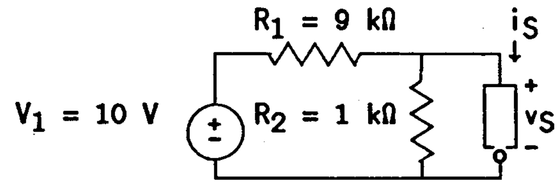
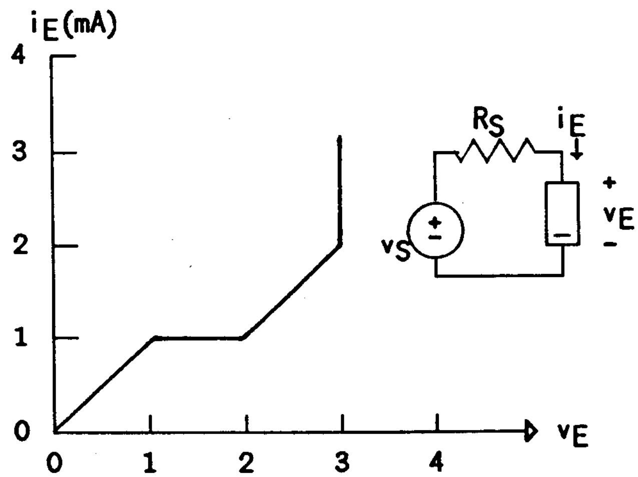
Chapter 3
Nonlinear Circuit Elements

3.1 A square-law device having parameters $A = 1\ mA/V^2$ and $V_{TR} = 2$ V is connected to the circuit shown below. Find its operating point.



3.2 The circuit shown above contains a square-law device, resistors, and voltage source. Find the value of V_1 that will cause a current i_S = 1 mA to flow through the square-law device.

3.3 A circuit element with the v-i characteristic shown below is connected to a variable voltage source in series with a 2-kΩ resistor. The source voltage is increased at the rate of 1 V per second, beginning with zero at t = 0. Plot the device voltage versus time.



3.4 The v-i equation of a PN junction diode is given by

$$i_D = I_S\ (e^{v_D/\eta V_T} - 1)$$

The diode passes 1 mA of current at room temperature when v_D = 0.7 V. The emission coefficient η for the diode is equal to 2.

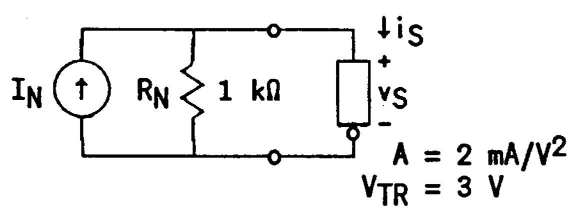
a) Find I_S for this diode.
b) Find the value of v_D that will double the diode current.
c) Find the value of v_D that will halve the diode current.
d) Find the value of v_D that will increase i_D by a factor of ten.

3.5 A PN junction diode is made with $I_S = 10^{-15}$ A and is operated at a temperature of 20 °C. Over the relevant current range, $\eta = 1$. For a diode made from this particular semiconductor, I_S increases by 15% for every °C rise in temperature. Calculate the voltage drop across the diode at 20 °C and 30 °C when i_D = 1 mA. Estimate the temperature coefficient of v_D versus T at this current level.

3.6 The voltage drop across a PN junction diode, measured at room temperature, is found to be 0.71 V at 5 mA and 0.764 V at 15 mA. Find the values of η and I_S for this diode.

3.7 A silicon PN junction diode has a temperature coefficient of -2 mV/°C. At 100 µA of current and at 20 °C, the diode drop is 0.70 V. What will the voltage drop be at 0 °C and at 100 °C?

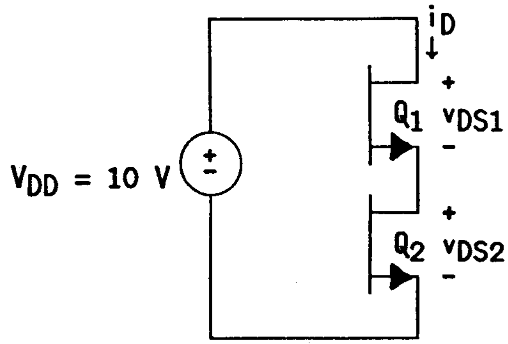
3.8 A square-law device is connected to a Norton circuit consisting of a current source I_N and 1 kΩ parallel resistor R_N, as shown below. The square-law device has parameters V_{TR} = 3 V and A = 2 mA/V^2. At what I_N will the current through the square-law device equal 1, 2, and 4 mA?



