# RC, RL, and LCR Circuits

## EK307 Lab

**Note: This is a two week lab. Most students complete part A in week one and part B in week two.**

## Introduction:

Inductors and capacitors are energy storage devices. They differ in that a capacitor stores energy as accumulated charge (voltage potential) and an inductor stores energy in a magnetic field that is due to current.

We learned that in a resistor the ratio of voltage across the terminals to the current through them is the resistance, R = V/I.
In the inductor and capacitor this ratio depends on the rate of change, not the instantaneous values of voltage and current. The voltage to current ratios have a rate dependency. For the capacitor this is I = CdV/dt. For the inductor it is V = Ldi/dt. This means that if the voltage across a capacitor is not changing then there is zero current flowing through it. For the inductor if the current isn't changing there is zero voltage across it.

Note that although these conditions of zero current and voltage can exist it does not mean the capacitor or inductor is not storing energy. For example if you short circuit an ideal inductor that has a current flowing at the instant before the short occurred the current will flow in the loop forever. Of course this is an ideal situation and in reality the current will decrease due to the internal resistance of the inductor and the wires short circuiting it. Similarly a capacitor charged to a voltage will retain that voltage if it is open circuited. Over time the leakage of the capacitor dielectric will cause the voltage across its plates to decrease.

Although the details are past the scope of this class, it is important to know that real circuit elements (as opposed to schematics on paper) have intrinsic inductance, capacitance, and resistance due to the physical properties of the components. For example, real inductors have significant resistance and capacitance in the windings which limits their use at high frequencies. Real capacitors have resistance in series and parallel with the capacitor which limits their current characteristics. What this means to the circuit designer is that there will be limits on the precision and frequency response of circuits you make.

## PreLab

A. Calculate the RC time constant of the circuit in figure 2. Where R is 12k and C is 100 nF. Draw a sketch of a graph of voltage vs time of the voltage across the capacitor in response to a unit step voltage source. Label the time constant on the graph. A time range (x axis) of 10 time constants is reasonable.

B. Calculate the RL time constant of the circuit in figure 3.Where R is 47k and L is 100 mH. Draw a sketch of a graph of the voltage across the inductor in response to a unit step voltage source. Label the time constant on the graph.

Your sketches should have two traces (lines) on each graph. One is the step input voltage, the other is the voltage across the capacitor or inductor. The sketch will mimic what you would see on an oscilloscope if you were monitoring the respective voltages. The traces are aligned in the time dimension. It is common to refer to this as a timing graph or diagram. See figure 1 for an example timing diagram sketch.

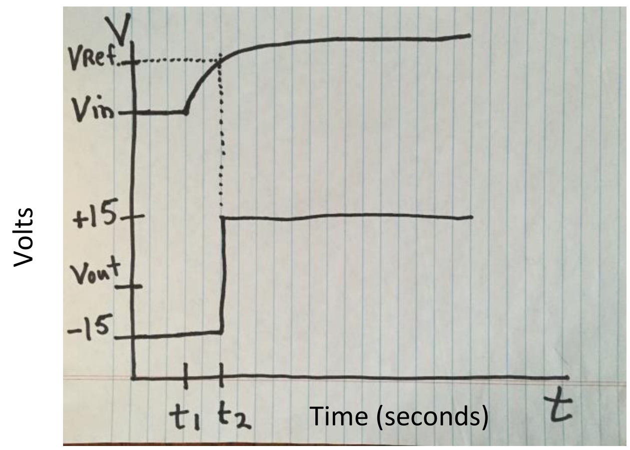


**Figure 1:** **<u>Sample</u> hand sketch of a timing graph or diagram. Yours will not look exactly like this! This figure communicates cause and effect in the time dimension. It is basically a cartoon of a two trace oscilloscope. Notice it takes up a whole page in my lab notebook. That leaves space to modify it, annotate, or add traces.**

