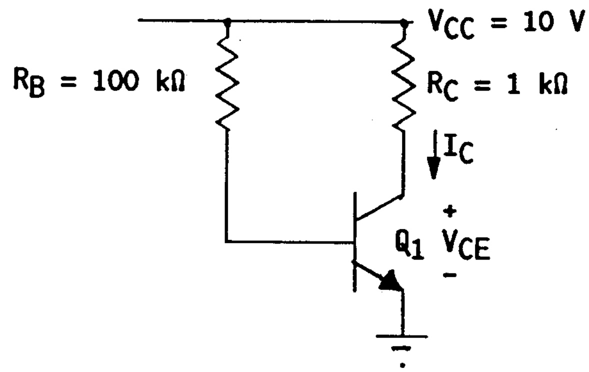
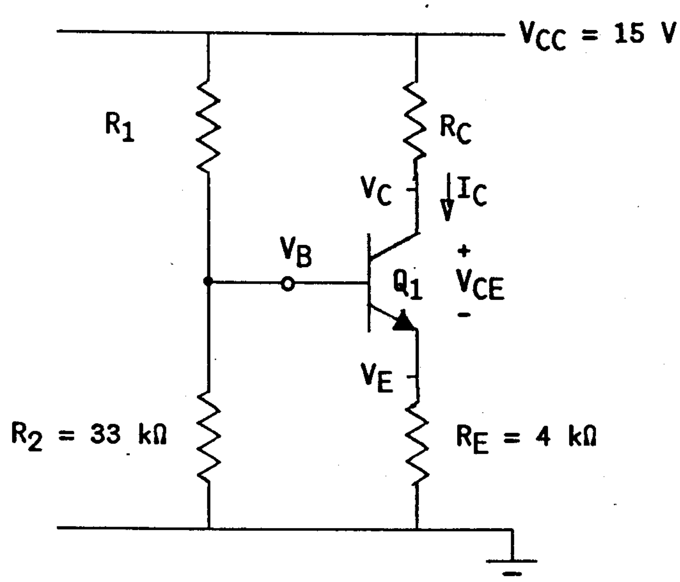
Chapter 7
Biasing

7.1 A BJT is biased in the fixed bias configuration shown below. What is the maximum value of β_F that will still allow Q_1 to be biased in the constant-current region? For this BJT, $V_f = 0.6$ V and $V_{sat} = 0.2$ V.

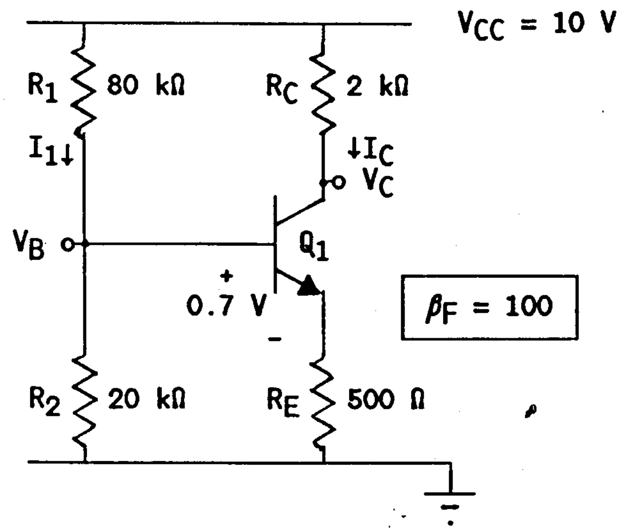


7.2 The operating point of the BJT in the feedback bias configuration shown below is to be set to $V_{CE} = 6$ V and $I_C = 1.7$ mA. β_F is large, but its value is not known. Choose appropriate values of R_1 and R_C.



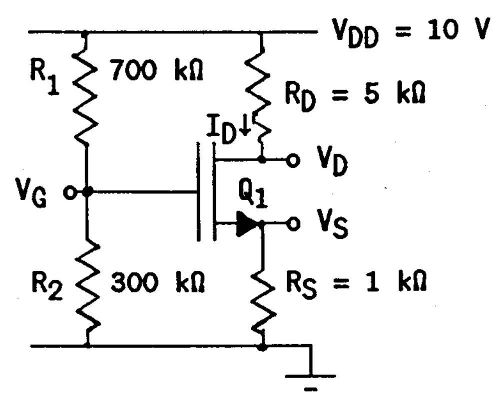
7.3 Repeat the previous problem if the BJT is replaced by an n-channel MOSFET with $K = 0.1$ mA/V^2 and $V_{TR} = 0.5$ V. In this case, let $R_2 = 1$ MΩ. The current through the device is to be set to $I_D = 1.7$ mA with $V_{DS} = 6$ V.

7.4 Consider the BJT bias circuit shown below. Find the dc values of I_C, I_1, V_B, and V_C. For this BJT, $\beta_F = 100$ and $V_f = 0.7$ V.