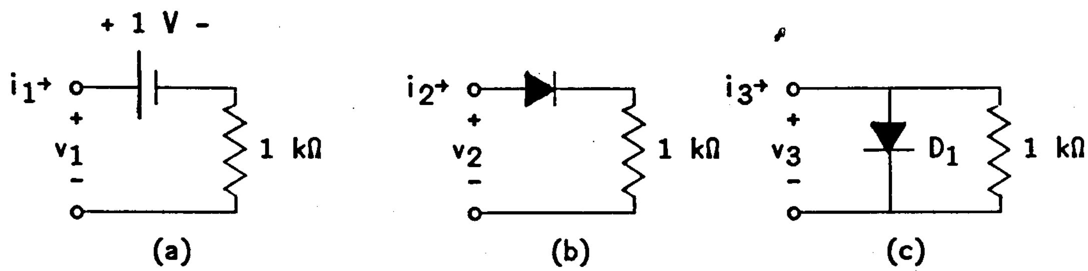
3.9 An FET channel has parameters I_{DSS} = 12 mA and V_A = 8 V. The device is connected to a Thevenin circuit consisting of a 10 V voltage source in series with a 1 kΩ resistor. Using graphical methods, find the current that flows through the FET channel.

3.10 Consider the FET channel and circuit described in the previous problem. Find the current through the FET channel by direct calculation.

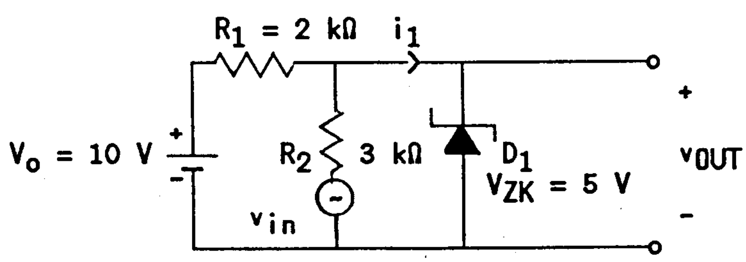
3.11 Two FET channels are connected in series with a 10-V voltage source, as shown below. The devices have parameters K_1 = 1 mA/V^2, K_2 = 0.5 mA/V^2, and $V_{A1} = V_{A2}$ = 2 V. Find the voltages v_{DS1} and v_{DS2}. Note that the same current i_D flows through both devices.



3.12 Plot the v-i characteristic of each of the following circuits as seen at the input terminals.



3.13 The circuit shown below contains an ac and a dc voltage source. The zener has a V_{ZK} of 5 V and a perfectly vertical slope to its v-i characteristic in the reverse breakdown region (i.e., $r_Z \simeq 0$). Plot the output voltage v_{OUT} and current i_1 as functions of time for $v_{in} = 5 \sin \omega t$ V.



3.14 Consider the zener circuit described above. In this case a load resistor of value R_L = 1.2 kΩ is connected across the output terminals. Plot v_{OUT}, i_1, and the current i_L to the load as functions of time.

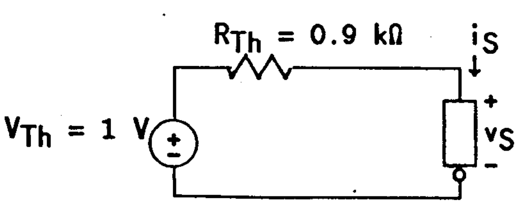
Solutions

Chapter 3

3.1 First find the Thevenin equivalent of V_1, R_1, and R_2:

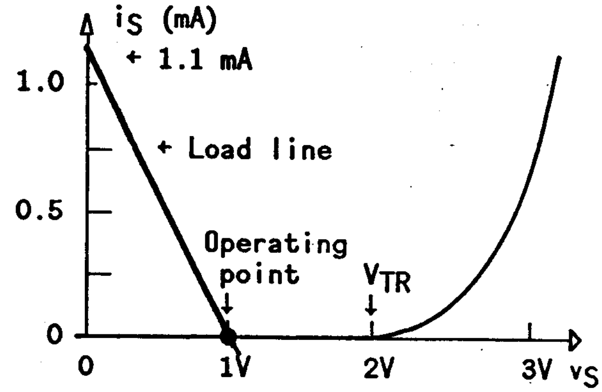
$$V_{Th} = V_1 \frac{R_2}{R_1 + R_2} = 10\ \text{V}\ \frac{1\ \text{k}\Omega}{1\ \text{k}\Omega + 10\ \text{k}\Omega}$$

$$= 1\ \text{V}; \quad R_{Th} = R_1 || R_2 = 0.9\ \text{k}\Omega$$



• Graphical method:

Plot the load line of V_{Th} and R_{Th} over the v-i characteristic of the square-law device:

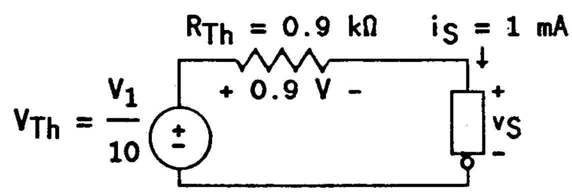


From the above plot, it is evident that $v_S < V_{TR}$, i.e., the resistive circuit has insufficient voltage to turn on the square-law device. Hence $v_S = 1$ V (open-circuit voltage of Thevenin circuit) and $i_S = 0$.

