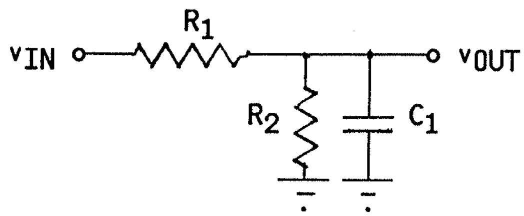
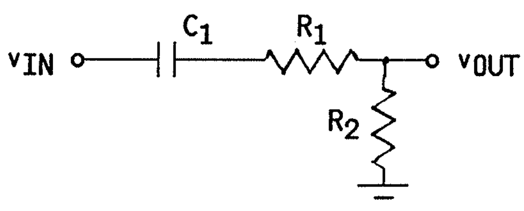
Chapter 9
Time-Dependent Circuit Behavior and Frequency Response

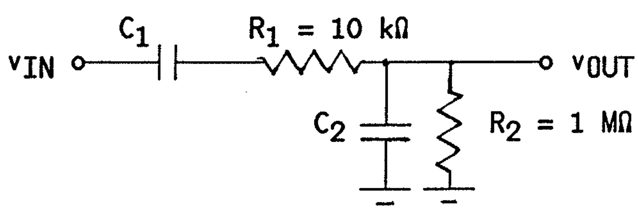
9.1 Consider the resistor-capacitor circuit shown below. Choose component values so that v_{IN} is attenuated by a frequency-independent factor of 2/3 at low frequencies and is reduced toward zero above 1 kHz.



9.2 Choose R_1, R_2 and C_1 in the circuit shown below so that v_{OUT} is equal to half of v_{IN} at frequencies well above 10 kHz and is reduced toward zero at lower frequencies. Note that v_{OUT} is exactly equal to zero at dc, where C_1 behaves as an open circuit.



9.3 The following circuit is to have a constant response between 100 Hz and 1 MHz. Choose appropriate values for C_1 and C_2, where $C_1 >> C_2$. What is the magnitude of v_{OUT}/v_{IN} in the midband?

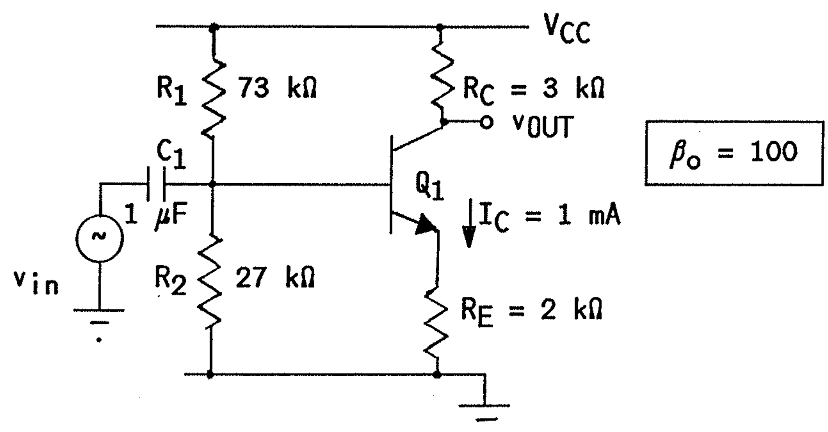


9.4 A network has the following frequency-dependent voltage transfer function:

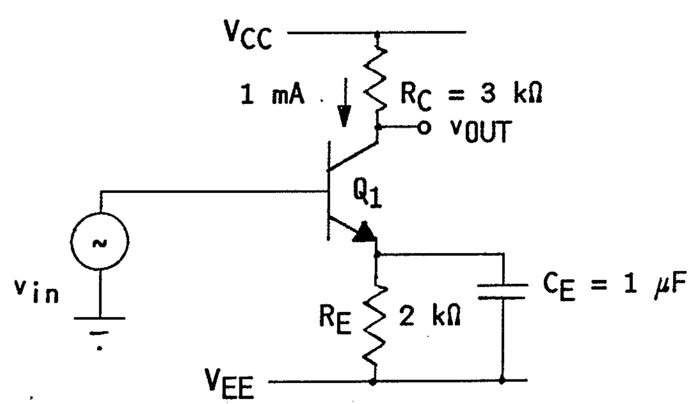
$$\frac{V_{out}}{V_{in}} = \frac{100\left[1 + \frac{j\omega}{10^2}\right]\left[1 + \frac{j\omega}{10^3}\right]\left[1 + \frac{j\omega}{10^5}\right]}{\left[1 + \frac{j\omega}{10^4}\right]\left[1 + \frac{j\omega}{10^5}\right]\left[1 + \frac{j\omega}{10^6}\right]\left[1 + \frac{j\omega}{10^7}\right]}$$

Draw the magnitude and angle Bode plots of V_{out}/V_{in}.

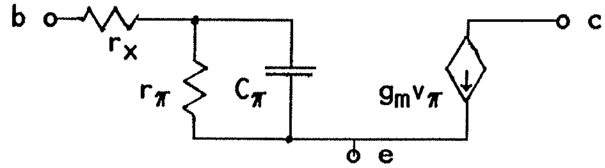
9.5 Consider the BJT amplifier shown below. The input signal v_{in} is coupled to the circuit via a series capacitor C_1. The BJT is biased with I_C = 1 mA, ηV_T = 25 mV, and β_o = 100. Derive an expression for the gain that includes C_1. Find ω_L, the low-frequency -3-dB endpoint of the midband region. Sketch the $|V_{out}/V_{in}|$ magnitude Bode plot.



9.6 The BJT in the amplifier shown below is biased in the active region at room temperature. Derive an expression for the small-signal gain V_{out}/V_{in} as a function of frequency. Your expression should be valid within and below the midband region. Find ω_L, the low-frequency -3-dB endpoint of the midband region. Sketch the magnitude Bode plot of $|V_{out}/V_{in}|$.

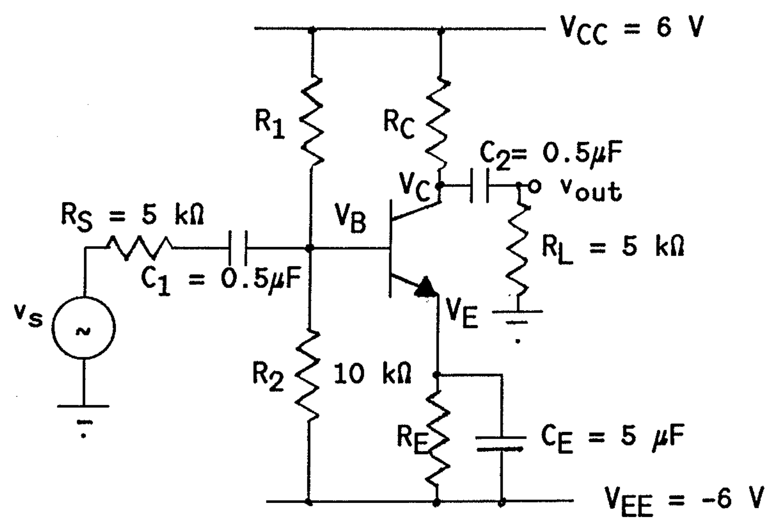


9.7 The BJT in the amplifier of the previous problem is biased in the active region. In this problem, the high-frequency portion of the Bode plot is considered. For small excursions of i_C and v_{CE} about the bias point, represent the BJT by the following small-signal model:



Note that the transverse capacitance C_μ and output resistance r_o have been omitted from this model, i.e., C_μ is assumed to be very small and r_o large. If $r_x = 50\ \Omega$, $r_\pi = 2.5\ \text{k}\Omega$, and $C_\pi = 20$ pF, obtain an expression for the gain within and above the midband region. Identify the -3-dB high-frequency endpoint of the midband region and sketch the high-frequency portion of the Bode plot.

9.8 The BJT in the discrete-design amplifier shown below is to be biased in the constant-current region at room temperature. The internal BJT parameters are listed in the box.



$\beta_o = 100$	$f_T = 250$ MHz	$r_o = \infty$
$V_f = 0.7$ V	$C_\mu = 2$ pF	$r_x = 2\ \Omega$

a) Find values of R_C, R_E, R_1 and the node voltages V_B, V_C, and V_E that will establish the following conditions with R_L disconnected:

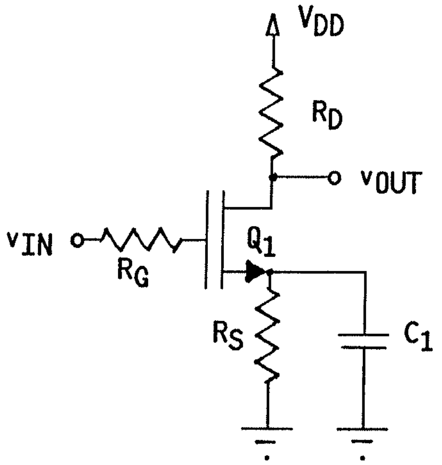
- Intrinsic small-signal gain v_c/v_b = 40 dB
- Bias current I_C = 0.5 mA
- Bias voltage V_B = -2 V.

b) Find the overall midband gain v_{out}/v_s with v_s, R_S, and R_L connected

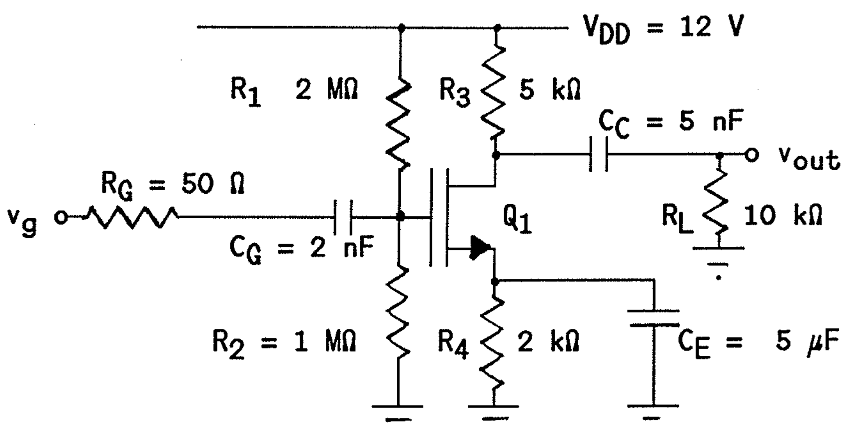
c) Identify the low- and high-frequency capacitors.

d) Compute low- and high-frequency endpoints of the midband region.

9.9 For the MOSFET inverter shown below, find expressions for the time constants that affect the high frequency rolloff and rise time. Derive in terms of the internal MOSFET parameters.



9.10 An n-channel MOSFET is connected in the feedback bias configuration, as shown below. The device parameters are listed in the box.



$K = 2\ mA/V^2$	$C_{gs} = 2$ pF
$V_{TR} = 2.5$ V	$C_{gd} = 5$ pF
$r_o = \infty$	$C_{ds} = 0$

a) Verify that $I_D = 0.5$ mA

b) Find a value for the MOSFET transconductance g_m.

c) Determine the low-frequency -3-dB endpoint of the midband region.

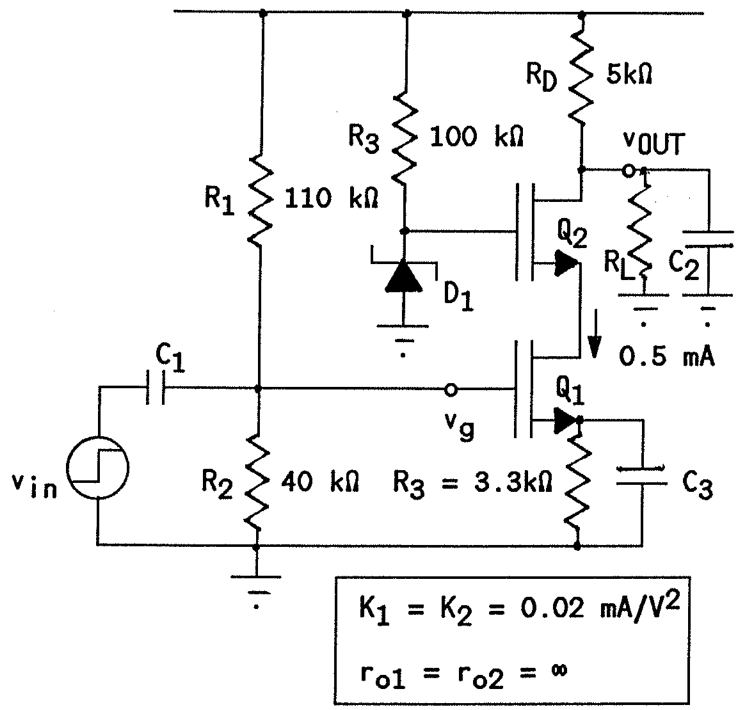
d) Determine the high-frequency -3-dB endpoint of the midband region.

9.11 The MOSFET pulse circuit shown below is driven by a step function. The MOSFET is not driven out of the constant current region by v_{in}.

a) Find the value of C_1 such that the incremental gate voltage v_g falls ("droops") with exponential time constant τ_1 = 1 ms. To what voltage will C_1 eventually be charged?

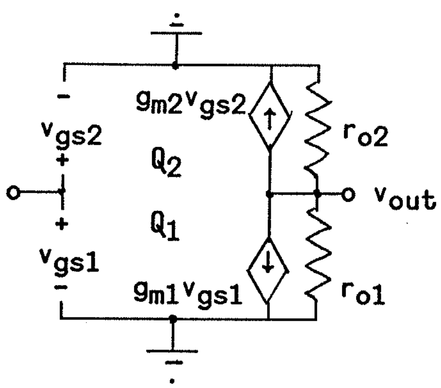
b) Find the value of C_2 such that v_{OUT} rises with exponential time constant τ_2 = 1 μs when R_L = 10 kΩ.

c) Find C_3 such that the source terminal of Q_1 is held at incremental ground over the time span of τ_2 and τ_1. These latter time constants define the pulse rise and fall time.