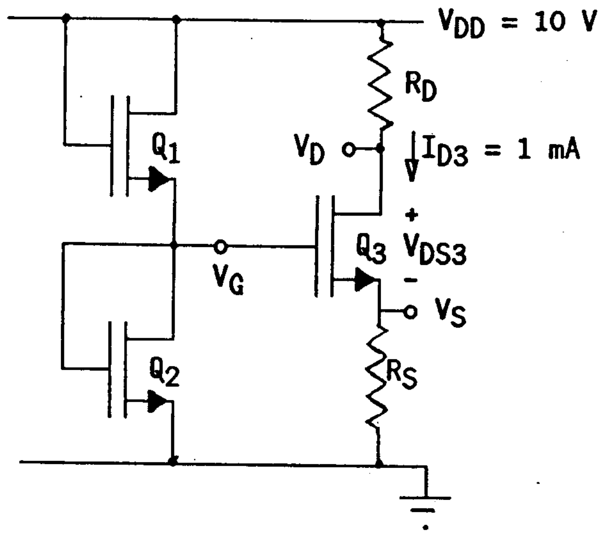


7.5 Repeat the previous problem for the case $\beta_F = 50$.

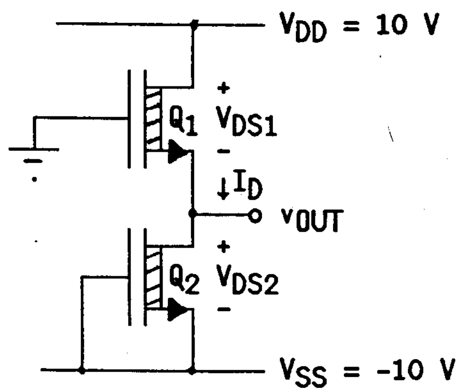
7.6 Find the dc operating point of the MOSFET circuit shown below if $K = 4\ mA/V^2$ and $V_{TR} = 1.5$ V.



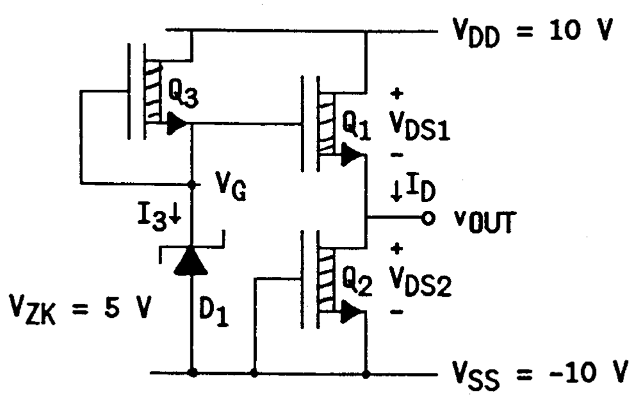
7.7 Consider the MOSFET circuit shown below. Choose R_D and R_S so that Q_3 operates in the constant-current region with $I_{D3} = 1$ mA. What value of V_{DS3} results from your design? The MOSFETs are all matched with $K = 2\ mA/V^2$ and $V_{TR} = 2$ V.



7.8 Consider the depletion-mode MOSFET circuit shown below. Both devices have parameters $K = 0.2\ mA/V^2$ and $V_{TR} = -2$ V. Find the bias current I_D and the voltages v_{OUT}, V_{DS1} and V_{DS2}.



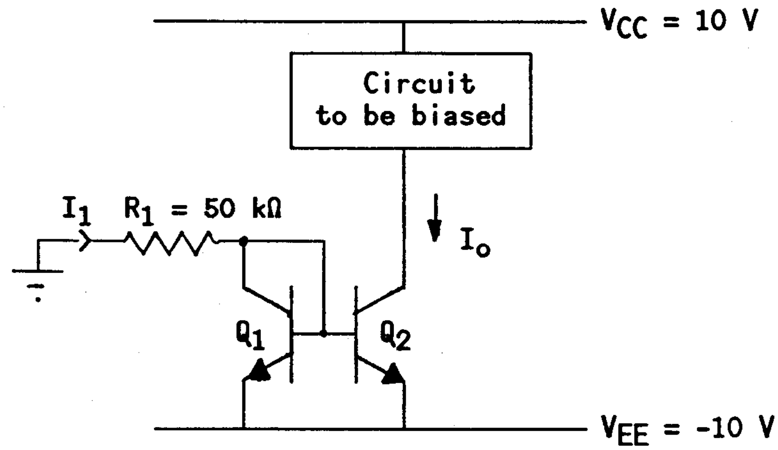
7.9 Repeat the previous problem for the MOSFET circuit shown below.



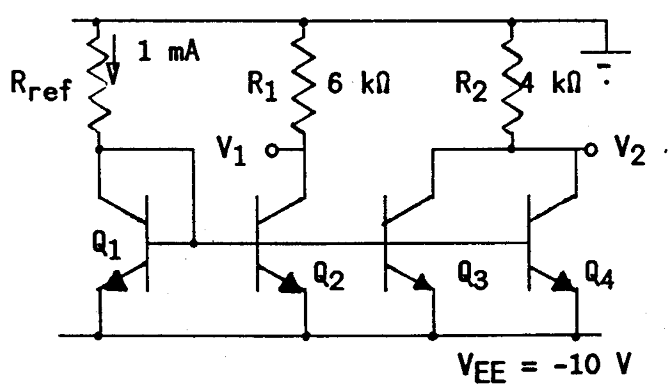
7.10 Consider the BJT current mirror bias circuit shown below. Q_1 and Q_2 are perfectly matched with $V_f \simeq 0.7$ V.

a) Find an approximate value for I_o.

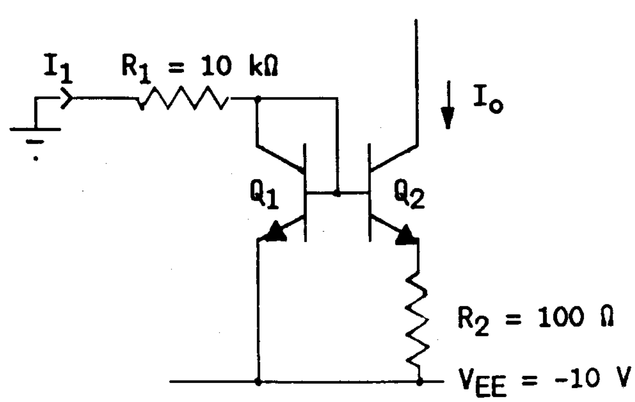
b) Compute a more accurate value for I_o at room temperature (ηV_T = 25 mV) if $I_{E0} = 10^{-12}$ A and $I_{B0} = 10^{-14}$ A. Do not assume a value for V_f.