3.2 First find the Thevenin equivalent of V_1, R_1, and R_2. In this case,

$$V_{Th} = V_1 \frac{R_2}{R_1 + R_2} = \frac{V_1}{10}$$

$$R_{Th} = R_1 || R_2 = 0.9\ \text{k}\Omega$$



If the current i_S is to be 1 mA, a voltage of $v_S = 3$ V must be developed across the square-law device, e.g.

$$i_S = A(v_S - V_{TR})^2$$

$$= (1\ \text{mA/V}^2)(3\text{V} - 2\text{V})^2 = 1\ \text{mA}$$

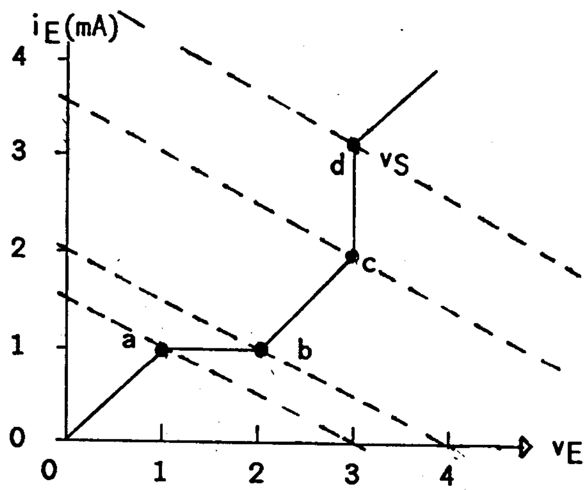
Similarly, a drop of

$$i_S R_{Th} = (1\ \text{mA})(0.9\ \text{k}\Omega) = 0.9\ \text{V}$$

is required across R_{Th} if the current through it is to be 1 mA. The Thevenin voltage V_{Th} must therefore be

0.9 V + 3 V = 3.9 V. The voltage V_1, which is ten times larger, should be set to 3.9 V.

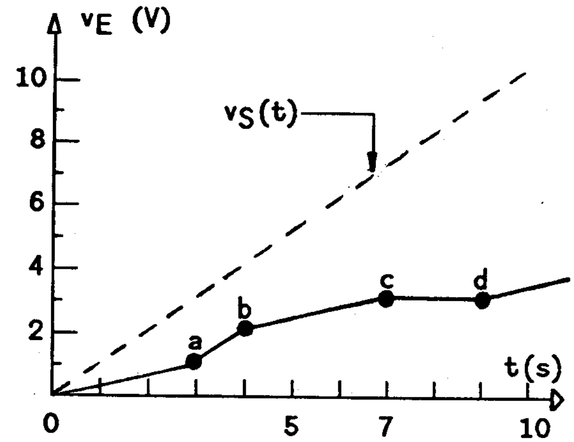
3.3 Plot the load line imposed by the voltage source and 2-kΩ resistor over the v-i characteristic of the element. Note that the v_E-axis intercept of the load line begins at zero and increases with time at the rate 1 V/s. The load line and operating point at four key transition points *a,b,c,* and *d* are shown below. Since the v-i characteristic is linear between these points and the load line moves linearly with time, the operating point will change linearly with time between the various points.



For each of the operating points shown, the v-axis intercept of the corresponding load line (i.e., the time in seconds) can be computed by adding the drop i_ER_S to v_E. The v_E and i_E values at each of the points *a* through *d* are found graphically:

pnt	i_E	v_E	i_ER_S	t
a	1 mA	1 V	2 V	3 s
b	1 mA	2 V	2 V	4 s
c	2 mA	3 V	4 V	7 s
d	3 mA	3 V	6 V	9 s

Here is a plot of v_E versus time. Note that the slope of $v_E(t)$ beyond point *d* is the same as that between points 0 and *a* and beyond point *d*.



