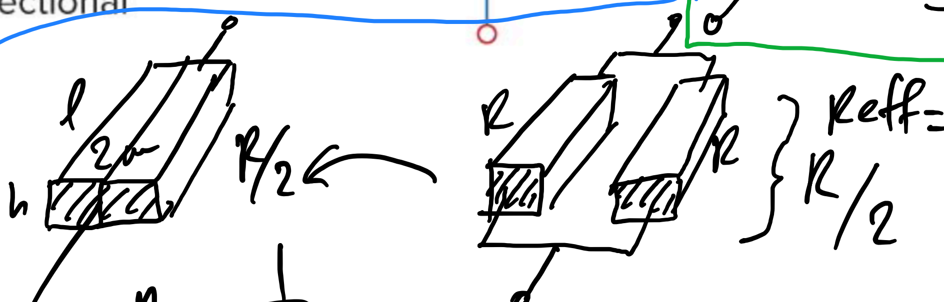
$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$



$R_{eff} = 2R$

$2R$

$R \propto l$



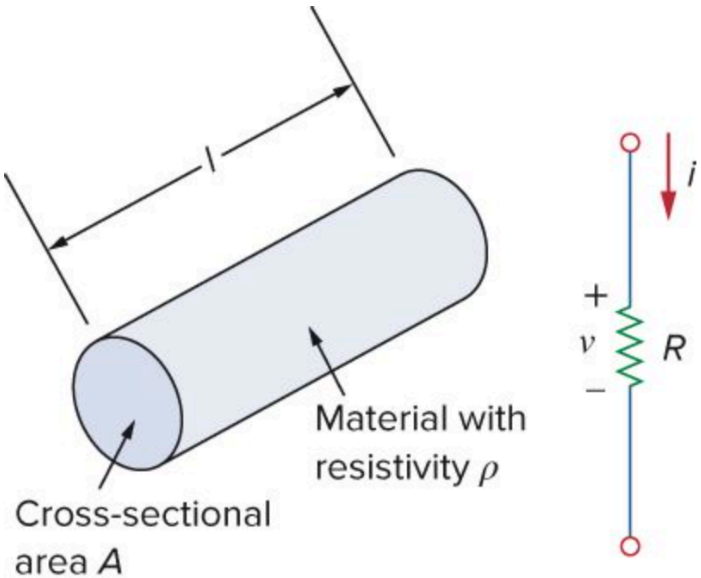
$R_{eff} = R/2$

$R \sim \frac{1}{w}$

$R \sim \frac{1}{h}$

$R \propto \frac{l}{wh} = \frac{l}{A}$

Ohm's law



resistivity

$$R = \rho \frac{l}{A}$$

m

m^2

Ω $\quad$ Ωm

conductivity

$$\sigma = \frac{1}{\rho}$$

Design a carbon resistor

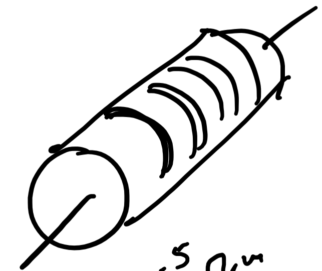


TABLE 2.1

Resistivities of common materials.

Material	Resistivity (Ω•m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	$\rho = 4 \times 10^{-5}\ \Omega m$	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

$4 \cdot 10^{-5}\ \Omega m$

$R = 1k\Omega$

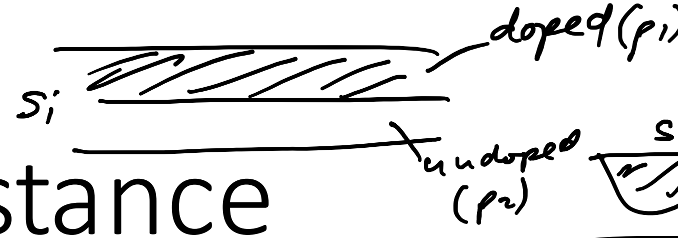
$$R = 1k\Omega = \rho \frac{l}{A}$$

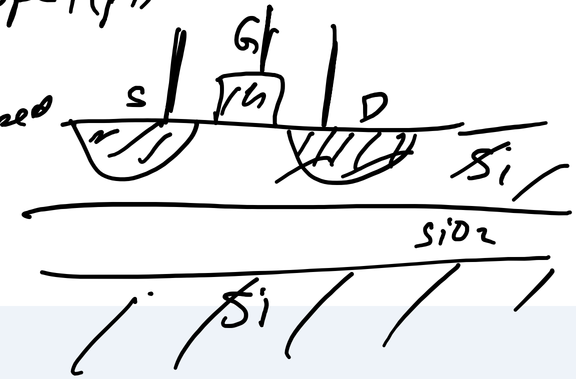
$$A = \pi r^2 = \pi (0.5\,mm)^2$$
$$= 3.14(5e-4)^2$$
$$= 75e-8\ m^2$$