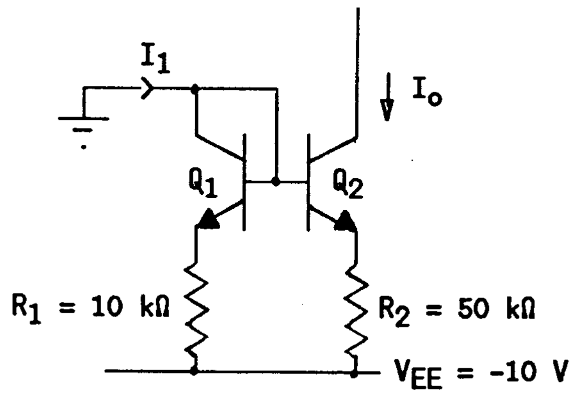
7.11 In the following current mirror all transistors are matched. Find R_{ref}, V_1, and V_2 if $V_f \simeq 0.7$ V. Neglect base currents with respect to collector currents.



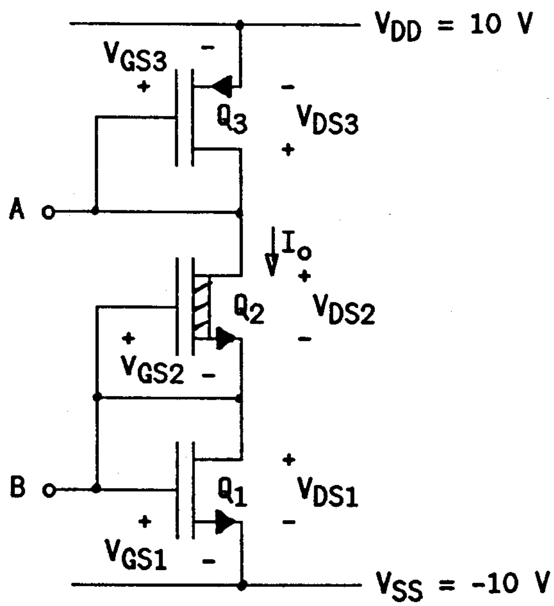
7.12 In the Widlar source shown below, find the bias current I_o at room temperature (ηV_T = 25 mV) if $V_f \simeq 0.7$ V



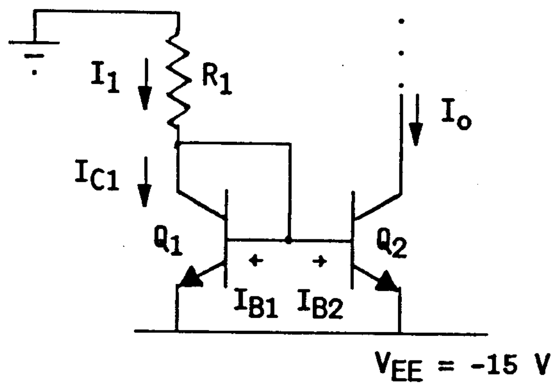
7.13 Find the approximate bias current I_o in the Wilson source shown below if $V_f \simeq 0.7$ V.



7.14 The hybrid CMOS circuit shown below contains both an enhancement-mode and a depletion-mode n-channel device and a p-channel pullup load. If $K_1 = K_3 = 1$ mA/V^2, $K_2 = 4$ mA/V^2, and $|V_{TR}| = 1$ V for all transistors, find the bias current I_o and the node voltages V_A and V_B. Ignore the "body" effect.



7.15 The transistors in the following circuit are biased with $\beta_F = 10$. For the base-emitter junction of each device, $V_f = 0.7$ V.



a) Find the required value of the reference current I_1 so that the mirrored current I_o becomes 5 mA.

b) The circuit functions poorly because I_1 includes the large base current components required of Q_1 and Q_2. Using one extra transistor, design a modified current mirror that will improve this situation.

Solutions

Chapter 7

7.1 For the bias configuration shown,

$$I_B = \frac{V_{CC} - V_f}{R_B} = \frac{10\text{ V} - 0.6\text{ V}}{100\text{ k}\Omega} = 94\ \mu\text{A}$$

The current I_C will be equal to $\beta_F I_B$ when the BJT operates in the constant current region. As β_F increases, I_C will increase, reducing V_{CE}. The BJT will enter saturation when V_{CE} is reduced to V_{sat}. This point will be reached when

$$I_C = \frac{V_{CC} - V_{sat}}{R_C} = \frac{10\text{V} - 0.2\text{V}}{1\text{ k}\Omega} = 9.8\text{ mA}$$

The β_F responsible for creating this I_C and forcing the BJT into saturation is equal to

$$\frac{I_C}{I_B} = \frac{9.8\text{ mA}}{0.094\text{ mA}} \simeq 104$$

7.2 With β_F large, the emitter and collector currents will be approximately equal:

$$I_E = \frac{\beta_F + 1}{\beta_F} I_C \simeq I_C$$

If I_C is to equal 1.7 mA, the voltage V_E (relative to ground) must be set to $I_E R_E$ = (1.7 mA)(4 kΩ) = 6.8 V. The voltage V_B must be set to $V_E + V_f$. For $V_f \simeq 0.7$ V, V_B = 7.5 V.