3.4 a) Rewrite the diode equation in the following form:

$$I_S = \frac{i_D}{e^{v_D/\eta V_T} - 1} \simeq \frac{10^{-3}\text{ A}}{\exp\,(0.7\text{V}/50\text{mV})}$$

$$= 8.3 \times 10^{-10}\text{ A}$$

where ηV_T = 2(25 mV) = 50 mV at room temperature. The -1 term in the diode equation becomes insignificant compared to the exponential term and has been omitted.

b) Solve the diode equation for v_D in terms of i_D. Under forward bias conditions, the -1 in the diode equation becomes negligible compared to the exponential term, so that

$$i_D \simeq I_S\, e^{v_D/\eta V_T} \Longrightarrow v_D = \eta V_T \ln\left[\frac{i_D}{I_S}\right]$$

Substituting numbers with $i_D = 2(1\ mA) = 2\ mA$ yields

$$v_D = (50mV)\ \ln\left[\frac{2 \times 10^{-3}\ A}{8.3 \times 10^{-10}\ A}\right] = 0.73\ V$$

c) Similarly, a diode current of (1 mA)/2 = 0.5 mA will result for

$$v_D = (50mV)\ \ln\left[\frac{0.5 \times 10^{-3}\ A}{8.3 \times 10^{-10}\ A}\right] = 0.67\ V$$

d) A tenfold increase in current to i_D = 10 mA will require a voltage of

$$v_D = (50\ mV)\ln\left[\frac{10 \times 10^{-3}\ A}{8.3 \times 10^{-10}\ A}\right] = 0.82\ V$$

3.5 Solve the diode equation for v_D in terms of i_D. Under forward bias conditions, the -1 in the diode equation becomes negligible compared to the exponential term, so that

$$i_D \simeq I_S\ e^{v_D/\eta V_T} \Longrightarrow v_D = \eta V_T\ \ln\left[\frac{i_D}{I_S}\right]$$

At 20 °C ≡ 293 K (Note: 0 °C = 273 K), V_T becomes

$$\frac{kT}{q} = \frac{(1.38\text{x}10^{-23}\ J/K)(293\ K)}{1.6 \times 10^{-19}\ C} = 25.3\ mV$$

Thus, at i_D = 1 mA, v_D becomes

$$(25.3\ mV)\ \ln\left[\frac{10^{-3}\ A}{10^{-15}\ A}\right] = 0.70\ V$$

At T = 30 °C ≡ 303 K (increase in T of 10 °C), V_T becomes

$$25.3\ mV \times \frac{303\ K}{293\ K} = 26.1\ mV$$

and I_S increases to

$$10^{-15}\ A \times (1.15)^{10} = 4 \times 10^{-15}\ A$$

so that v_D becomes

$$(26.1\ mV)\ \ln\left[\frac{10^{-3}\ A}{4 \times 10^{-15}\ A}\right] = 0.68\ V$$

The temperature coefficient of v_D can be estimated by noting that

$$\frac{\Delta v_D}{\Delta T} = \frac{0.68\ V - 0.70\ V}{10\ °C} = -1.5\ mV/°C$$

3.6 Under forward bias conditions, the -1 term in the diode equation becomes negligible compared to the exponential term, so that

$$i_D = I_S\ (e^{v_D/\eta V_T} - 1) \simeq i_D\ e^{v_D/\eta V_T}$$

The ratio of two diode currents can be expressed as

be expressed as

$$\frac{i_{D1}}{i_{D2}} = \frac{e^{v_{D1}/\eta V_T}}{e^{v_{D2}/\eta V_T}} = e^{(v_{D1} - v_{D2})/\eta V_T}$$

Taking the logarithm of both sides of this equation leads to

$$\eta = \frac{v_{D1} - v_{D2}}{V_T\ \ln\ (i_{D1}/i_{D2})}$$

For the data given,

$$\eta = \frac{710\ mV - 764\ mV}{(25\ mV)\ \ln\ [(5\ mA)/(15\ mA)]} = 1.97$$

Next take the reciprocal of the diode equation (neglect the -1 term):

$$I_S = i_D\ e^{-v_D/\eta V_T}$$

Given the value η = 1.97 (ηV_T = 49.2 mV at room temperature), this equation can be evaluated at any known value of i_D to determine I_S. From the data at i_D = 5 mA, for example, the equation yields

$$I_S = (5\ mA)\ e^{-710/49.2} \simeq 2.7 \times 10^{-9}\ A$$

Alternatively, I_S can be evaluated at the 15 mA current value, yielding the same result:

$$I_S = (15\ mA)\ e^{-764/49.2} \simeq 2.7 \times 10^{-9}\ A$$

3.7 Reference all calculations involving changes in temperature to 20 °C, where v_D is known. If the current through the diode remains constant, the diode voltage will change with temperature according to the relation