$$1e3\,\cancel{\Omega} = (4e-5\ \cancel{\Omega m}) \frac{l}{75e-8\,\cancel{m^2}}$$

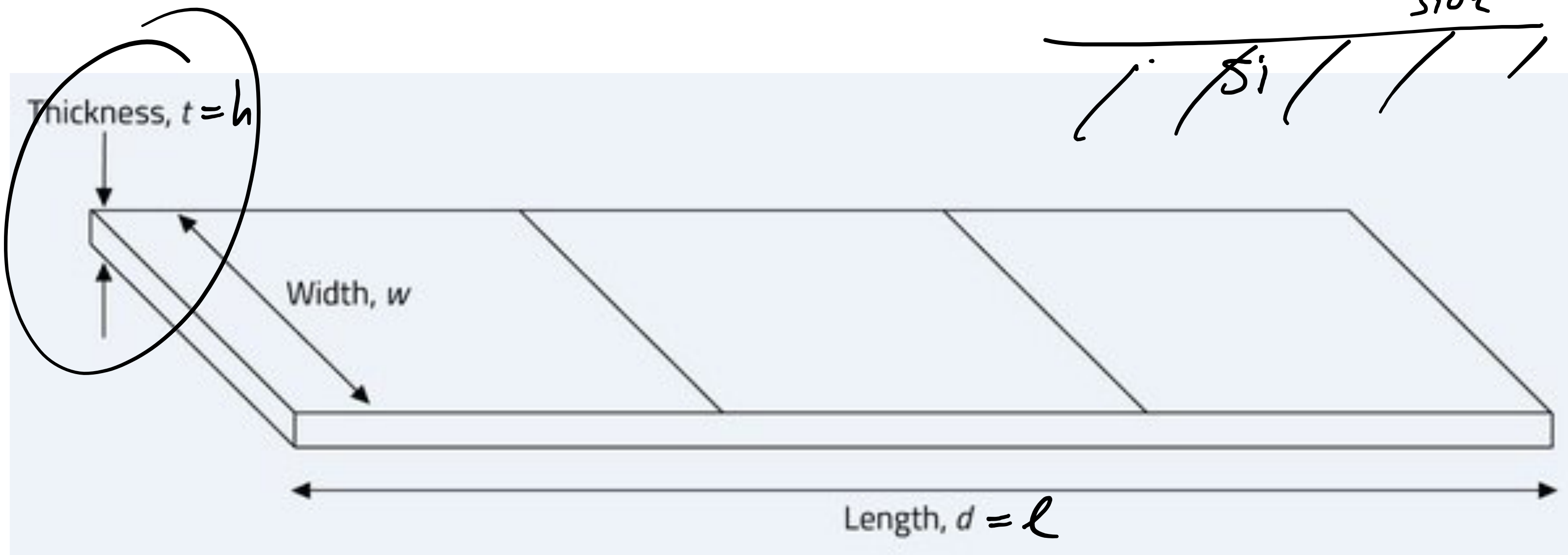
$$l = \frac{1e3 \cdot 75e-8}{4e-5} = \frac{75e-5}{4e-5}$$

$$l = 20\,m$$

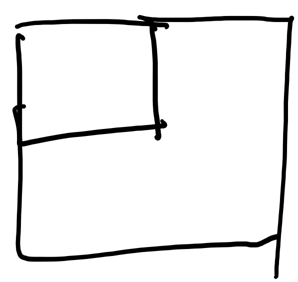




Sheet resistance

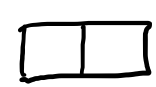


$$R = \rho \frac{\ell}{w\boxed{h}}$$

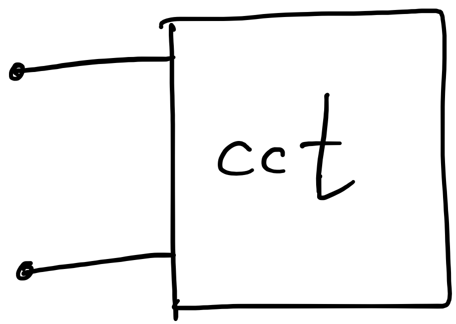


$R_\square$ Resistance per square

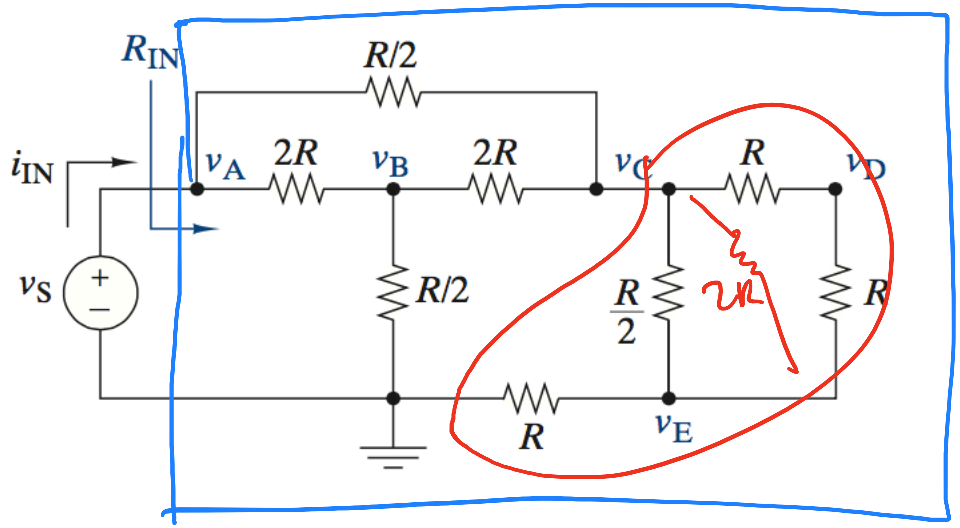
$R_\square = 10\Omega$

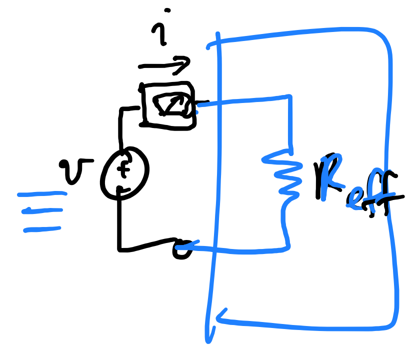
20Ω: 

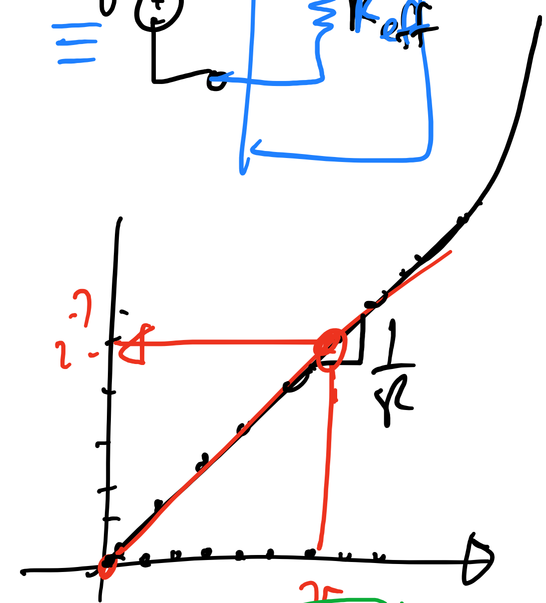
Input resistance



Find input resistance:



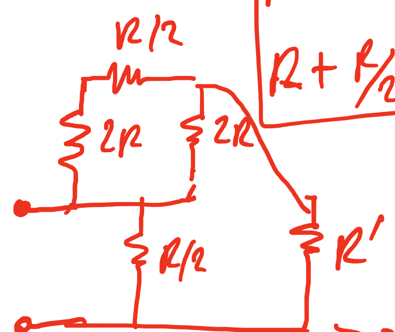


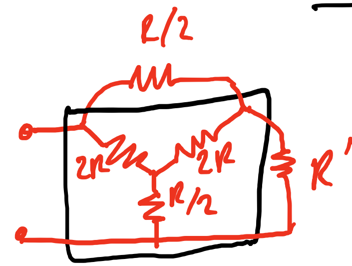


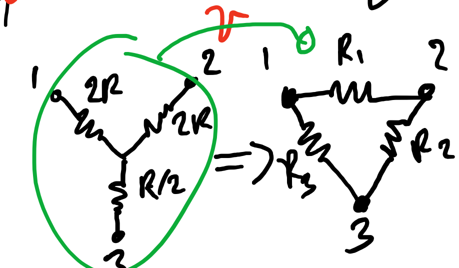
- Apply v_S
- Solve i_{IN}

↳ $R_{IN} = \frac{v_S}{i_{IN}}$

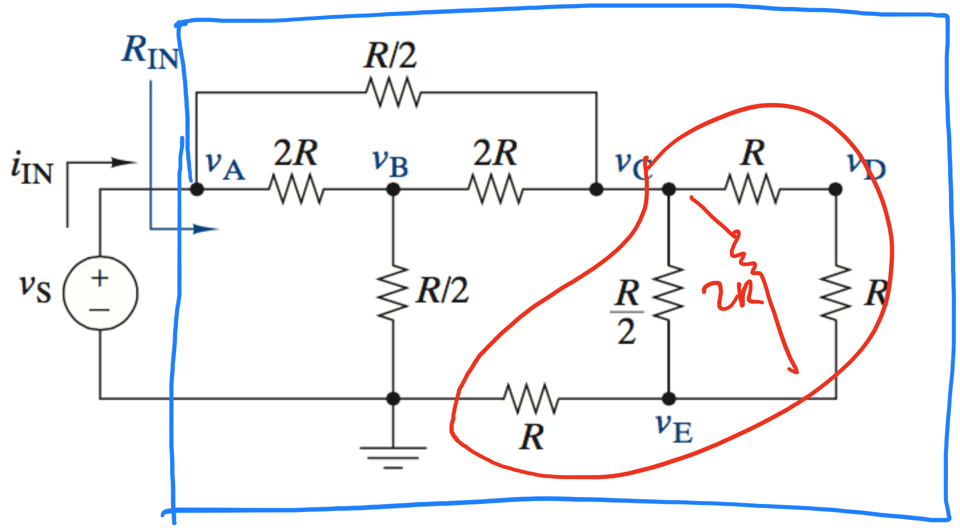
$R' = R + R/2 \,||\, 2R$

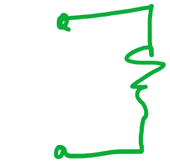






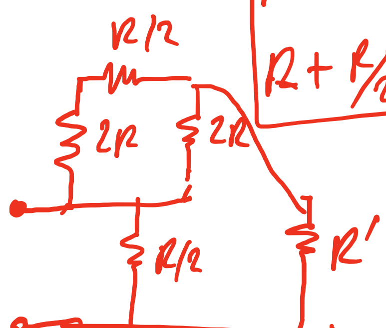
Find input resistance:



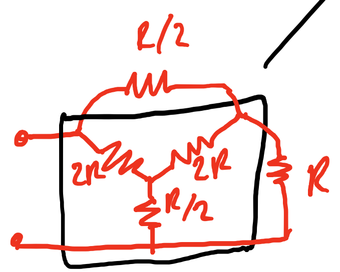


- Apply v_S
- Solve i_{IN}

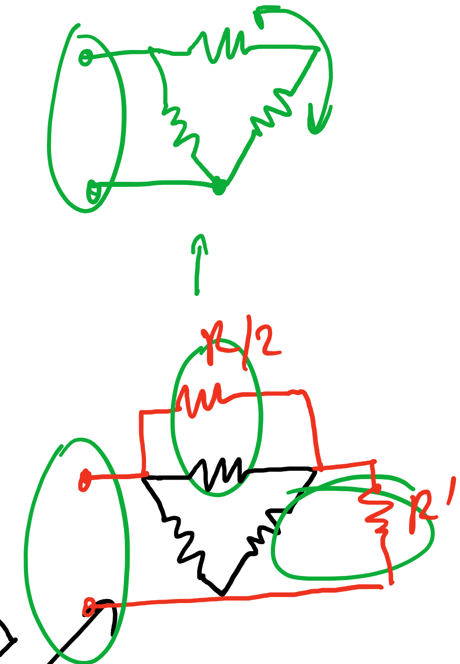
↳ $R_{IN} = \frac{v_S}{i_{IN}}$

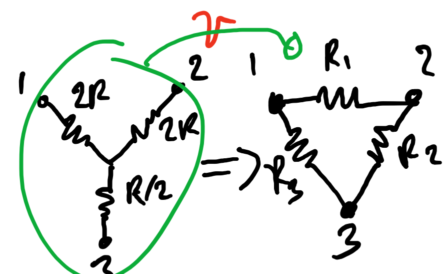


$R' = R + R/2 \,\|\, 2R$

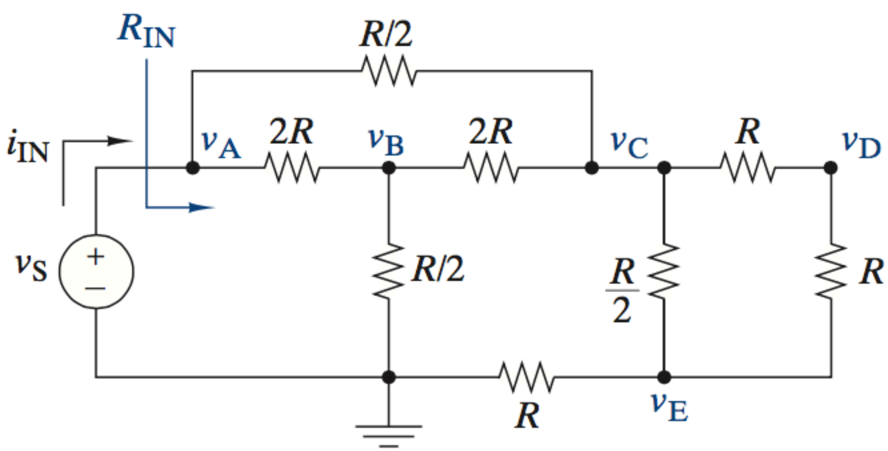


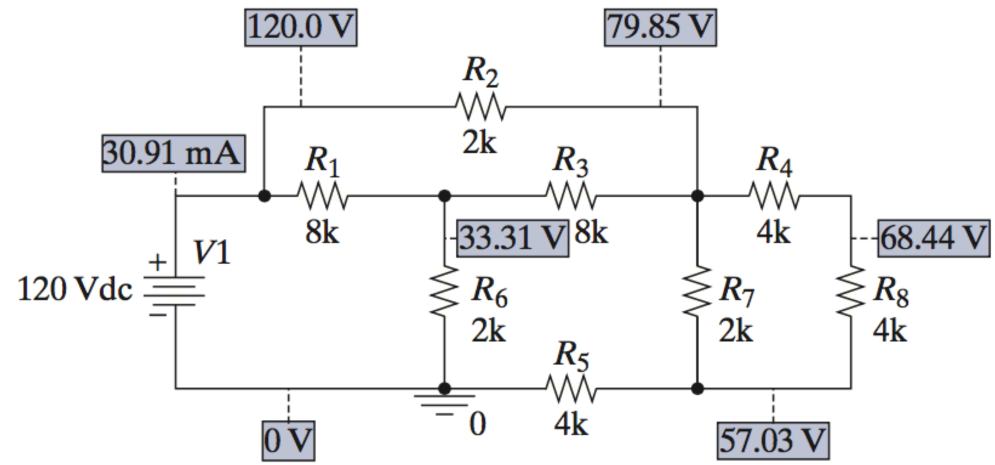
Y to Δ





Find input resistance:





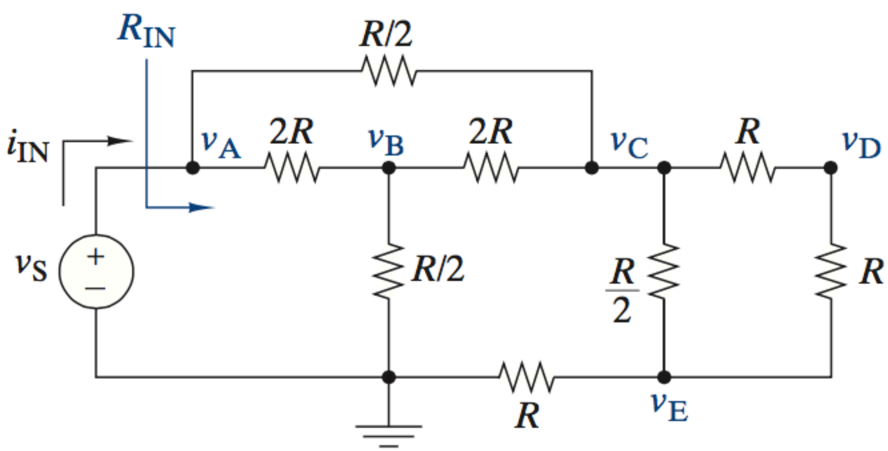
$$\text{Node B: } -v_A\left(\frac{1}{2R}\right) + v_B\left(\frac{1}{2R}+\frac{2}{R}+\frac{1}{2R}\right) - v_C\left(\frac{1}{2R}\right) = 0$$

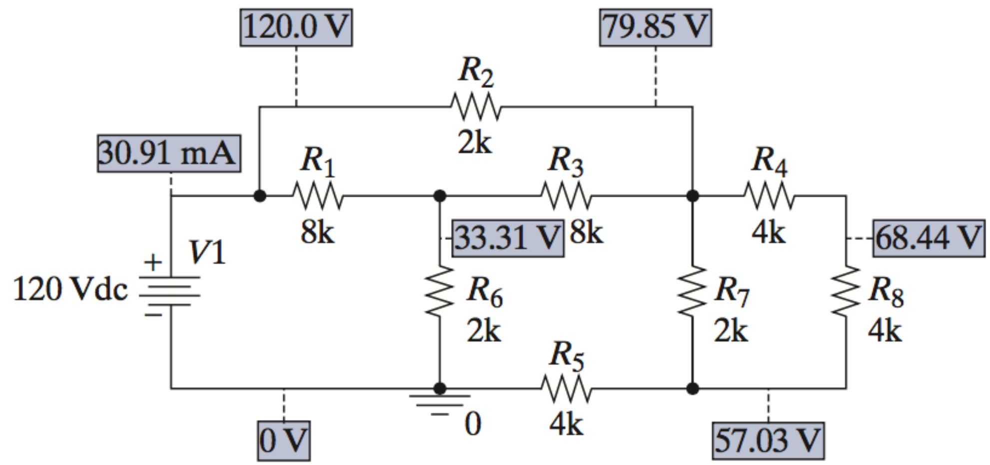
$$\text{Node C: } -v_A\left(\frac{2}{R}\right) - v_B\left(\frac{1}{2R}\right) + v_C\left(\frac{2}{R}+\frac{1}{2R}+\frac{2}{R}+\frac{1}{R}\right) - v_D\left(\frac{1}{R}\right) - v_E\left(\frac{2}{R}\right) = 0$$

$$\text{Node D: } -v_C\left(\frac{1}{R}\right) + v_D\left(\frac{1}{R}+\frac{1}{R}\right) - v_E\left(\frac{1}{R}\right) = 0$$

$$\text{Node E: } -v_C\left(\frac{2}{R}\right) - v_D\left(\frac{1}{R}\right) + v_E\left(\frac{1}{R}+\frac{1}{R}+\frac{2}{R}\right) = 0$$

Find input resistance:





Node B: $-v_A\left(\frac{1}{2R}\right)+v_B\left(\frac{1}{2R}+\frac{2}{R}+\frac{1}{2R}\right)-v_C\left(\frac{1}{2R}\right)=0$

Node C: $-v_A\left(\frac{2}{R}\right)-v_B\left(\frac{1}{2R}\right)+v_C\left(\frac{2}{R}+\frac{1}{2R}+\frac{2}{R}+\frac{1}{R}\right)-v_D\left(\frac{1}{R}\right)-v_E\left(\frac{2}{R}\right)=0$

Node D: $-v_C\left(\frac{1}{R}\right)+v_D\left(\frac{1}{R}+\frac{1}{R}\right)-v_E\left(\frac{1}{R}\right)=0$

Node E: $-v_C\left(\frac{2}{R}\right)-v_D\left(\frac{1}{R}\right)+v_E\left(\frac{1}{R}+\frac{1}{R}+\frac{2}{R}\right)=0$

$$\begin{bmatrix} \frac{3}{R} & -\frac{1}{2R} & 0 & 0 \\ -\frac{1}{2R} & \frac{11}{2R} & -\frac{1}{R} & -\frac{2}{R} \\ 0 & -\frac{1}{R} & \frac{2}{R} & -\frac{1}{R} \\ 0 & -\frac{2}{R} & -\frac{1}{R} & \frac{4}{R} \end{bmatrix} \begin{bmatrix} v_B \\ v_C \\ v_D \\ v_E \end{bmatrix} = \begin{bmatrix} \frac{v_S}{2R} \\ \frac{2v_S}{R} \\ 0 \\ 0 \end{bmatrix}$$

Matlab code

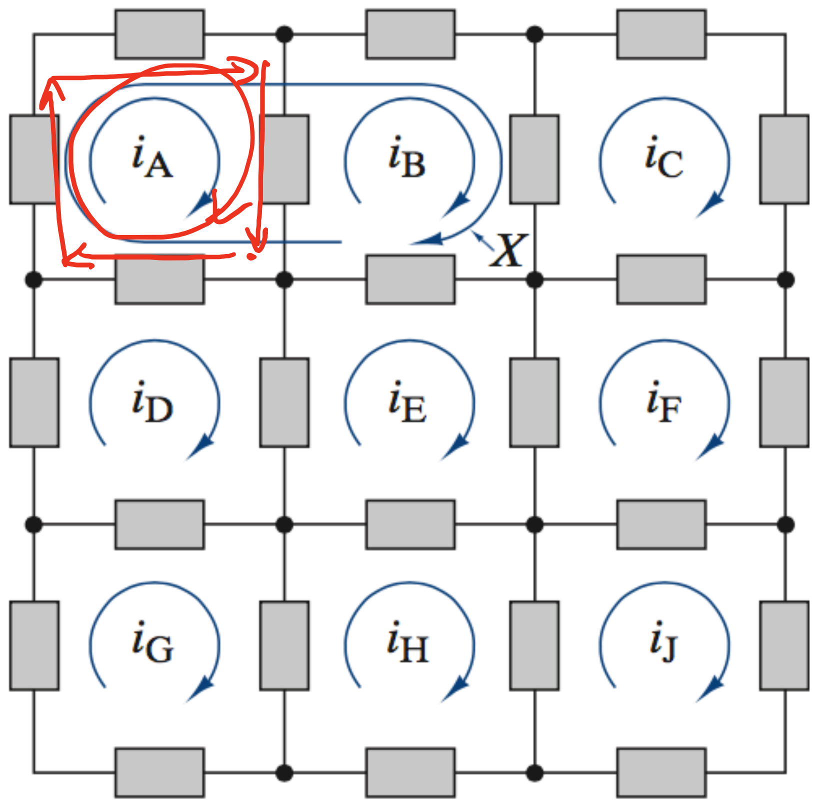
```
syms vS vB vC vD vE R
A = [3/R -1/2/R 0 0;...
-1/2/R 11/2/R -1/R -2/R; . . . 0 -1/R 2/R -1/R;...
0 -2/R -1/R 4/R];
B = [vS/2/R; 2*vS/R; 0; 0]; x = A\B;
vB = x(1)
vC = x(2)
vD = x(3) vE = x(4)
```

$$i_{\mathrm{IN}} = \frac{v_{\mathrm{S}} - v_{\mathrm{C}}}{R/2} + \frac{v_{\mathrm{S}} - v_{\mathrm{B}}}{2R}$$

$$R_{\mathrm{IN}} = \frac{v_{\mathrm{IN}}}{i_{\mathrm{IN}}} = \frac{263R}{271}$$

Mesh current analysis

Mesh current analysis



$$\bar{x} = \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \\ \vdots \\ i_J \end{bmatrix}$$

$$\bar{\bar{A}} \cdot \bar{x} = \bar{b}$$

$$\bar{x} = \bar{\bar{A}}^{-1} \cdot \bar{b}$$

To be continued…