If the base current is assumed negligible compared to the current through R_1 (an assumption that must be confirmed), the voltage divider can be used to approximate V_B:

$$V_B \simeq \frac{R_2}{R_1 + R_2} V_{CC}$$

This equation can be rearranged to yield

$$\frac{R_2}{R_1 + R_2} = \frac{V_B}{V_{CC}} = \frac{7.5\text{ V}}{15\text{ V}} = 0.5$$

This ratio will be realized for the values $R_1 = R_2$ = 33 kΩ. To the extent that i_B is neglected, the current through these resistors will be on the order of (15 V)/(66 kΩ) = 230 μA. This current will be at least five times greater than i_B for β_F greater than about 40.

With I_C set to 1.7 mA by the proper choice of R_1, the value of V_E will be 6.8 V, as discussed above. Setting V_{CE} to 6 V thus requires that V_C be set at 12.8 V, i.e. the voltage drop across R_C must be set to

$$V_{CC} - V_C = 15\text{ V} - 12.8\text{ V} = 2.2\text{ V}$$

With I_C = 1.7 mA, this voltage drop will be realized with

$$R_C = \frac{2.2\text{ V}}{1.7\text{ mA}} = 1.3\text{ k}\Omega$$

7.3 Assume the MOSFET operates in the constant-current region. This assumption must be confirmed later. If I_D is to equal 1.7 mA, the voltage V_{GS} must be set to

$$V_{GS} = \sqrt{\frac{I_D}{K}} + V_{TR} = \sqrt{\frac{1.7\ \text{mA}}{0.1\ \text{mA/V}^2}} + 0.5\ \text{V}$$

$\simeq$ 4.6 V. If I_D is set to the desired value, the voltage across R_E will be $I_D R_E$ = (1.7 mA) x (4 kΩ) = 6.8 V. Setting V_{GS} to 4.6 V thus requires a V_G (gate voltage to ground) of 6.8 V + 4.6 V = 11.4 V.

Applying the voltage divider relationship to R_1 and R_2 results in

$$V_G = \frac{R_2}{R_1 + R_2} V_{CC}$$

For this circuit, the voltage divider can be applied without approximation, since $I_G = 0$. The above equation can be rearranged, yielding

$$\frac{R_2}{R_1 + R_2} = \frac{V_G}{V_{CC}} = \frac{11.4\ \text{V}}{15\ \text{V}} = 0.76$$

or

$$R_2 = 0.76(R_1 + R_2)$$

$$(1 - 0.76)R_2 = 0.76\ R_1$$

$$R_1 = 0.316\ R_2 = 316\ \text{k}\Omega$$

Setting V_{DS} to 6 V requires that V_D be biased at 12.8 V, as in the previous problem. Thus the voltage drop across R_C must again be set to

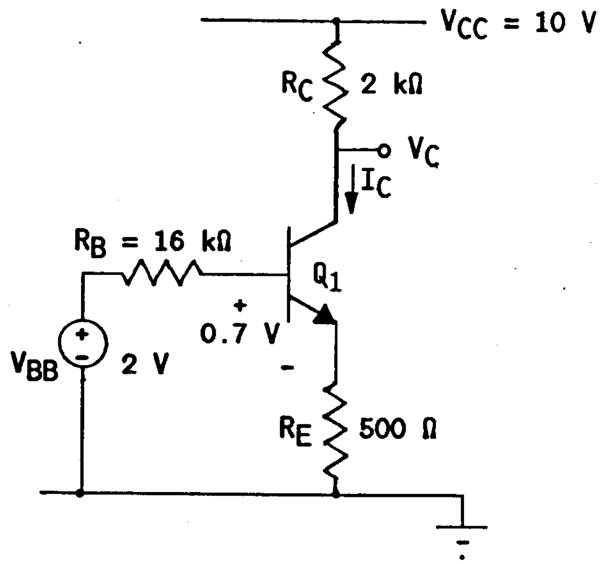
$$V_{CC} - V_D = 15\ \text{V} - 12.8\ \text{V} = 2.2\ \text{V}$$

With I_D = 1.7 mA, this voltage drop is realized with

$$R_C = \frac{2.2\ \text{V}}{1.7\ \text{mA}} = 1.3\ \text{k}\Omega$$

Note that the condition $v_{DS} > v_{GS} - V_{TR}$ = 4.1 V is satisfied in this circuit; the MOSFET is confirmed to operate in the constant current region.

7.4 First form a Thevenin equivalent of R_1, R_2, and V_{CC}. In this case, $R_B = R_1 || R_2$ = 16 kΩ and $V_{BB} = V_{CC}R_2/(R_1 + R_2)$ = 2 V:



From this representation of the circuit, we note, via KVL, that

$$I_B = \frac{V_{BB} - V_f}{R_B + (\beta_F + 1)R_E} = \frac{2\ \text{V} - 0.7\ \text{V}}{16\ \text{k}\Omega + (101)(500\ \Omega)} = 19.5\ \mu\text{A}$$

and

$$I_C = \beta_F I_B = 100(19.5\ \mu\text{A}) = 1.95\ \text{mA}$$

$$I_E = \frac{\beta_F + 1}{\beta_F} = \frac{101}{100}\ 1.95\ \text{mA} = 1.97\ \text{mA}$$

For this value of I_E, V_E becomes $I_E R_E$ = (1.97 mA)(500Ω) = 0.99 V $\simeq$ 1 V. Adding V_f to V_E yields V_B = 1.7 V.