$$v_D = v_D(20\ ^oC) - (2\ mV/^oC)\Delta T$$

At 0 ^oC, $\Delta T = -20\ ^oC$, so that the diode voltage becomes

$$v_D(0\ ^oC) = 700\ mV - (2\ mV/^oC)(-20\ ^oC) = 740\ mV.$$

Similarly, at 100 ^oC, $\Delta T = 80\ ^oC$ ==>

$$v_D(100\ ^oC) = 700 mV - (2\ mV/^oC)(80\ ^oC) = 540\ mV.$$

3.8 If an i_S of 1 mA is to flow through the square-law device, its voltage must become

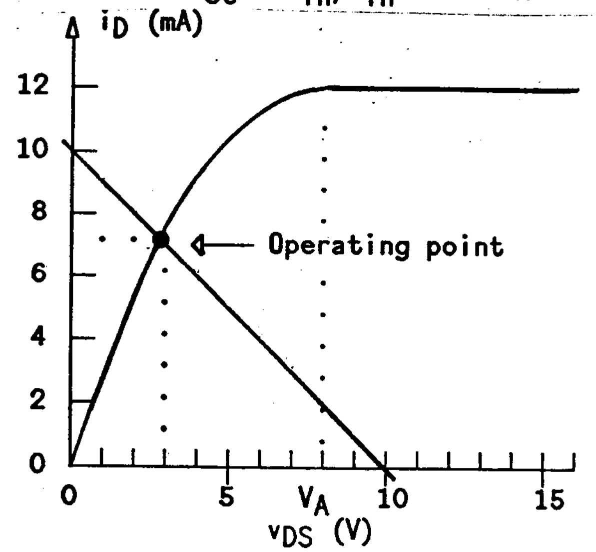
$$v_S = V_{TR} + \sqrt{i_S/A}$$
$$= 3\ V + \sqrt{(1\ mA)/(2\ mA/V^2)} = 3.7\ V$$

The current through R_N will be determined by the voltage developed across the square-law device. At the calculated voltage, 3.7 mA of current will flow through the 1 kΩ resistor R_N, so that I_N must be set to 4.7 mA. Note that the voltage provided by the current source will be whatever value is needed to meet the joint requirements of R_N and the square-law device.

By similar calculation, an i_S of 2 mA will require a v_S of 3 V + $[(2\ mA)/(2\ mA/V^2)]^{1/2}$ = 4 V, a resistor current of (4 V)/(1 kΩ) = 4 mA, and an I_N of 2 mA + 4 mA = 6 mA.

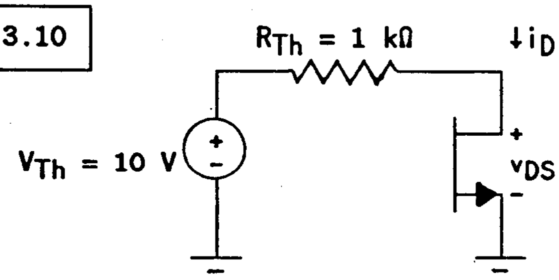
An i_S of 4 mA will require a v_S of 3 V + $[(4\ mA)/(2\ mA/V^2)]^{1/2}$ = 4.4 V, a resistor current of (4.4 V)/(1 kΩ) = 4.4 mA, and an I_N of 4 mA + 4.4 mA = 8.4 mA.

3.9 The v-i characteristic of the FET channel can be drawn using the given values of I_{DSS} = 12 mA and V_A = 8 V. The load line imposed on the device passes through the points V_{Th} = 10 V and $i_{SC} = V_{Th}/R_{Th}$ = 10 mA:



The graphical point of intersection, which occurs at $i_D \simeq 7.1$ mA and $v_{DS} \simeq$ 2.9 V, defines the current through the FET channel.

3.10



Begin with the constraint imposed on v_{DS} by the Thevenin circuit:

$$v_{DS} = V_{Th} - i_D R_{Th}$$

or

$$i_D = \frac{V_{Th} - v_{DS}}{R_{Th}}$$

We note that the FET channel cannot operate in the constant current region with $i_D = I_{DSS}$. If it did, the above equation would yield

$$v_{DS} = 10\ V - (12\ mA)(1\ k\Omega) = -2\ V$$

The value of v_{DS} must lie between zero and V_{Th}, and cannot be negative. Hence the FET channel must operate in its triode region, where

$$i_D = K[2V_Av_{DS} - (v_{DS})^2]$$

In this case, $K = I_{DSS}/V_A^2 = (12\ \text{mA})/(8\ \text{V})^2 = 0.188\ \text{mA/V}^2 \simeq 0.19\ \text{mA/V}^2$. The operating point in the triode region can be found by equating the current through the Thevenin circuit with the current through the FET channel, yielding

$$\frac{V_{Th} - v_{DS}}{R_{Th}} = K[2V_Av_{DS} - (v_{DS})^2]$$

Substitution of numbers yields the following numerical equation:

$$10 - v_{DS} = 3v_{DS} - 0.188v_{DS}^2$$

or

$$v_{DS}^2 - 21.3v_{DS} + 53.3 = 0$$

where v_{DS} is in volts, K in mA/V^2, and R_{Th} in kΩ. Applying the quadratic formula to this equation yields $v_{DS} = [21.3 - \sqrt{(21.3)^2 - 4(53.3)}]/2 = 2.9\ \text{V}$ (The second solution to the quadratic yields a v_{DS} larger than V_{Th} and does not represent a true electronic solution. It corresponds to the point where the load line would intersect the fictitious extension of the triode-region parabola beyond the apex voltage V_A).