9.12 An amplifier has two high-frequency time constants τ_1 = 500 ns and τ_2 = 250 ns. What is the value of ω_H, the high-frequency endpoint to its midband region?

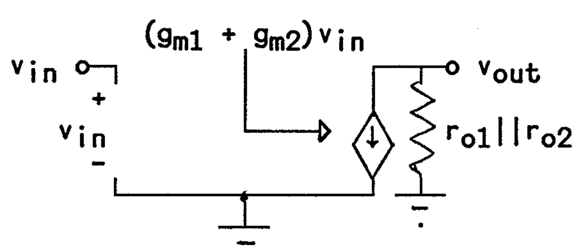
8.14 The small-signal model of the circuit is shown below. The input source v_{in} is connected to the gates of both Q_1 and Q_2. The incremental output resistances r_{o1} and r_{o2} must be included in the model because no smaller external resistors appear in parallel with them.



For this circuit $v_{gs1} = v_{gs2} = v_{in}$. The total resistance between v_{out} and ground becomes $r_{o1}||r_{o2}$. The current pulled up from ground through this parallel combination is equal to

$$g_{m1}v_{gs1} + g_{m2}v_{gs2} = (g_{m1} + g_{m2})v_{in}$$

The above circuit can thus be simplified by the following equivalent model:



From this simplified version of the circuit, the output is seen to be

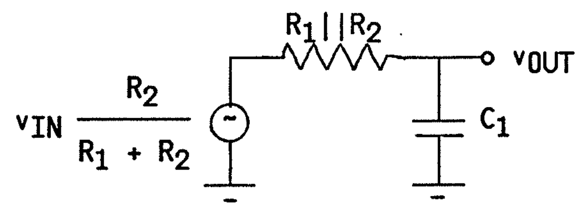
$$v_{out} = -(g_{m1} + g_{m2})(r_{o1}||r_{o2})v_{in}$$

Chapter 9

9.1 At very low frequencies (i.e., at dc), the capacitor behaves as an open circuit and the circuit reduces to a simple voltage divider. Choosing resistors in the ratio $R_2 = 2R_1$ will yield an attenuation factor of

$$\frac{v_{OUT}}{v_{IN}} = \frac{R_2}{R_1 + R_2} = \frac{2R_1}{R_1 + 2R_1} = \frac{2}{3}$$

One choice, for example, might be $R_1 = 10$ kΩ; $R_2 = 20$ kΩ. At higher frequencies, the impedance of the capacitor will begin to short out the voltage across R_2, forcing v_{OUT} to be reduced toward zero (v_{OUT} will reach zero exactly only at "infinite" frequency). The pole frequency at which this attenuation begins to take place can be computed by forming the Thevenin equivalent of the circuit seen by the capacitor:



From the above circuit, we note that

$$f_p = \frac{\omega_p}{2\pi} = \frac{1}{2\pi(R_1||R_2)C_1}$$

Setting this pole frequency to 1 kHz with the suggested resistor values would require a capacitor of value.

$$C_1 = \frac{1}{2\pi(R_1||R_2)f_p}$$

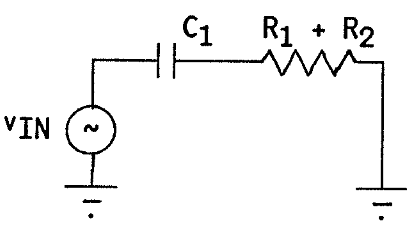
$$= \frac{1}{2\pi(6.67\ \text{k}\Omega)(1\ \text{kHz})} = 0.024\ \mu\text{F}$$

9.2 At high frequencies, the impedance of the capacitor becomes small enough such that C_1 begins to look like a short circuit. Under such a condition, R_1 and R_2 form a simple voltage divider, so that

$$\frac{v_{OUT}}{v_{IN}} = \frac{R_2}{R_1 + R_2}$$

Setting this ratio to 1/2 requires that $R_1 = R_2$, e.g. $R_1 = R_2 = 1\ \text{k}\Omega$.

The frequency at which the impedance of C_1 begins to become negligible compared to R_1 and R_2 can be found by examining the Thevenin resistance seen by C_1:



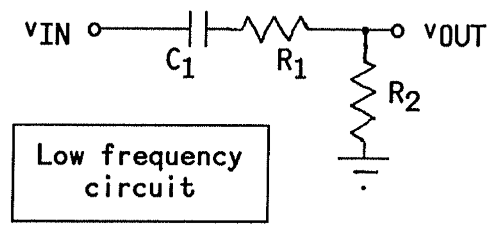
From this representation of the circuit, it is evident that

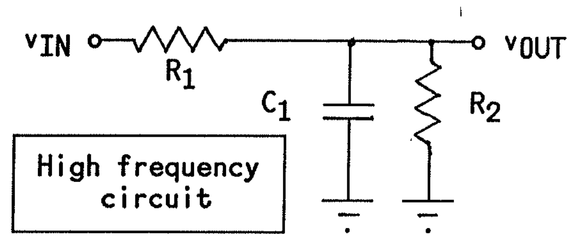
$$f_p = \frac{1}{2\pi(R_1 + R_2)C_1}$$

For the suggested values of R_1 and R_2, the required value of C_1 would be

$$\frac{1}{2\pi(R_1 + R_2)f_p} = \frac{1}{2\pi(2\ \text{k}\Omega)(10\ \text{kHz})} \simeq 8\ \text{nF}$$

9.3 The frequency range between 100 Hz and 1 MHz constitutes the "midband" region. In the circuit shown, C_1 is a low-frequency capacitor that behaves as a short circuit in-and-above the "midband" region. Similarly, C_2 is a high-frequency capacitor that behaves as an open circuit in-and-below the midband region. The poles associated with each capacitor can be found by invoking the dominant pole principle using the following simplified circuits:





From the above circuits, the required capacitor values can be found. In the low-frequency circuit, ($C_2 \equiv$ open), C_1 sees R_1 and R_2 in series. The low-frequency pole f_L of the circuit can be set to 100 Hz by choosing

$$C_1 = \frac{1}{2\pi(10\ \text{k}\Omega + 1\ \text{M}\Omega)(100\ \text{Hz})} \simeq 1.6\ \text{nF}$$

In the high-frequency circuit ($C_1 \equiv$ short), C_2 sees an equivalent Thevenin resistance of $R_1 || R_2 = 9.9\ \text{k}\Omega$. The high-frequency pole f_H of the circuit can be set to 1 MHz by choosing

$$C_2 = \frac{1}{2\pi(9.9\ \text{k}\Omega)(1\ \text{MHz})} \simeq 16\ \text{pF}$$

Note that $C_1 \gg C_2$, as suggested in the problem statement. In this case, $C_1/C_2 = 100$.