From the original circuit, it follows that

$$I_1 = \frac{V_{CC} - V_B}{R_1} = \frac{10\ \text{V} - 1.7\ \text{V}}{80\ \text{k}\Omega} \simeq 104\ \mu\text{A}$$

Finally, $V_C = V_{CC} - I_C R_C$ = 10 V - (1.95 mA)(2 kΩ) = 6.1 V.

7.5 As before (see solution to the

previous problem)

$$I_B = \frac{V_{BB} - V_f}{R_B + (\beta_F + 1)R_E}$$

In this case $\beta_F = 50$, so that

$$I_B = \frac{2\ V - 0.7\ V}{16\ k\Omega + (51)(500\Omega)} = 31.3\ \mu A$$

and

$$I_C = \beta_F I_B = 50(31.3\ \mu A) = 1.57\ mA$$

$$I_E = \frac{\beta_F + 1}{\beta_F} = \frac{51}{50}\ 1.57\ mA = 1.6\ mA$$

For this value of I_E, V_E becomes $I_E R_E$ $= (1.6\ mA)(500\Omega) = 0.8\ V$. Adding V_f to V_E yields $V_B = 1.5\ V$.

From the original circuit, it follows that

$$I_1 = \frac{V_{CC} - V_B}{R_1} = \frac{10\ V - 1.5\ V}{80\ k\Omega} \simeq 106\ \mu A$$

and $V_C = V_{CC} - I_C R_C = 10\ V -$ $(1.6\ mA)(2\ k\Omega) = 6.8\ V$.

7.6 Here is an iterative solution method that is somewhat different from the solution methods given in the text. From the voltage divider,

$$V_G = V_{DD} \frac{R_2}{R_1 + R_2}$$

$$= (10\ V) \frac{300\ k\Omega}{700\ k\Omega + 300\ k\Omega} = 3\ V$$

This equation is exact because $I_G = 0$ into the gate of the MOSFET.

Assume that Q_1 operates in the constant-current region with

$$I_D = K(V_{GS} - V_{TR})^2$$

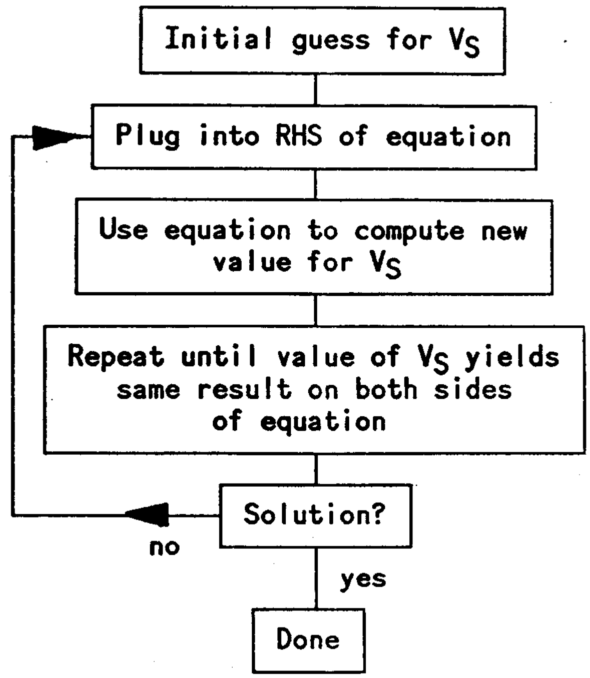
For the circuit shown, $I_D = V_S/R_S$, and $V_{GS} = V_G - V_S$, so that the constant-current region equation becomes

$$V_S/KR_S = (V_G - V_S - V_{TR})^2$$

or

$$V_S = V_G - V_{TR} - \sqrt{V_S/KR_S}$$

This equation cannot be solved in closed form but can be solved numerically via successive iteration:



Apply the method to this problem:

- Initial guess: $V_S = 0$, with
  $V_G - V_{TR} = 3\ V - 1.5\ V = 1.5\ V$
  $KR_S = (4\ mA/V^2)(1\ k\Omega) = 4\ V^{-1}$
- Here are the results of the successive iterations:

$$V_G - V_{TR} - \sqrt{V_S/KR_S} = 1.5V - \sqrt{0} = 1.5\ V$$

$$1.5V - \sqrt{1.5V/4V} = 0.89\ V$$

$$1.5V - \sqrt{0.89V/4V} = 1.03\ V$$

$$1.5V - \sqrt{1.03V/4V} = 0.99\ V$$

$$1.5V - \sqrt{0.99V/4V} = 1.0\ V$$

$$1.5V - \sqrt{1.0V/4V} = 1.0\ V$$

V_S repeats itself, indicating that the iteration is complete. For this value of V_S,

$$I_D = V_S/R_S = (1\ V)/(1\ k\Omega) = 1\ mA$$

$$V_{GS} = V_G - V_S = 3\ V - 1\ V = 2\ V$$

• Check the current equation:

$I_D = K(V_{GS} - V_{TR})^2$

$= (4\ mA/V^2)(2\ V - 1.5\ V)^2 = 1\ mA$

This answer is consistent with the results of the iteration.

For the computed value of I_D:

$V_D = V_{DD} - I_D R_D$

$= 10\ V - (1\ mA)(5\ k\Omega) = 5\ V$

$V_{DS} = V_{DD} - I_D(R_D + R_S)$

$= 10\ V - (1\ mA)(5\ k\Omega + 1\ k\Omega) = 4\ V$

Note that this value of V_{DS} exceeds $V_{GS} - V_{TR} = 1.5\ V$; the initial assumption of constant-current region operation is correct.