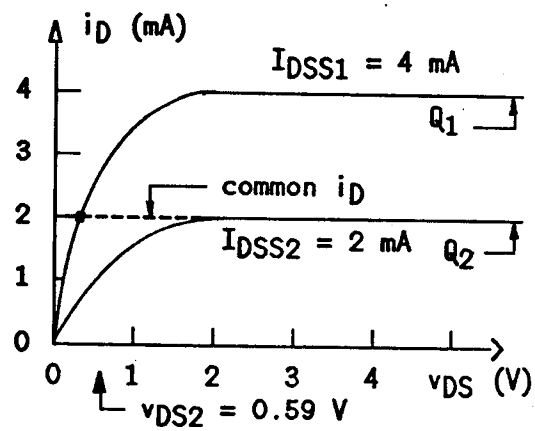
For the computed value of v_{DS},

$$i_D = \frac{V_{Th} - v_{DS}}{R_{Th}} = \frac{10\text{V} - 2.9\text{V}}{1\ \text{k}\Omega} = 7.1\ \text{mA}$$

3.11 For these two devices,

$$I_{DSS1} = K_1V_A^2 = (1\ \text{mA/V}^2)(2\text{V})^2 = 4\ \text{mA}$$
$$I_{DSS2} = K_2V_A^2 = (0.5\ \text{mA/V}^2)(2\text{V})^2 = 2\ \text{mA}$$

From KVL, $v_{DS1} + v_{DS2} = 10\ \text{V}$. This voltage sum is greater than $2V_A$, hence at least one device must operate with $v_{DS} > V_A$ (constant current region operation with $i_D = I_{DSS}$). On the other hand, *only* one device can operate with $i_D = I_{DSS}$, because the devices have different values of I_{DSS}. Since $I_{DSS2} < I_{DSS1}$, Q_2 will be the device that operates in the constant-current region with $v_{DS} > V_A$, as the following diagram indicates.



The value of v_{DS1} can be found by solving the triode-region v-i equation of Q_1 with $i_{D1} = I_{DSS2} = 2\ \text{mA}$:

$$i_{D1} = K_1(2V_Av_{DS1} - v_{DS1}^2)$$

$$2\ \text{mA} = (1\ \text{mA/V}^2)[2(2\ \text{V})v_{DS1} - v_{DS1}^2]$$

Rearranging leads to

$$v_{DS1}^2 - (4\ \text{V})v_{DS1} + 2\ (\text{V}^2) = 0$$

where v_{DS1} is in units of volts. Applying the quadratic formula yields

$$v_{DS1} = \frac{4\ \text{V} - \sqrt{16\ \text{V}^2 - 8\ \text{V}^2}}{2} \simeq 0.59\ \text{V}$$

A second solution, obtained using the positive root in the numerator, yields $v_{DS1} = 3.4\ \text{V}$. This solution represents the mathematical intersection of the I_{DSS2} line with the continuation of Q_1's triode region beyond the parabolic apex. It is not a true electronic solution in the triode region because it yields a v_{DS1} larger than V_A.

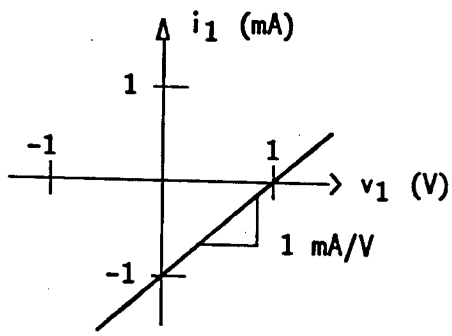
Finally, v_{DS2} can be found from

$$v_{DS1} + v_{DS2} = 10\ V$$

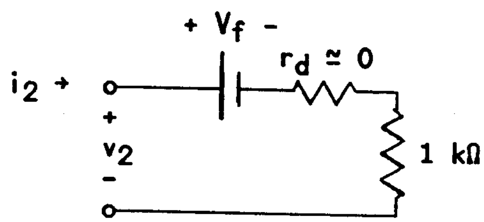
$$\Longrightarrow v_{DS2} = 10\ V - 0.59\ V = 9.41\ V$$

3.12 a) This circuit is resistive, hence its v-i characteristic will be a straight line. Its v-i characteristic will not pass through the origin because the circuit contains a voltage source.