In the midband ($C_1 \equiv$ short, $C_2 \equiv$ open), the output becomes

$$v_{OUT} = \frac{R_2 v_{IN}}{R_1 + R_2} = \frac{1\ \text{M}\Omega}{1.01\ \text{M}\Omega} v_{IN} = 0.99 v_{IN}$$

9.4 The transfer function has zeros at $\omega = 10^2$, 10^3, and 10^5 rad/s; it has poles at $\omega = 10^4$, 10^5, 10^6, and 10^7 rad/s. The expression has no single factor of $j\omega$ in the numerator, hence the magnitude Bode plot begins at low frequencies with a horizontal slope. Evaluating the expression for $\omega \ll 10^2$, which is the lowest breakpoint (pole or zero) in the system function, yields $|V_{out}/V_{in}| \simeq 100 \equiv 40$ dB.

As the frequency of V_{in} is raised through the various breakpoints in the system function, the slope of the Bode plot will shift upward when a zero is encountered and downward when a pole is encountered. At $\omega = 10^2$, for example, the zero in the numerator shifts the slope upward by 20 dB/dec; the next zero at 10^3 repeats the upward shift for a total slope of 40 dB/dec.

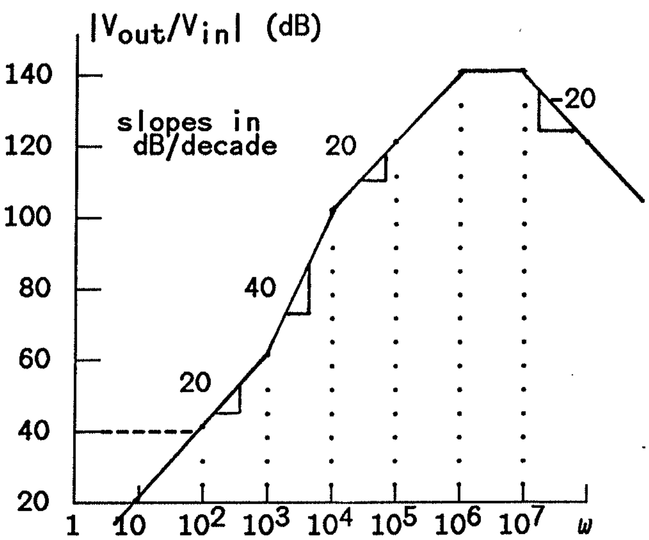
At $\omega = 10^4$, the first pole in the denominator is encountered, which shifts the slope downward by -20 dB/dec for a total of +20 dB/dec. At $\omega = 10^5$, a pole and zero are encountered simultaneously; their effects cancel. The remaining two poles at $\omega = 10^6$ and 10^7 each add successive downward shifts in the slope of -20 dB/dec, for a total slope at high frequencies (well above the highest breakpoint of 10^7 r/s) of -20 dB/dec.

The Bode plot is briefly horizontal between 10^6 and 10^7 rad/s. This portion of the plot represents the "midband" region over which the transfer function has its maximum height. The magnitude of V_{out}/V_{in} over this frequency range can be estimated by noting that each binomial of the form $(1 + j\omega/\omega_n)$ will contribute a factor of $1/\omega_n$ to $|V_{out}/V_{in}|$ in the midband. The one exception will be the binomial $(1 + j\omega/10^7)$, which will contribute a factor of 1 because its pole lies above the midband.

Thus $|V_{out}/V_{in}|$ in the midband will be approximately equal to

$$100\,\frac{(10^4)(10^5)(10^6)}{(10^2)(10^3)(10^5)} = 10^7 \equiv 140 \text{ dB}$$

Using these basic guidelines, the magnitude Bode plot of the system function can be drawn:

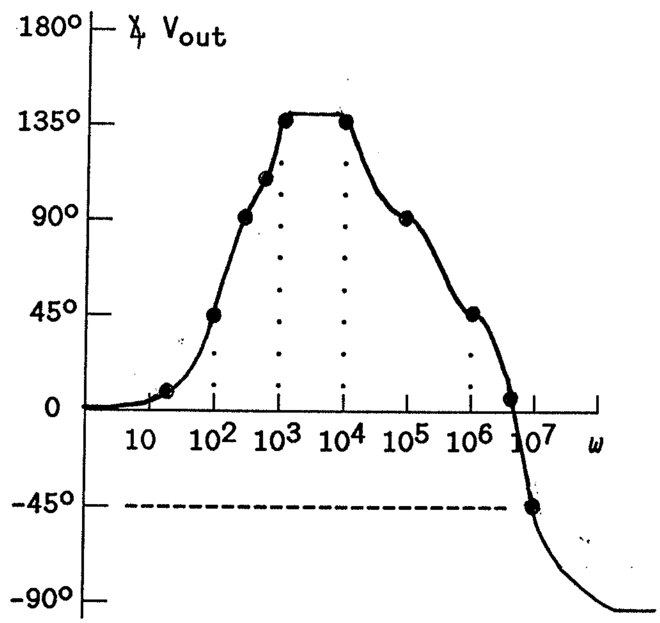


The angle plot can be formed in a similar manner. Below $\omega = 10^2$, where the magnitude plot has a horizontal slope, the angle is zero. As the frequency of V_{in} is raised through $\omega = 10^2$, the angle of V_{out}, measured relative to V_{in}, undergoes a shift of $+90^\circ$. Its phase angle just at $\omega = 10^2$ becomes $+45^\circ$. As ω passes through 10^3, the total phase shift changes toward 180°, reaching the value $90^\circ + 45^\circ = 135^\circ$ just at $\omega = 10^3$.

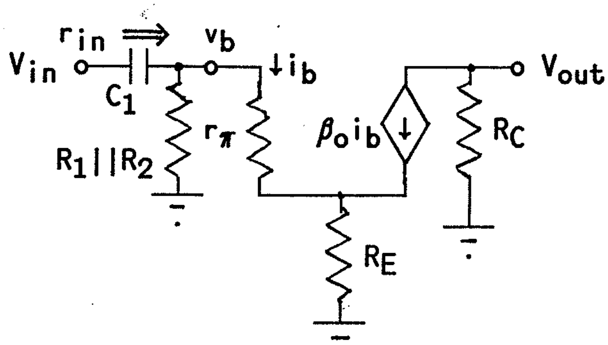
Above $\omega = 10^4$, the net phase shift of V_{out} is reduced again toward 90°; it is decreased toward zero as ω passes through 10^6 rad/sec. Note that the phase angle of V_{out} reaches zero over the region where the magnitude Bode plot is flat.

Beyond $\omega = 10^7$, the phase angle of V_{out} heads assymptotically toward -90^o for all further increases in the frequency of V_{in}, reaching the value -45^o just at $\omega = 10^7$.

Here is an approximate angle Bode plot for the given system function:



9.5 Here is the incremental representation of the amplifier, including C_1, with v_{IN} and v_{OUT} expressed in phasor notation:



The base current i_b can be expressed in terms of the node voltage v_b. Application of KVL yields

$$v_b = i_b r_\pi + (\beta_o + 1) i_b R_E$$

or

$$i_b = \frac{v_b}{r_\pi + (\beta_o + 1)R_E}$$

Note that the "effective" value of R_E, as seen from the base, becomes $(\beta_o + 1)R_E$. This concept was explored in Chapter 8.

The dependent source pulls a small-signal current $\beta_o i_b$ up from ground through R_C, so that $v_{out} = -\beta_o i_b R_C$. The ratio v_{out}/v_b, the "intrinsic" gain of the amplifier, thus becomes

$$\frac{v_{out}}{v_b} = \frac{-\beta_o R_C}{r_\pi + (\beta_o + 1)R_E}$$

Now find a frequency-dependent expression for V_b (phasor notation) in terms of V_{in}. The small-signal input resistance r_{in} seen by C_1 is $R_1||R_2||[r_\pi + (\beta_o + 1)R_E]$. Expressing the impedance of the capacitor as $1/j\omega C_1$ yields, via voltage division,

$$\frac{V_b}{V_{in}} = \frac{r_{in}}{r_{in} + 1/j\omega C_1} = \frac{j\omega r_{in} C_1}{1 + j\omega r_{in} C_1}$$

Combining the above equations lead to

$$\frac{V_{out}}{V_{in}} = \frac{-\beta_o R_C}{r_\pi + (\beta_o + 1)R_E} \quad \frac{j\omega r_{in} C_1}{1 + j\omega r_{in} C_1}$$

where $r_{in} = R_1||R_1||[r_\pi + (\beta_o + 1)R_E]$. The low-frequency midband endpoint occurs at the pole $\omega_L = 1/r_{in}C_1$.

Substitution of numbers results in

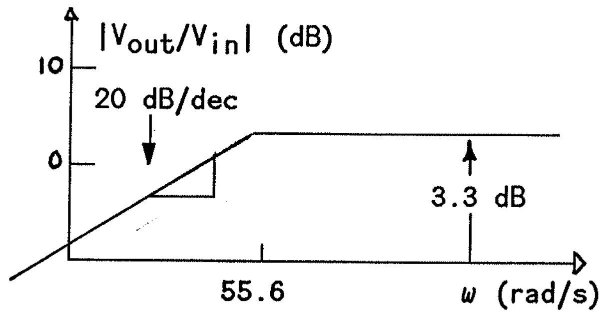
$r_\pi = \beta_o/g_m = \beta_o \eta V_T/I_C = 2.5\ \text{k}\Omega$

$r_{in} = 73\text{k}\Omega||27\text{k}\Omega||[2.5\text{k}\Omega + 101(2\text{k}\Omega)]$
$= 18\ \text{k}\Omega$

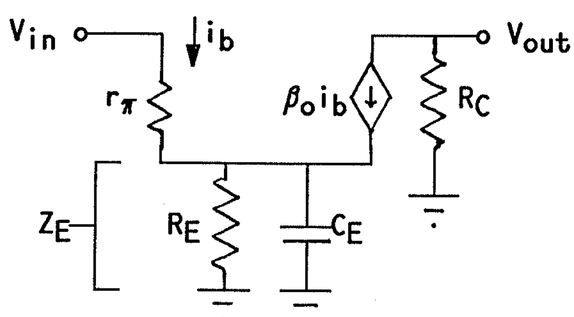
$\omega_L = [(18\ \text{k}\Omega)(1\ \mu\text{F})]^{-1} = 55.6\ \text{rad/s}$

$|v_{out}/v_b| = 100(3\text{k}\Omega)/(2.5\text{k}\Omega + 202\text{k}\Omega)$
$= 1.47 \equiv 3.3\ \text{dB}$

Here is a Bode plot of $|V_{out}/V_{in}|$:



9.6 Here is the incremental representation of the amplifier with v_{IN} and v_{OUT} expressed in phasor notation:



From KVL around the input loop,

$$I_b = \frac{V_{in}}{r_\pi + (\beta_o + 1)Z_E}$$

where

$Z_E = R_E||(1/j\omega C_E) = R_E/(1 + j\omega R_E C_E)$

V_{out} is equal to $-\beta_o I_b R_C$, so that

$$\frac{V_{out}}{V_{in}} = \frac{-\beta_o R_C}{r_\pi + (\beta_o + 1)R_E/(1 + j\omega R_E C_E)} = \frac{-\beta_o R_C(1 + j\omega R_E C_E)}{r_\pi + (\beta_o + 1)R_E + j\omega r_\pi R_E C_E}$$

Dividing numerator and denominator by $r_\pi + (\beta_o + 1)R_E$ and manipulating results in

$$\frac{V_{out}}{V_{in}} = \frac{-\beta_o R_C}{r_\pi + (\beta_o+1)R_E} \; \frac{(1 + j\omega R_E C_E)}{1 + j\omega(r_e||R_E)C_E}$$

where $r_e = r_\pi/(\beta_o + 1) \simeq 1/g_m \equiv \eta V_T/I_C$. This gain expression has a zero at

$$f_z = 1/2\pi R_E C_E = 79.6 \text{ Hz}$$

and a pole at

$$f_p = 1/2\pi(r_e||R_E)C_E = 6.45 \text{ kHz}$$

Note that $f_z < f_p$. At low frequencies ($f \ll f_z$), the binomial factors containing $j\omega$ in the above equation reduce to unity, and that the gain approaches the value

$$\frac{v_{out}}{v_{in}} = \frac{-\beta_o R_C}{r_\pi + (\beta_o + 1)R_E} \simeq \frac{-R_C}{R_E} = -1.5$$

The magnitude of this expression is equivalent to 3.5 dB.

In the midband region ($f \gg f_p$), the frequency-dependent binomials approach the values $j\omega R_E C_E$ and $j\omega(r_e||R_E)C_E$, respectively. The latter factor can be put in the form

$$j\omega\frac{[r_\pi/(\beta_o+1)]\,R_E}{r_\pi/(\beta_o+1) + R_E}C_E = j\omega\frac{r_\pi R_E\, C_E}{r_\pi + (\beta_o+1)R_E}$$

The gain expression for $f \gg f_p$ thus approaches the value

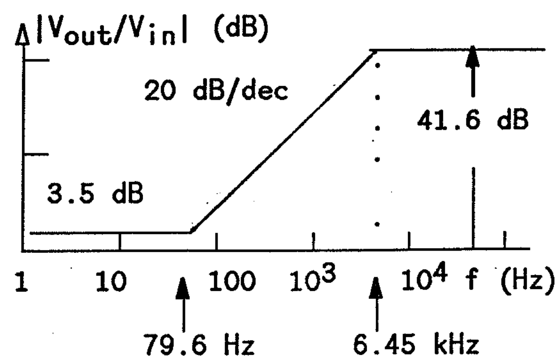
$$\frac{V_{out}}{V_{in}} = \frac{-\beta_o R_C}{r_\pi + (\beta_o + 1)R_E} \; \frac{j\omega R_E C_E}{j\omega(r_e||R_E)C_E}$$

$$= \frac{-\beta_o R_C}{r_\pi + (\beta_o+1)R_E} R_E C_E \frac{r_\pi + (\beta_o+1)R_E}{r_\pi R_E\, C_E}$$

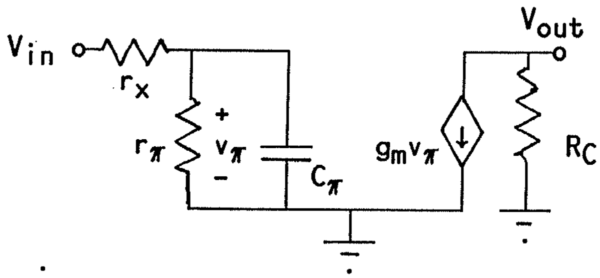
$$= \frac{-\beta_o R_C}{r_\pi} = -g_m R_C = \frac{-I_C R_C}{\eta V_T} = -120$$

The magnitude of this gain expression is equivalent to 41.6 dB.

A magnitude Bode plot of the amplifier gain, showing the low-frequency, midband, and transition regions, is drawn below.



9.7 In and above the midband region, the low-frequency capacitor C_E behaves as a short circuit. Here is an appropriate small-signal model for the circuit that includes r_x and C_π:



For this circuit, $v_{out} = -g_m v_\pi R_C$, as before. With r_x and C_π present, the phasor V_π becomes, via the impedance form of the voltage divider,

$$V_{in}\left[\frac{r_\pi||(1/j\omega C_\pi)}{r_x + r_\pi||(1/j\omega C_\pi)}\right]$$

Algebraic reduction of the above expression leads to

$$V_\pi = V_{in}\frac{r_\pi}{r_x + r_\pi + j\omega r_\pi r_x C_\pi}$$

$$= \underbrace{V_{in}\frac{r_\pi}{r_x + r_\pi}}_{V_{th}} \frac{1}{1 + j\omega \underbrace{(r_x||r_\pi)}_{r_{th}} C_\pi}$$

Note that the same result can be obtained by representing V_{in}, r_x, and r_π by a Thevenin equivalent circuit and computing the pole associated with r_{th} and C_π.

Substituting this expression for V_π into the expression for v_{out} yields

$$\frac{V_{out}}{V_{in}} = -g_m R_C \frac{r_\pi}{r_x + r_\pi}\frac{1}{1 + j\omega(r_x||r_\pi)C_\pi}$$

Note that at frequencies well below $\omega_H = [(r_x||r_\pi)C_\pi]^{-1}$, the gain becomes

$$\frac{v_{out}}{v_{in}} = -g_m R_C \frac{r_\pi}{r_x + r_\pi}$$

which is the midband gain expression with r_x included in the model. For the element values given, the midband gain becomes

$$\frac{v_{out}}{v_{in}} = \frac{-1\text{ mA}}{25\text{ mV}}\, 3\text{ k}\Omega\, \frac{2.5\text{ k}\Omega}{50\ \Omega + 2.5\text{ k}\Omega}$$

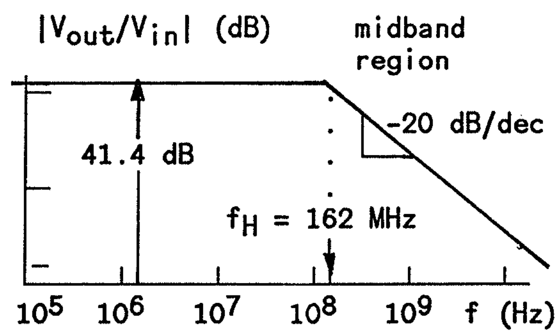
$$= -118 \equiv 41.4\text{ dB}$$

Because of the attenuation introduced by including r_x in the small-signal model, the computed gain is slightly smaller than the midband gain of -120 found in the previous problem.

The high-frequency pole occurs at

$$f_H = \frac{1}{2\pi(r_x||r_\pi)C_\pi} = \frac{1}{2\pi(49\ \Omega)(20\text{ pF})} =$$

162 MHz. Here is a magnitude Bode plot of the midband and high frequency regions of the circuit's response:



9.8 a) The "intrinsic" gain v_c/v_b (with R_L disconnected) will be equal to $-g_m R_C$. Setting this gain to 40 dB (magnitude of 100) with $I_C = 0.5$ mA requires R_C to be chosen so that

$$g_m R_C = \frac{I_C}{\eta V_T} R_C = 100$$

or

$$R_C = \frac{100\eta V_T}{I_C} = \frac{100\ (25\text{mV})}{0.5\text{ mA}} = 5\text{ k}\Omega$$

Note that this choice is tantamount to setting the voltage drop across R_C to 100 times the thermal voltage ηV_T. With this value of R_C, the node voltage V_C becomes $V_{CC} - I_C R_C = 6\ V - (0.5\ mA)(5\ k\Omega) = 3.5\ V$.

Now find the required value of R_E. V_B is to be biased at -2 V via the proper choice of R_1 and R_2. The node voltage V_E will thus be $V_B - V_f = -2.7$ V and the voltage drop across R_E equal to $V_E - V_{EE} = -2.7\ V - (-6\ V) = 3.3\ V$. For the condition $I_E \simeq I_C = 0.5$ mA, the chosen value of R_E must be $(3.3\ V)/(0.5\ mA) = 6.6\ k\Omega$.

The values of R_1 and R_2, which set V_B, are most easily chosen by neglecting I_B compared to the current through R_1. In such a case, V_B will be given approximately by the voltage divider relation, which can be written in the following form:

$$\frac{R_2}{R_1 + R_2} = \frac{V_B - V_{EE}}{V_{CC} - V_{EE}} = \frac{-2V - (-6V)}{6V - (-6V)} = \frac{1}{3}$$

In this equation, $V_{CC} - V_{EE}$ is the voltage drop across $R_1 + R_2$, and $V_B - V_{EE}$ is the drop across R_2 alone. With $R_2 = 10\ k\Omega$, the conditions of the equation can be met by choosing $R_1 = 20\ k\Omega$.

b) From the two-port small-signal amplifier model, the overall gain in the midband with v_s, R_S, and R_L connected becomes

$$\frac{r_{in}}{r_{in} + R_S} a_v \frac{R_L}{r_{out} + R_L}$$

where $r_{in} = R_1 || R_2 || r_\pi$ and $r_{out} = R_C$. These incremental resistance values are computed in the midband where C_1, C_2, and C_E behave as short circuits.

For this BJT, which is biased with $I_C = 0.5$ mA,

$$r_\pi = \frac{\beta_o}{g_m} = \frac{\beta_o \eta V_T}{I_C} = \frac{100(25\ mV)}{0.5\ mA} = 5\ k\Omega$$

so that $r_{in} = (20\ k\Omega)||(10\ k\Omega)||(5\ k\Omega) \simeq 2.9\ k\Omega$. Substitution of values for r_{in} and r_{out} into the expression for the overall midband gain yields

$$\frac{2.9\ k\Omega}{2.9k\Omega + 5k\Omega}(-100)\frac{5\ k\Omega}{5k\Omega + 5k\Omega} \simeq -18.2$$

The gain magnitude has a decibel value of $20 \log_{10}(18.2) = 25.2$ dB.

c) The external capacitors C_1, C_2, and C_E are all low-frequency capacitors; the internal capacitances C_π and C_μ are high-frequency capacitors.

d) The low- and high-frequency endpoints of the midband region can be computed by finding the poles of the dominant low- and high-frequency capacitors. To use the dominant pole computation method, find the Thevenin resistance seen by each capacitance with all other capacitances in their midband state (i.e., low-frequency capacitors set to short circuits and high-frequency capacitances set to open circuits). Under these conditions, for example, the capacitor C_1 sees a Thevenin resistance of $r_{th1} = R_S + R_1||R_2||r_\pi = 7.9\ k\Omega$. The computed pole of C_1 thus becomes

$$\omega_1 = (r_{th1}C_1)^{-1} = [(0.5\ \mu F)(7.9\ k\Omega)]^{-1} = 254\ rad/s$$

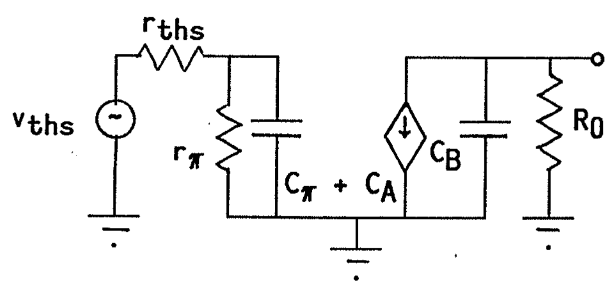
Similarly, the Thevenin resistance seen by C_2 under "midband" conditions is given by $r_{th2} = R_C + R_L = 5\ k\Omega + 5\ k\Omega = 10\ k\Omega$ so that

$$\omega_2 = (r_{th2}C_2)^{-1} = [(0.5\ \mu F)(10\ k\Omega)]^{-1} = 200\ rad/s$$

The Thevenin resistance seen by C_E with C_1 & C_2 set to short circuits is $R_E||(r_x + r_\pi + R_S||R_1||R_2)/(\beta_o + 1) = (6.6\ k\Omega)||(2\ \Omega + 5\ k\Omega + 2.9\ k\Omega)/101 \simeq 77\ \Omega$, so that $\omega_E = (r_{thE}C_E)^{-1} = 2600$ rad/s.

Of the three low-frequency capacitor poles, ω_E is dominant (highest computed pole frequency) and represents the low-frequency endpoint of the midband region. Note that r_{th1}, which was computed with C_E set to a short, must be re-computed with C_E set to an open if the true value of ω_1 is desired. The value of r_{th2} is not affected by the status of C_E, hence ω_2 is correct as computed.

The high-frequency end of the midband can be evaluated by setting all low-frequency capacitors to short circuits and using Miller's theorem on the incremental circuit model:



In the above circuit diagram $r_{ths} = R_S||R_1||R_2 + r_x \simeq 2.9$ kΩ and $R_0 = R_C||R_L = 2.5$ kΩ.

The equivalent Miller capacitances C_A and C_B are evaluated as follows:

$C_A = C_\mu[1 + g_m(R_C||R_L)] = 102$ pF and

$C_B = C_\mu[1 + (g_m R_C||R_L)^{-1}] \simeq 2$ pF

The capacitance C_π can be found by evaluating the expression

$g_m/2\pi f_T - C_\mu = (20\text{mA/V})/[2\pi(250\text{MHz})] - 2\text{pF} \simeq 11$ pF

The computed high-frequency poles thus become

$\omega_A = [(C_\pi + C_A)(r_\pi||r_{ths})]^{-1}$

$= [(113 \text{ pF})(5 \text{ k}\Omega||2.9 \text{ k}\Omega)]^{-1}$

$= 4.8 \times 10^6$ rad/s

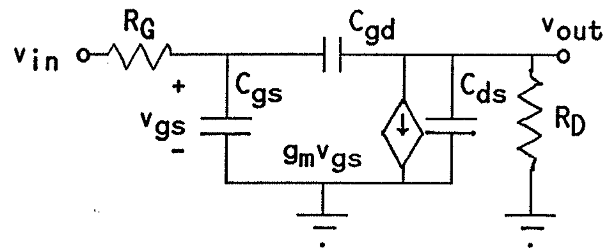
and

$\omega_B = (C_\mu R_0)^{-1}$

$= [(2\text{pF})(2.5\text{k}\Omega)]^{-1} = 2 \times 10^8$ rad/s

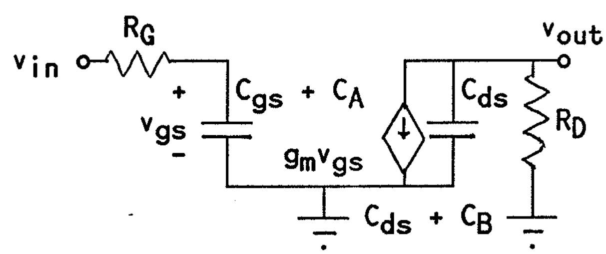
Of these two computed values, ω_A is the lowest and represents the high-frequency endpoint of the midband region. Note that the use of the Miller approximation is justified in this case because $\omega_A << \omega_B$.

9.9 At high frequencies, C_1 behaves as a short circuit. The remaining high-frequency capacitors can be incorporated into the small-signal representation of the circuit:



The incremental MOSFET output resistance r_o appears directly in parallel with R_D. If $r_o||R_D \simeq R_D$, the resistance r_o can be omitted from the model.

Applying Miller's theorem transforms the circuit into the following form:



where $C_A \simeq C_{gd}(1 + g_m R_D)$

and $C_B \simeq C_{gd}[1 + (g_m R_D)^{-1}] \simeq C_{gd}$

Note that the Miller approximation must be justified, as noted below, if this substitution is to be made.

The Thevenin resistances seen by these capacitances determine the high-frequency time constants. With v_{in} set to zero, the resistance seen by $C_{gs} + C_A$ becomes just R_G, so that

$$\tau_1 = R_G[C_{gs} + C_{gd}(1 + g_m R_D)]$$

Similarly, with v_{in} set to zero, the dependent source becomes an open circuit and the resistance seen by $C_{ds} + C_B$ becomes just R_D, so that

$$\tau_2 = R_D(C_{ds} + C_{gd})$$

These time constant expressions are valid if the Miller approximation, which requires that $\tau_1 \gg \tau_2$, is justified.

9.10 a) If $I_D = 0.5$ mA, then

$$V_S = I_D R_4 = (0.5 \text{ mA})(2 \text{ k}\Omega) = 1 \text{ V}$$

From the voltage divider relation:

$$V_G = V_{CC}\frac{R_2}{R_1 + R_2} \simeq 12\text{V}\,\frac{1 \text{ M}\Omega}{3 \text{ M}\Omega} = 4 \text{ V}$$

Thus $V_{GS} = V_G - V_S \simeq 4 \text{ V} - 1 \text{ V} = 3$ V.

If Q_1 operates in the constant-current region, then $I_D = K(V_{GS} - V_{TR})^2 = (2 \text{ mA/V}^2)(3 \text{ V} - 2.5 \text{ V})^2 \simeq 0.5$ mA

b) $g_m = 2(KI_D)^{1/2} = 2$ mA/V.

c) Use the dominant pole technique:

$$\omega_G = \frac{1}{C_G(R_G + R_1||R_2)}$$

$$= \frac{1}{2 \text{ nF}(50 \ \Omega + 1 \text{ M}\Omega||2 \text{ M}\Omega)} \simeq 750 \text{ rad/s}$$

$$\omega_E = \frac{1}{C_E[R_4||(1/g_m)]}$$

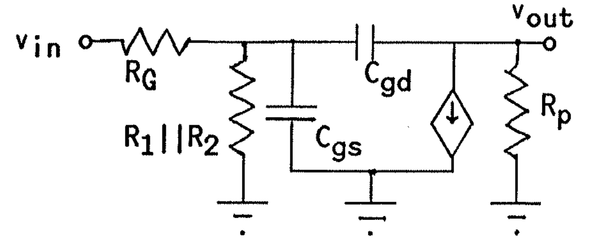
$$= \frac{1}{(5 \ \mu\text{F})(2 \text{ k}\Omega||500 \ \Omega)} = 500 \text{ rad/s}$$

$$\omega_C = \frac{1}{C_C(R_3 + R_L)} = \frac{1}{5 \text{ nF}(5 \text{ k}\Omega + 10 \text{ k}\Omega)}$$

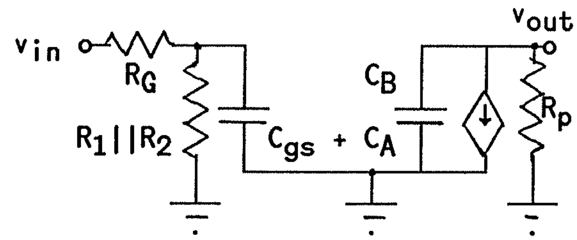
$$= 1.3 \times 10^4 \text{ rad/s}$$

The largest of these computed values, i.e. ω_C, represents the true low frequency pole. Note that R_G has little effect on the value of ω_G since $R_G \ll R_1||R_2$.

d) Here is a model of the circuit valid at high frequencies. The capacitors C_G, C_C, and C_E have been set to short circuits:



In this model, $R_p = R_3||R_L||r_o = 3.3$ kΩ. Applying Miller's theorem results in:



where

$$C_A = C_{gd}[1 + g_m R_p]$$

$$= (5 \text{ pF})[1 + (2 \text{ mA/V})(3.3 \text{ k}\Omega)]$$

$$\simeq 38 \text{ pF}$$

$$C_B \simeq C_{gd} = 5 \text{ pF}$$

Evaluate the tentative poles by computing the Thevenin resistance seen by $(C_{gs} + C_A)$ and C_B:

$$\omega_{H1} = [(C_{gs} + C_A)(R_1||R_2||R_G)]^{-1}$$

$$= [(40 \text{ pF})(50\Omega)]^{-1} = 500 \text{ Mrad/s}$$

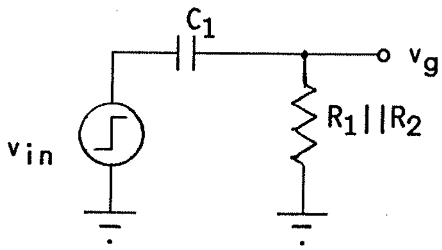
(Note that $R_1||R_2||R_G \simeq R_G = 50 \ \Omega$).

$$\omega_{H2} = (C_B R_p)^{-1}$$

$$= [(5\text{pF})(3.3\text{k}\Omega)]^{-1} \simeq 60.6 \text{ Mrad/s}$$

The lower of these two values, i.e. $\omega_{H2} = 60.6$ Mrad/s $\equiv 9.6$ MHz, represents the dominant, high frequency pole and the -3-dB high-frequency endpoint to the midband region.

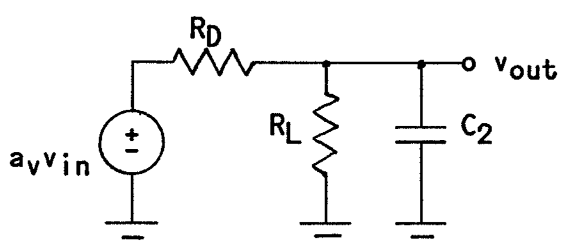
9.11 a) To the extent that the internal MOSFET capacitances can be ignored, the gate of Q_1 appears as an open circuit to v_{in}, R_1, R_2, and C_1. The rise time associated with C_1 can thus be evaluated by considering the following subcircuit:



where $R_1||R_2 = (110\ k\Omega)||(40\ k\Omega) = 29.3\ k\Omega$. Note that v_g will initially jump to the value of the step function v_{in} and will decay toward zero thereafter. The exponential time constant associated with this decay is $\tau_1 = (R_1||R_2)C_1$. Setting τ_1 to 1 ms requires that

$$C_1 = \frac{1\ ms}{29.3\ k\Omega} = 0.034\ \mu F \equiv 34\ nF$$

b) The incremental resistance appearing at the v_{OUT} terminal is equal to R_D. Thus C_2 sees the following incremental circuit, where a_v is the small-signal gain produced by Q_1 and Q_2:



In this case, v_{out} will rise exponentially toward the value

$$a_v v_{in} \frac{R_L}{R_L + R_D}$$

after the transition of the step function. The time constant associated with this transition is $\tau_2 = (R_D||R_L)C_2$. Setting τ_2 to 1 μs requires that

$$C_2 = \frac{1\ \mu s}{(5\ k\Omega)||(10\ k\Omega)} = 300\ pF$$

c) The incremental resistance seen by C_3 is $R_3||(1/g_{m1})$, where $g_{m1} = 2\sqrt{K_1 I_{D1}} = 2\sqrt{(0.02\ mA/V^2)(0.5\ mA)} = 0.2\ mA/V \Longrightarrow 1/g_{m1} = 5\ k\Omega$. If the source node of Q_1 is to be held at incremental ground over the time span of τ_1 and τ_2, the charging time constant of C_3 must be at least an order of magnitude longer than τ_1 and τ_2. In this case, $\tau_1 = 1$ ms, hence τ_3 must be at least, say 10 ms. This value of τ_3 can be realized if

$$C_3 = \frac{\tau_3}{R_3||(1/g_{m1})} = \frac{10\ ms}{3.3\ k\Omega||5\ k\Omega} \simeq 5\ \mu F$$

9.12 The time constant $\tau_2 = 500$ ns is determined by the product of a capacitance (say C_1) and the incremental Thevenin resistance r_{th1} seen by C_1. In the frequency domain, the pole associated with these elements becomes

$$\omega_1 = \frac{1}{r_{th1}C_1} = \frac{1}{500\ ns} = 2 \times 10^6\ rad/s$$

Similarly, for the elements associated with ω_2,

$$\omega_2 = \frac{1}{r_{th2}C_2} = \frac{1}{250\ ns} = 4 \times 10^6\ rad/s$$

Since these two poles are close in frequency, the value of ω_H must be found using superposition of poles. Specifically, ω_H will be equal to the "parallel combination" of ω_1 and ω_2:

$$\omega_H = \omega_1||\omega_2 = 1.33 \times 10^6\ rad/s$$

Note that this frequency is equivalent to

$$\frac{1}{\tau_1 + \tau_2} = \frac{1}{750\ ns} = 1.33 \times 10^6\ rad/s.$$