7.7 If I_{D3} is to equal 1 mA, then V_{GS3} must be set to the value

$$\sqrt{\frac{I_{D3}}{K_3}} + V_{TR} = \sqrt{\frac{1\ mA}{2\ mA/V^2}} + 2\ V = 2.7\ V$$

The matched MOSFETs Q_1 and Q_2 will produce a voltage $V_G = V_{DD}/2 = 5\ V$ at the gate of Q_3. The source voltage V_S of Q_3 must thus be set to 2.3 V, so that V_{GS3} will equal 2.7 V. With I_{D3} at 1 mA, V_S will be set to 2.3 V if $R_S = 2.3\ k\Omega$.

The value of R_D can be anything as long as Q_3 is not forced out of the constant-current region. The minimum value of V_{DS3} for constant-current region operation is

$V_{GS3} - V_{TR} = 2.7\ V - 2\ V = 0.7\ V$

With V_S at 2.3 V, the drain voltage V_D could be set as low as 3 V. This condition would occur if R_D were as large as

$$\frac{V_{DD} - V_D}{I_{D3}} = \frac{10\ V - 3\ V}{1\ mA} = 7\ k\Omega$$

7.8 The voltage sum $V_{DS1} + V_{DS2} = V_{DD} - V_{SS}$ is large, hence we can assume Q_1 and Q_2 to operate in the constant-current region. This assumption will be confirmed later.

With $V_{GS2} = 0$, the current I_D becomes $I_D = K_2(0 - V_{TR2})^2 = (0.2\ mA/V^2)(-2\ V)^2 = 0.8\ mA$. If this current is to flow through the matched device Q_1, then V_{GS1} must equal zero also. Hence $V_{OUT} = 0$ and

$V_{DS1} = V_{DD} = 10\ V$

$V_{DS2} = 0 - V_{SS}$

$= 0 - (-10\ V) = 10\ V$

For both devices, the condition $V_{DS} > V_{GS} - V_{TR} = 0 -(-2\ V) = 2\ V$ is met; Q_1 and Q_2 are confirmed to operate in the constant-current region.

7.9 See the solution to the previous problem. In this circuit, $V_{GS2} = 0$ also, so that I_D is again set to $K_2(0 - V_{TR2})^2 = 0.8$ mA and V_{GS1} is set to zero. With $V_{GS3} = 0$, the current I_3 through Q_3 is set to 0.8 mA; I_3 flows as reverse breakdown current through D_1.

With the zener well-established in reverse breakdown, V_G becomes

$V_{SS} + V_{ZK} = -10\ V + 5\ V = -5\ V$

so that $V_{OUT} = V_G - V_{GS1} = -5\ V$.

Finally, $V_{DS1} = V_{DD} - V_{OUT}$

$= 10\ V - (-5\ V) = 15\ V$

and $V_{DS2} = V_{OUT} - V_{SS}$

$= -5\ V -(-10\ V) = 5\ V$

All devices can be confirmed to operate in the constant-current region.

7.10 For $V_f = 0.7\ V$, KVL yields

$$I_1 = \frac{0 - V_f - (-V_{EE})}{R_1} = \frac{9.3\ V}{50\ k\Omega} = 186\ \mu A$$

Since V_{BE1} and V_{BE2} are identical, Q_1 and Q_2 will have the same emitter current. To the extent that base currents can be neglected compared to emitter currents, the collector current of Q_1 will be equal to I_1, so that $I_o \simeq 186\ \mu A$.

b) A more accurate value for I_1 can be obtained using the iterative numerical technique. In this case, I_1 is given exactly by

$$I_1 = \frac{-V_{EE} - V_{BE1}}{R_1} \qquad (7.1)$$

where, via the v-i equation for Q_1,

$$I_{E1} = I_{E01}(e^{v_{BE1}/\eta V_T} - 1)$$

and

$$I_{B2} = I_{B02}(e^{v_{BE2}/\eta V_T} - 1)$$

Since $v_{BE1} = v_{BE2}$, and given that that $I_1 = I_{E1} + I_{B2}$ (note that I_{E1} includes the current I_{B1}), the above becomes

$$I_1 = (I_{E01} + I_{B02})(e^{v_{BE1}/\eta V_T} - 1)$$

If the factor of -1 is neglected, this exponential equation can be inverted, yielding

$$V_{BE1} \simeq \eta V_T \ln \frac{I_1}{I_{E01} + I_{B02}} \qquad (7.2)$$

where $I_{E01} + I_{B02} = 10^{-12}$ A + 10^{-14} A = 1.01×10^{-12} A. Use of the iterative technique involves substituting an initial estimate of I_1 into Eqn. (7.2) to obtain a value for V_{BE1}. This V_{BE1} value is then substituted into Eqn. (7.1) to obtain an updated value for I_1. The procedure is repeated until the solution converges on a value for I_1.

Here is a table that shows the results of successive iterations. An initial guess of I_1 = 186 μA (the value of I_1 obtained in part a) yields an initial V_{BE1} of 0.4758 V. Thereafter,

From Eqn. (7.1)	From Eqn (7.2)
I_1 = 190.5 μA	V_{BE1} = 0.4764 V
= 190.5 μA	= 0.4764 V

The solution is seen to converge on the value I_1 = 190.5 μA after only two iterations.

The value of I_o can be found by noting that

$$I_1 = I_{C1} + I_{B1} + I_{B2} = I_{E1} + I_{B2}$$

Given $I_{E2} = I_{E1}$, this equation becomes

$$I_1 = I_{E1}\left[1 + \frac{1}{\beta_F + 1}\right] = I_{E1}\frac{\beta_F + 2}{\beta_F + 1}$$

Finally,

$$I_o = I_{C2} = \frac{\beta_F}{\beta_F + 1} I_{E2} = \frac{\beta_F}{\beta_F + 1} I_{E1}$$

$$= \frac{\beta_F}{\beta_F + 1}\,\frac{\beta_F + 1}{\beta_F + 2} I_1 = \frac{\beta_F}{\beta_F + 2} I_1$$

From the values of I_{E0} and I_{B0}, we note that

$$\frac{I_{E0}}{I_{B0}} = \frac{i_E}{i_B} = \beta_F + 1 \Longrightarrow \beta_F = \frac{I_{E0}}{I_{B0}} - 1$$

= 100 - 1 = 99. For this value of β_F,

$$I_o = \frac{99}{101}\ 190.5\ \mu A = 186.7\ \mu A$$

7.11 If base currents are neglected with respect to collector currents, an expression for the current through Q_1 can be found from KVL:

$$I_{ref}R_{ref} + V_f + V_{EE} = 0$$

or

$$I_{ref} = \frac{-V_{EE} - V_f}{R_{ref}}$$

Setting I_{ref} to 1 mA requires that

$$R_{ref} = \frac{-V_{EE} - V_f}{I_{ref}} = \frac{-(-10\ V) - 0.7\ V}{1\ mA}$$

$= 9.3\ k\Omega$. This current is replicated in Q_2, Q_3, and Q_4, so that

$$V_1 = 0 - I_{C2}R_1 = 0 - (1mA)(6k\Omega) = -6\ V$$

$$V_2 = 0 - (I_{C3} + I_{C4})R_2$$
$$= 0 - (2\ mA)(4\ k\Omega) = -8\ V.$$

Note that V_{CE2}, V_{CE3}, and V_{CE4} are all greater than V_{sat} (about 0.2 V to 0.3 V). Hence Q_2, Q_3, and Q_4 operate in the constant-current region.

7.12 Neglect base currents with respect to collector and emitter currents. The current I_1 through Q_1 is given by

$$I_1 = \frac{0 - V_f - V_{EE}}{R_1} = \frac{9.3\ V}{10\ k\Omega} = 0.93\ mA$$

As discussed in the text, I_o can be found by noting that

$$I_{E1} = I_{E0}(e^{V_{BE1}/\eta V_T} - 1) \simeq I_{E0}e^{V_{BE1}/\eta V_T}$$

and

$$I_{E2} = I_{E0}(e^{V_{BE2}/\eta V_T} - 1) \simeq I_{E0}e^{V_{BE2}/\eta V_T}$$

where $I_{E2} \simeq I_o$, $I_{E1} \simeq I_1$, and

$$V_{BE1} = V_{BE2} + I_oR_2$$

Taking the logarithm of the exponential equations yields expressions for V_{BE1} and V_{BE2}. Substituting these expressions into the above equation results in

$$\eta V_T \ln \frac{I_{E1}}{I_{E0}} = \eta V_T \ln \frac{I_{E2}}{I_{E0}} + I_oR_2$$

or

$$I_oR_2 = \eta V_T \ln \frac{I_{E1}}{I_{E2}} = \eta V_T \ln \frac{I_1}{I_o}$$

This last equation can be solved by iteration, with $I_1 = 0.93$ mA and $R_2 = 100\ \Omega$, yielding $I_o = 0.29$ mA. A check shows this answer to be correct:

$$I_oR_2 = (0.29\ mA)(100\ \Omega) = 29\ mV$$

$$\eta V_T \ln (I_1/I_o) = 25\ mV \ln (0.93/0.29) = 29\ mV.$$

7.13 The current I_1 is given by:

$$I_1 = \frac{0 - V_f - V_{EE}}{R_1} = \frac{9.3\ V}{10\ k\Omega} = 0.93\ mA$$

As discussed in the text, the emitter currents in Q_1 and Q_2 are related to V_{BE1} and V_{BE2} by the equations

$$I_{E1} = I_{E0}(e^{V_{BE1}/\eta V_T} - 1) \simeq I_{E0}e^{V_{BE1}/\eta V_T}$$

$$I_{E2} = I_{E0}(e^{V_{BE2}/\eta V_T} - 1) \simeq I_{E0}e^{V_{BE2}/\eta V_T}$$

If base currents are neglected with respect to emitter and collector currents, then KVL yields

$$V_{BE1} + I_1R_1 = V_{BE2} + I_oR_2$$

where $I_{E1} \simeq I_1$ and $I_{E2} \simeq I_o$.

Taking the logarithm of the exponential equations yields expressions for V_{BE1} and V_{BE2}. Substituting these expressions into the above equation results in

$$\eta V_T \ln \frac{I_{E1}}{I_{E0}} + I_1R_1 = \eta V_T \ln \frac{I_{E2}}{I_{E0}} + I_oR_2$$

or

$$I_oR_2 = \eta V_T \ln \frac{I_{E1}}{I_{E2}} + I_1R_1$$

$$= \eta V_T \ln \frac{I_1}{I_o} + I_1R_1$$

This last equation can be solved by iteration, with $I_1 = 0.93$ mA, $R_1 = 10\ k\Omega$, and $R_2 = 50\ k\Omega$, to yield $I_o = 186.8\ \mu A$. A quick check shows this answer to be correct:

$$I_oR_2 = (186.8\ \mu A)(50\ k\Omega) = 9.34\ V$$

$$\eta V_T \ln(I_1/I_o) = (25\ mV) \ln(930/186.8) = 0.04\ V.$$

$$I_1R_1 = (0.93\ mA)(10\ k\Omega) = 9.3\ V$$

$$9.34\ V = 0.04\ V + 9.3\ V$$

Note that a good approximation to this result can be obtained by assuming $V_{BE1} \simeq V_{BE2}$ and equating the voltage drops across R_1 and R_2: $I_1R_1 = I_oR_2 \implies$

$$I_o = \frac{R_1}{R_2}I_1 = \frac{10\ k\Omega}{50\ k\Omega}0.93\ mA = 186\ \mu A$$

7.14 Begin by assuming Q_2 to operate in the constant current region. This assumption must be confirmed later. Devices Q_1 and Q_3, which are connected so that $V_{GS} = V_{DS}$, automatically operate in the constant current region. This condition follows because

$$V_{DS1} > V_{GS1} - V_{TR1}$$

is satisfied for Q_1 and

$$V_{DS3} < V_{GS3} - V_{TR3}$$

is satisfied for the p-channel device Q_3. Note that V_{DS3}, V_{GS3}, and V_{TR3} are negative for the p-channel Q_3.

If Q_2 operates in the constant current region, then I_o will be given by

$$I_o = K_2(V_{GS2} - V_{TR2})^2 = K_2(-V_{TR2})^2$$

where $V_{GS2} = 0$. For the parameter values given, ($V_{TR2} = -1$ V for an n-channel depletion-mode device):

$$I_o = (4 \text{ mA/V}^2)(1 \text{ V})^2 = 4 \text{ mA}$$

The current I_o also flows through Q_1:

$$I_o = K_1(V_{GS1} - V_{TR1})^2$$
$$\equiv K_1(V_{DS1} - V_{TR1})^2$$

Solving this equation for V_{DS1} yields

$$V_{DS1} = (I_o/K_1)^{1/2} + V_{TR1}$$

$$= \sqrt{4\text{mA}/(1 \text{ mA/V}^2)} + 1 \text{ V} = 3 \text{ V}$$

so that

$$V_B = V_{SS} + V_{DS1} = -10 \text{ V} + 3 \text{ V} = -7 \text{ V}$$

Similarly, the current I_o also flows through Q_3. The current I_o is equivalent to $-i_{D3}$; the latter is defined as positive into the drain of Q_3, so that

$$-I_o = -K_3(V_{GS3} - V_{TR3})^2$$
$$\equiv -K_3(V_{DS3} - V_{TR3})^2 \quad \Longrightarrow$$

$$V_{DS3} = (I_o/K_3)^{1/2} + V_{TR3}$$

$$= -\sqrt{4\text{mA}/(1 \text{ mA/V}^2)} - 1 \text{ V} = -3 \text{ V}$$

Note that the negative root has been chosen to compute V_{DS3}. Choosing the positive root would yield $V_{DS3} = V_{GS3} = 1$ V which would place the p-channel device Q_3 in cutoff.

For the above value of V_{DS3},

$V_A = V_{DD} + V_{DS3} = 10 \text{ V} - 3 \text{ V} = 7 \text{ V}$.

Finally, we note that

$V_{DS2} = V_A - V_B = 7 \text{ V} - (-7 \text{ V}) = 14 \text{ V}$

The condition

$$V_{DS2} > V_{GS2} - V_{TR2} = 1 \text{ V}$$

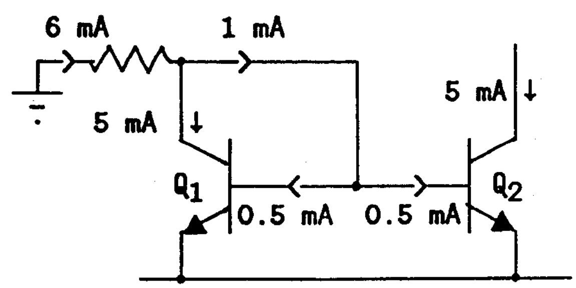
is satisfied; Q_2 is confirmed to operate in the constant-current region.

7.15 a) For $I_{C2} = I_o = 5$ mA, $I_{B2} = I_{C2}/\beta_F = (5 \text{ mA})/10 = 0.5$ mA. The two transistors have the same V_{BE}, hence they will have the same I_B, I_C, and I_E. It thus follows that

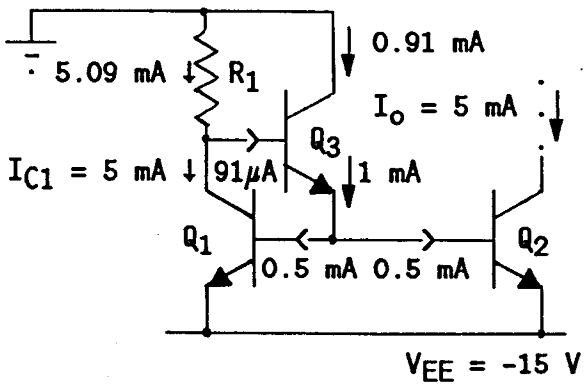
$$I_1 = I_{C1} + I_{B1} + I_{B2}$$
$$= 5 \text{ mA} + 0.5 \text{ mA} + 0.5 \text{ mA} = 6 \text{ mA}$$

This current can be set by choosing

$$R_1 = \frac{0 - V_f - V_{EE}}{I_1} = \frac{14.3 \text{ V}}{6 \text{ mA}} \simeq 2.4 \text{ k}\Omega$$



b) The following circuit configuration provides a buffer to the bases of Q_1 and Q_2.



For this configuration,

$$R_1 = \frac{0 - 2V_f - (-15\text{ V})}{5.09\text{ mA}} = 2.67\text{ k}\Omega$$

One V_f drop each is contributed by Q_1 and Q_3 to the above equation.