For $i_1 = 0$, v_1 becomes 1 V (open circuit voltage). For $v_1 = 0$, $i_1 = -1$ mA (i_1 is positive defined *into* the circuit).

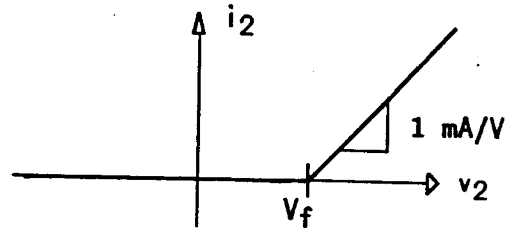


b) Under forward biased conditions ($v_2 > V_f$), the diode will conduct current. The v-i characteristic of the circuit over this region can be found by substituting an appropriate piecewise linear model for the diode:

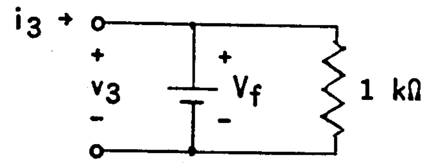


Assume r_d to be small enough to be neglected. The circuit shown above is identical in form to that of part a), hence it will have the same v-i characteristic for $v_2 > V_f$.

For $v_2 < V_f$, the diode becomes reverse biased, so that $i_2 = 0$. The v-i characteristic for the circuit over the entire range of v_2 thus looks like this:

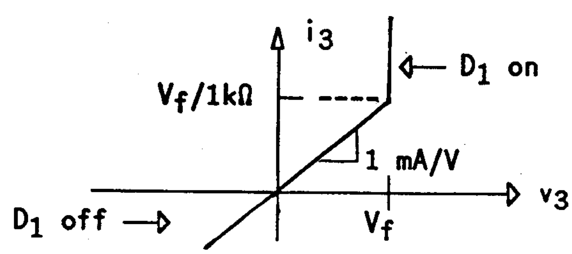


c) As v_3 is increased from zero, D_1 will at first appear as an open circuit, causing the circuit to have the v-i characteristic of the 1-kΩ resistor. When v_3 reaches V_f, the diode will become forward biased. Substituting an appropriate piecewise linear model (with r_d assumed zero) yields:



Over this region of operation, v_3 will remain clamped at V_f. The current i_3 will be determined by the Thevenin resistance or v-i characteristic of the source connected to the v_3-i_3 terminals.

For $v_3 < V_f$, the diode will be reverse biased (open circuit), and the circuit will again have the v-i characteristic of the 1-kΩ resistor. Here is a plot of i_3 versus v_3 over the complete range of v_3 values:

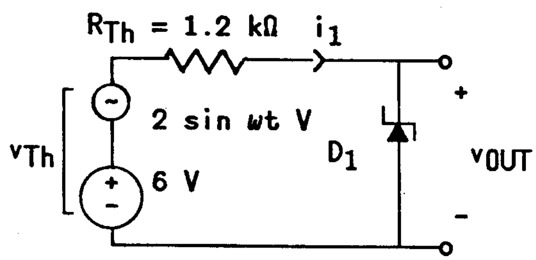


3.13 a) Form the Thevenin equivalent of the resistive part of the circuit consisting of V_o, v_{in}, R_1, and R_2. From superposition of V_o and v_{in}:

$$v_{Th} = V_o \frac{R_2}{R_1 + R_2} + v_{in} \frac{R_1}{R_1 + R_2}$$
$$= 6\ \text{V} + 2 \sin \omega t\ \text{V}$$

With V_o and v_{in} set to zero, $R_{Th} = R_1 || R_2 = (2\ \text{k}\Omega) || (3\ \text{k}\Omega) = 1.2\ \text{k}\Omega$

Now connect the zener to the Thevenin equivalent representation of the resistive circuit:



The zener D_1 will be forced into reverse breakdown when the total v_{Th} exceeds the value $V_{ZK} = 5$ V. Under these conditions v_{OUT} will equal V_{ZK} and i_1 will equal $(v_{Th} - V_{ZK})/R_{Th}$. At the peak value of $v_{Th} = 8$ V, which occurs when $\sin \omega t = 1$, i_1 will reach the value