## Background Material:

### Resistor - Capacitor Step Response:

A capacitor is a device that stores potential energy as charge. In lecture we learned that the response of a series *RC* circuit to a step change in excitation involves a decaying exponential function of the form $e^{-t/RC}$. For the circuit shown in figure 1, if the capacitor is initially uncharged, and if $v_{IN}$ undergoes a step change from 0 to V, the capacitor Voltage for t > 0 will be given by

$$V_{OUT} = V(1 - e^{-t/(RC)})$$

Where R is in Ohms, C is in Farads, and t is in seconds

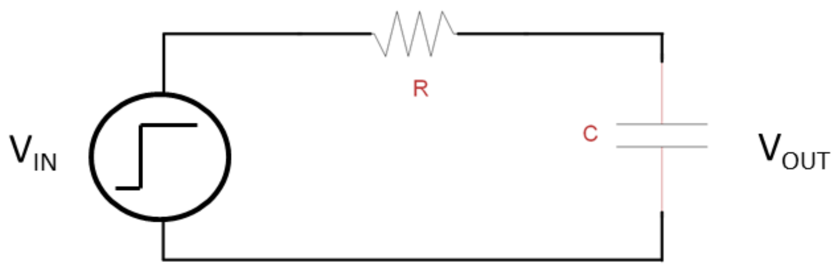


**Figure 2.** RC circuit driven by an ideal step voltage source. The source Voltage VIN can provide positive or negative going steps.

Similarly, if the capacitor is precharged to *V*, and if $V_{IN}$ steps from *V* to zero, the capacitor Voltage will be given by

$$V_{OUT} = Ve^{-t/(RC)}$$

What does the behavior of an RC circuit mean intuitively? The voltage across the capacitor will be delayed relative to the source voltage. This is because the capacitor needs to charge or discharge before the voltage across its terminals change. If the source voltage is simply a constant DC value, then the voltage across the capacitor will asymptote to the source voltage. When the capacitor voltage is equal to the source voltage no current will flow and the circuit will behave like an open circuit where you can imagine the capacitor is simply taken out of the circuit leaving the terminals open. If the source voltage is constantly changing, the voltage across the capacitor will be lagging behind proportionally to the rate of change of the source Voltage.

**Resistor - Inductor Step Response:**

An inductor stores potential energy in the form of current. Similar to the RC circuit, the RL circuit's dynamic behavior follows an exponential function, $e^{-tR/L}$. If the circuit in figure 2 has zero initial current, $V_{IN}$ equals zero, and a step voltage of value V is switched on at t = 0, the Voltage across the inductor ($V_{OUT}$) will behave according to this equation.

$$V_{OUT} = Ve^{-tR/L}$$

Where R is in Ohms, L in Henries, and t in seconds.

If $V_{IN}$ is a constant (DC) non – zero value for a long time, the current in the inductor will asymptote to V/R. Why? Because the voltage across the inductor is approaching zero and therefore the source voltage will appear across the resistor. You can model an ideal inductor as a short circuit as time goes to infinity. If you have an RL circuit that has been supplied with a constant voltage V for long enough that the voltage across the inductor is asymptotic to zero and the Voltage of the source undergoes a step change from V to zero Volts then the Voltage will behave according to this equation.

$$V_{OUT} = -Ve^{-tR/L}$$

The same instant the step voltage changes, the voltage across the inductor changes by the same magnitude as the step change in VIN then decays toward zero. In this example VOUT goes negative relative to the voltage source ground. It goes negative because before the negative step the voltage across the inductor was zero, then the step made a transition of magnitude -V. Since the current across an inductor can't change instantaneously the current through the resistor could not change thus to satisfy Ohm's law for the resistor, KCL, and KVL, the voltage across the inductor had to go negative. This is one of the methods of generating negative Voltages from positive Voltages.

## Level 1: There is a part A and part B

**(It is expedient to perform the level 1 and 2 simultaneously if you choose to complete level 2)**

### Part A:

**Goal: Measure the time constant of an RC and an RL circuit, compare to the theoretical calculations. Use the schematics of figure 2 and 3 as your circuits.**

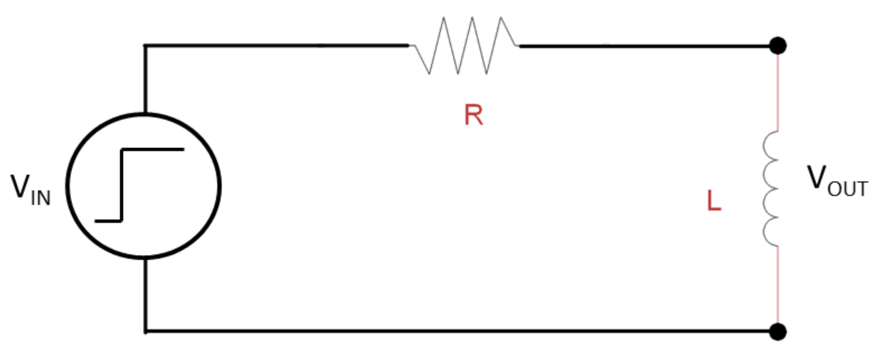


**Figure 3: R-L circuit driven by an ideal step Voltage source. $V_{IN}$ can undergo a step change from zero to $V_{IN}$ or from $V_{IN}$ to zero**

### Procedure:

a. Choose components that have time constants that are in an easy to measure range. For our lab this would be tens of microseconds to seconds (10E-6 s to 1 s). Your lab kit contains at least one inductor with a value of 33mH or 100mH. Capacitors with values of 100pF to 100 uF are available in the lab or your kit. Many different resistor values are available in the spinner (rotating thing with little drawers) near the door. When measuring the RL time constant we recommend using R values greater than 10kΩ to lessen the effects of the non-ideal behavior of the inductor.

b. The function generator square wave output is a convenient way to simulate a step function. It can be used as your signal source (or VIN referring to figures 1 and 2). You may recall from an earlier lab that the function generator has a 50 Ohm output resistance. You need to add this resistance in series with your R value to get accurate results when computing your time constants. See figure 4 for a schematic of the equivalent circuit.

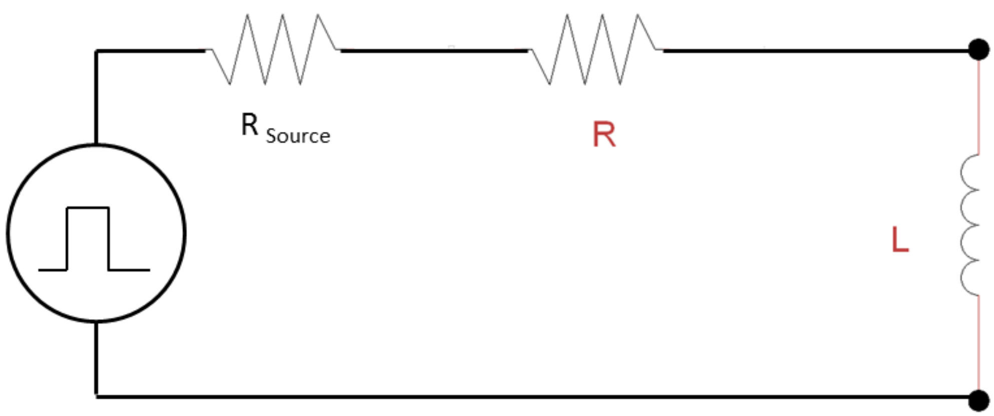


**Figure 4. Function generator connected to an RL circuit. The value of $R_{Source}$ is 50 Ohms. $R_{Source}$ is the Thevenin output resistance of the function generator. R and L are values you choose. If your R is much less than 10k than the effect of $R_{Source}$ on your measurement will be significant.**

c. If you use a square wave from the function generator to simulate a step function, the time constant measurement is easier if you choose a frequency that allows enough time for the voltage across your L and C to asymptote to the final value before the waveform changes states. A safe number would be greater than 5 time constants in duration. The period of a wave in seconds equals 1/Frequency, and for a square wave the voltage is constant between transitions for half of a period. Perform the calculation to figure out a good frequency.

d. To measure the time constant use an oscilloscope to measure the voltage vs time across your L and C components of each circuit. Recall from lecture that the voltage at t = time constant is a certain percent of the step voltage.

e. Draw a sketch of the waveforms for each circuit in your notebook. Label the axes and the voltage at one time constant from the step and five time constants.

**Level 1 Part B**

**Goal: Measure the voltage across the capacitor for the LCR circuit in figure 5 in response to a step response on the input. Repeat this measurement with three different resistors. One that makes the circuit critically damped, one overdamped, and one underdamped.**

Procedure:

a. Use your 100mH inductor as the L, choose a capacitor value between 1000pF and 100nF.
b. Calculate the resistor size that will make the circuit critically damped. Hint: if your calculated R value is lower than 500 ohms you may want to change C in order to increase the R value. The reason is the function generator has a Thevenin output resistance of 50 ohms. If your R is close to 50 ohms the function generator resistance must be considered. Using a larger value for R will avoid this issue.
c. Measure the step response of the critical damped circuit and sketch a timing plot.
d. Change the value of the resistor to make the circuit underdamped. For dramatic results choose a resistor that deviates at least by a factor of 10 from the critical solution.
e. Measure the step response and add the underdamped trace to your sketch.
f. Calculate the natural frequency of the circuit by measuring the frequency of the ringing.
g. Change the value of the resistor to make the circuit overdamped. For a measureable effect change the resistor by a factor of ten.
h. Measure the time it takes the capacitor to charge to 90% of its asymptotic value.
i. Add a trace showing the overdamped response to your sketch.

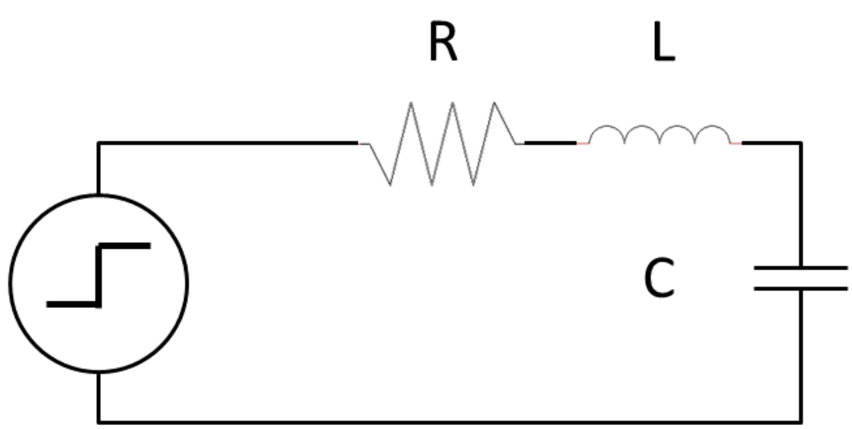


**Figure 5: RLC circuit. For level one you will be measuring voltage across the capacitor in response to a step input. Recall the square wave function on the generator is a good approximation of a step if the frequency is much lower than resonance.**

## Level 2

**(To be completed in addition to level one. It is expedient to perform the level one and two simultaneously if you choose to complete level 2)**

Measure the voltage across the resistor for the three circuits (RC, RL, LCR) that you completed in level one. Draw sketches of the results and compare them to the Level 1 measurements. You may have to rearrange the circuit elements because the oscilloscope needs one terminal to be grounded to make this work. For example swap the R and C in the RC circuit, R and L in the RL circuit.