$$i_1 = \frac{8\ \text{V} - 5\ \text{V}}{1.2\ \text{k}\Omega} = 2.5\ \text{mA}$$

When $\sin \omega t = 0$, i_1 retains the value

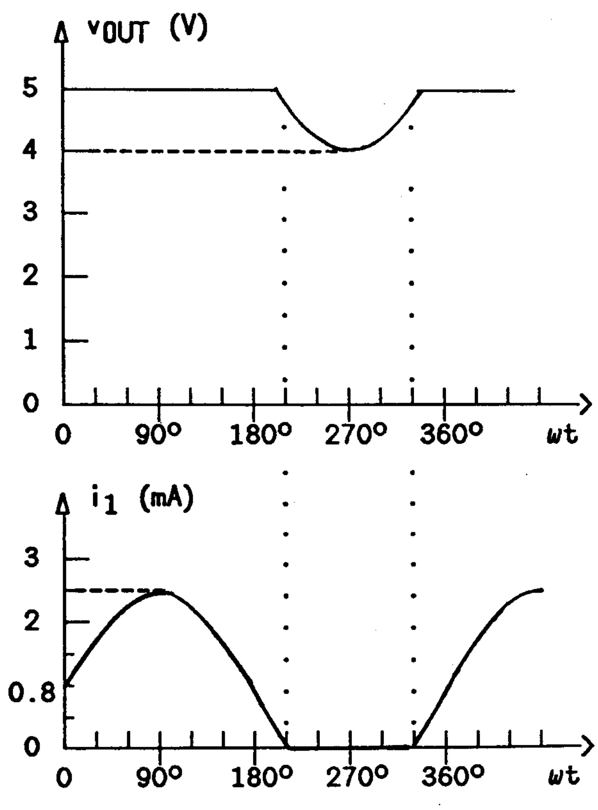
$$\frac{6\ \text{V} - 5\ \text{V}}{1.2\ \text{k}\Omega} = 0.8\ \text{mA}$$

because has a dc component of 6 V.

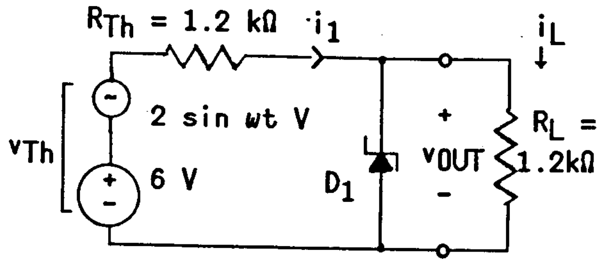
Over the time interval where $\sin \omega t$ drops below -1/2 ($210^\circ < \omega t < 330^\circ$), the total v_{Th} will drop below 5 V and the zener will no longer be forced into reverse breakdown. Under these conditions, the zener will behave as an open circuit, so that $v_{OUT} = v_{Th}$ and $i_1 = 0$. The lowest value of v_{OUT}, which occurs at when $\sin \omega t = -1$, is 6 V - 2 V = 4 V.

Note that the magnitude of the sinusoidal component of v_{Th} is smaller than its dc component. Hence v_{Th} will never become negative and the zener will never become forward biased.

Here are plots of v_{OUT} and i_1 versus time with the time scale expressed in units of ωt. Over the interval $210^\circ < \omega t < 330^\circ$, $\sin \omega t$ is less that -0.5.



3.14 Here is the circuit with R_L connected and with V_o, v_{in}, R_1, and R_2 represented by a Thevenin equivalent circuit:



With R_L connected, the Thevenin equivalent of the resistive portion of the circuit can be further simplified. The modified Thevenin circuit seen by the zener becomes

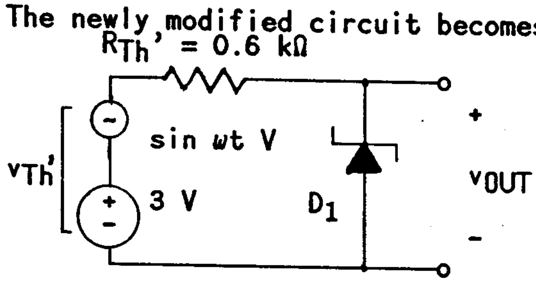
$$v_{Th}' = v_{Th}\frac{R_L}{R_{Th} + R_L} = 3 + \sin \omega t \text{ V}$$

and

$$R_{Th}' = R_{Th} || R_L$$

$$= (1.2 \text{ k}\Omega) || (1.2 \text{ k}\Omega) = 0.6 \text{ k}\Omega$$

The newly modified circuit becomes:



Under these conditions, v_{Th}' reaches a peak value of only 4 V, which is insufficient to drive the zener into reverse breakdown. Thus, for all t, the zener functions as an open circuit.

From the original Thevenin circuit explicitly containing R_L, we note that $i_1 = i_L$ when the zener functions as an open circuit. Under these conditions,

$$i_L = \frac{v_{Th}}{R_{Th} + R_L} = \frac{6 + 2 \sin \omega t \text{ V}}{2.4 \text{ k}\Omega}$$

$$= 2.5 + 0.83 \sin \omega t \text{ mA}$$

and

$$v_{OUT} = \frac{v_{Th}}{2} = 3 + \sin \omega t \text{ V}$$

Here are plots of v_{OUT} and i_L with R_L connected: