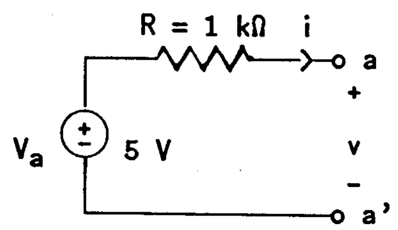
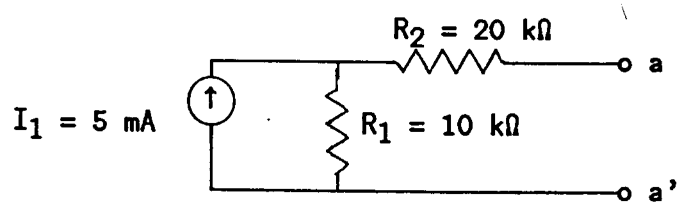
Chapter 1
Review of Linear Circuit Theory

1.1 Plot the v-i characteristic of the following circuit when $V_a = 5$ V.

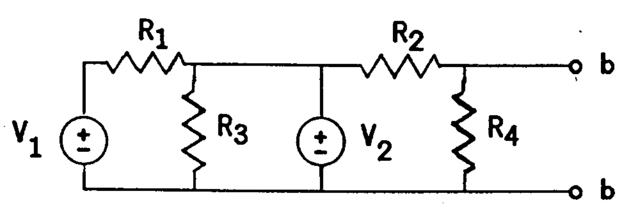


1.2 A 2 kΩ resistor is connected across the terminals a-a' in the circuit shown above. Find the operating point of the resistor using the graphical method. Verify the result using Ohm' Law.

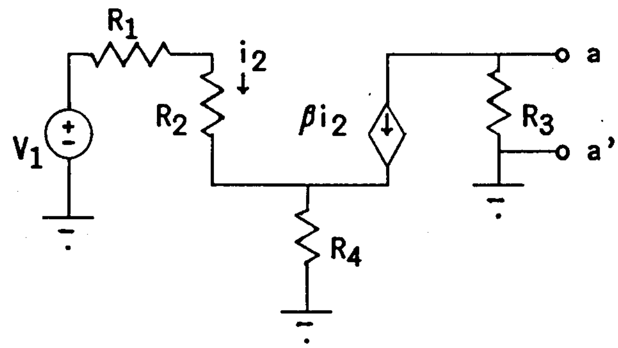
1.3 Find the Thevenin equivalent of the circuit shown below, as seen from the terminals a-a'.



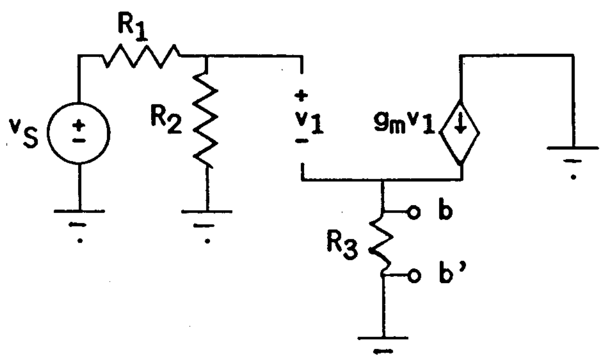
1.4 In the following circuit, use superposition to find the Thevenin equivalent seen at the terminals b-b'



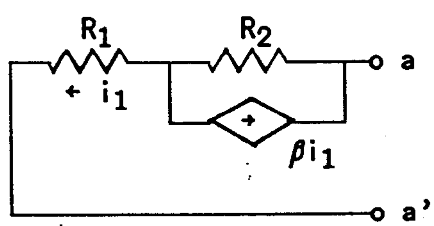
1.5 The circuit shown below contains a dependent source. Find the Thevenin equivalent of the circuit as seen at the terminals a-a'



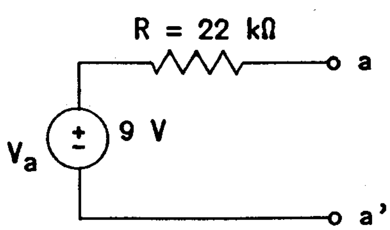
1.6 The circuit shown below contains a dependent source. Find the Thevenin equivalent of the circuit as seen at the terminals b-b'. Use the "test source" method to find R_{Th}.



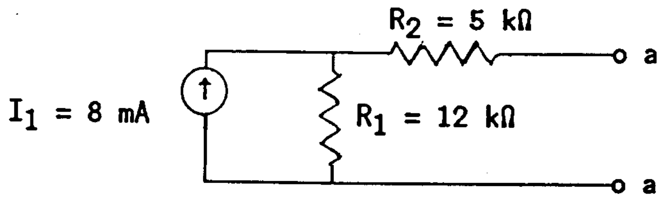
1.7 The circuit shown below contains a current-dependent current source. Find the equivalent resistance seen looking into the terminals a-a'. Use the "test source" method.



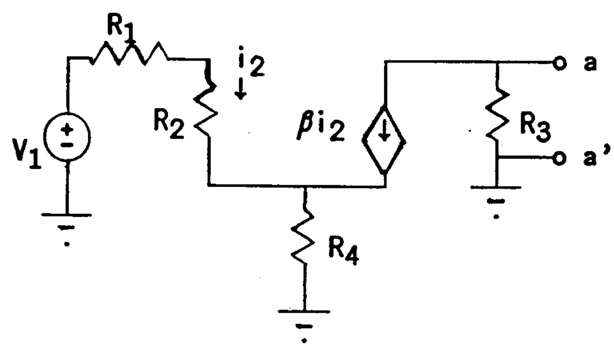
1.8 Find the Norton equivalent of the circuit shown below.



1.9 Find the Norton equivalent of the following circuit:

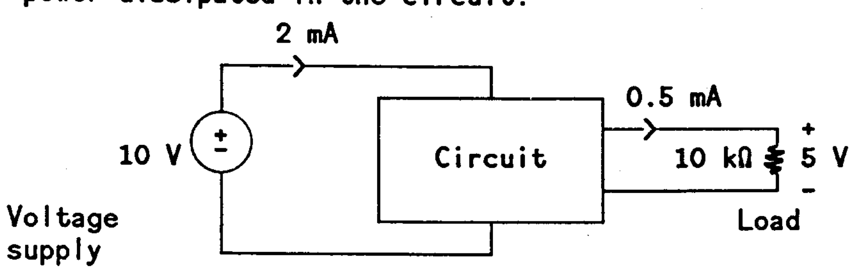


1.10 The circuit shown below contains a dependent source. Find the Norton equivalent of the circuit as seen at terminals a-a'. This circuit is the same one analyzed in Prob. 1.5.



1.11 Measurements with a voltmeter are made at the terminals of a circuit that contains only resistors, dc current sources, and fixed voltage sources. The meter has an internal resistance of 1 MΩ. With no other loads applied, the measured voltage is 12 V. With an additional external 1 MΩ resistance connected in parallel with the meter, the measured voltage is 10 V. Find the Thevenin equivalent of the resistive circuit as seen at the measured terminals.

1.12 A circuit is powered from a 10-V voltage supply, as shown below. When the circuit drives a 10 kΩ resistive load at 5 V, it draws 2 mA from this supply. Find the power dissipated in the load; find the power dissipated in the circuit.

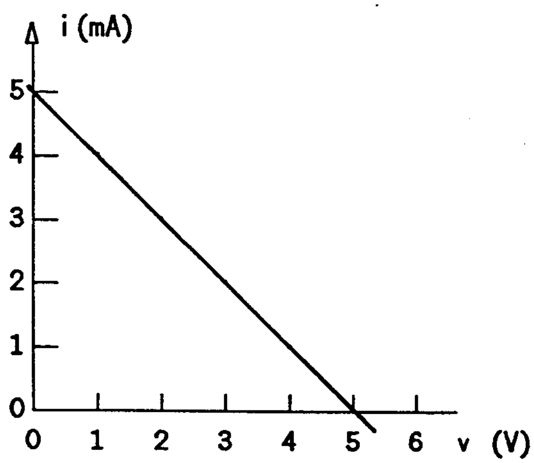


SOLUTIONS TO PROBLEMS

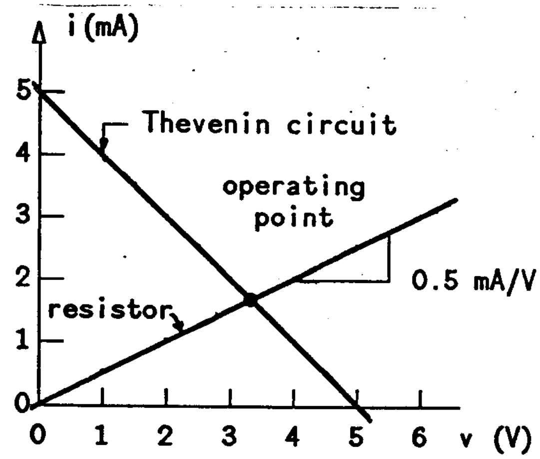
Chapter 1

1.1 Since the circuit is resistive (i.e., contains only a resistor and a constant voltage source), it's v-i characteristic is a straight line. The v-i characteristic can be plotted by finding two points.

For i = 0 (open circuit), $v = V_a =$ 5 V (v-axis intercept of the v-i characteristic). For v = 0 (short circuit), $i = V_a/R$ = 5 mA (i-axis intercept). The resulting v-i characteristic is shown below.



1.2 The v-i characteristic of a 2 kΩ resistor consists of a straight line with slope (1 mA)/(2 V) = 0.5 mA/V passing through the origin:



The point of intersection, i.e., the operating point, is seen to be 3.3 V; 1.6 mA. An analytical solution confirms this result. From Ohm's Law:

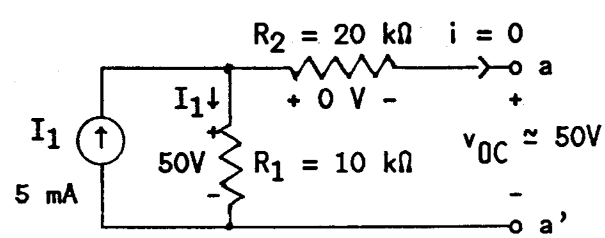
$$i = \frac{5\ \text{V}}{1\ \text{k}\Omega + 2\ \text{k}\Omega} = 1.67\ \text{mA}$$

$$v = (1.66\ \text{mA})(2\ \text{k}\Omega) = 3.33\ \text{V}$$

Alternatively, from the voltage divider relation:

$$v = 5\ \text{V}\ \frac{2\ \text{k}\Omega}{1\ \text{k}\Omega + 2\ \text{k}\Omega} = 3.33\ \text{V}$$

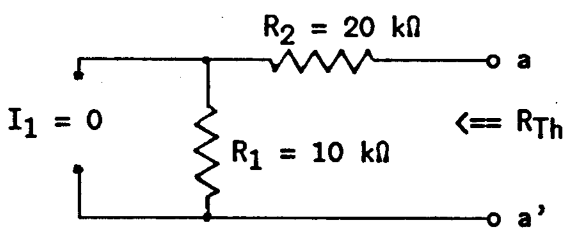
1.3 The Thevenin voltage is equal to the voltage at the terminals a-a' under open-circuit conditions (i.e., with no other elements connected to the terminals):



Under open circuit conditions, all of the current I_1 flows through R_1, causing a voltage drop

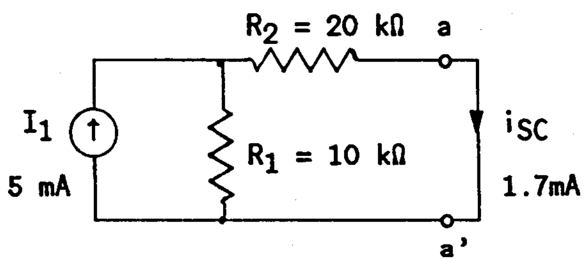
$$I_1R_1 = (5\text{mA})(10\text{k}\Omega) = 50 \text{ V}$$

to develop across R_1. No voltage drop is developed across R_2, since the current through it is zero. Thus, $v_{OC} = 50$ V. The Thevenin resistance is equal to the resistance seen at the terminals a-a' with I_1 set to zero (open circuit):



By inspection, $R_{Th} = R_1 + R_2 = 30$ kΩ.

- Alternative method for finding R_{Th}: Find the current flowing through a short-circuit applied to the terminals a-a':



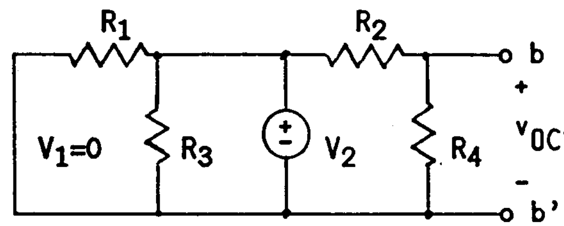
The current flowing through the short-circuit is also equal to the current flowing through R_2. From the current divider relation:

$$i_{SC} = I_1 \frac{R_1}{R_1 + R_2}$$

$$= 5 \text{ mA} \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 20 \text{ k}\Omega} = 1.67 \text{ mA}$$

$$R_{Th} = \frac{v_{OC}}{i_{SC}} = \frac{50 \text{ V}}{1.67 \text{ mA}} = 30 \text{ k}\Omega$$

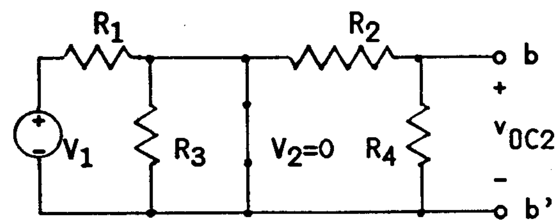
1.4 To find v_{Th}, find the separate contributions of V_1 and V_2 to v_{OC}. Here is the circuit with V_1 set to zero and V_2 "on":



From the voltage divider relation

$$v_{OC1} = V_2 \frac{R_4}{R_2 + R_4}$$

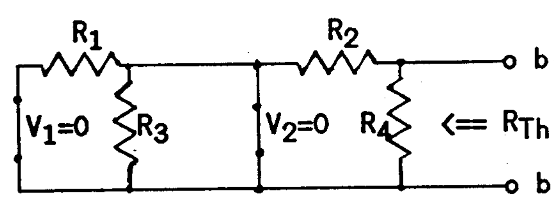
Here is the circuit with V_2 set to zero and V_1 "on":



Because a short circuit appears directly across the series combination of R_2 and R_4 (voltage drop equal to zero), $v_{OC2} = 0$. Thus v_{Th} is equal to v_{OC1} alone:

$$v_{Th} = v_{OC1} = V_2 \frac{R_4}{R_2 + R_4}$$

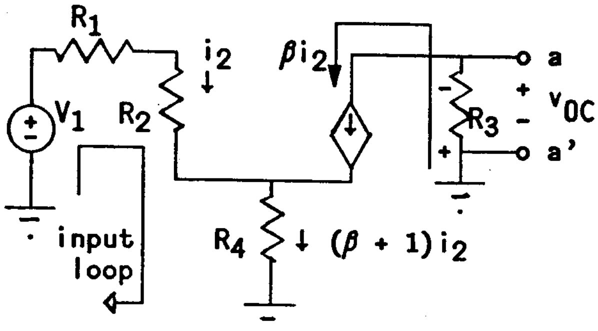
To find R_{Th}, set both V_1 and V_2 to zero (short circuits) and find the resistance seen at the terminals b-b'



By inspection, $R_{Th} = R_4||R_2$. Note that the $R_3||R_1$ combination is bypassed by the V_2=0 short circuit.

Alternatively, R_{Th} could be found by applying a short circuit to the terminals b-b' and using superposition to find the contributions of V_1 and V_2 to i_{SC}.

1.5 Find the open-circuit voltage at the terminals a-a'.



The dependent source pulls a current βi_2 up from ground through R_3, so that $v_{OC} = -\beta i_2 R_3$. Find i_2 by applying KVL to the "input loop" of the circuit. Note that a total current of $(\beta + 1)i_2$ flows through R_4, with i_2 entering R_4 via R_2 and βi_2 entering R_4 via the dependent source. From KVL:

$$V_1 = (R_1 + R_2)i_2 + R_4(\beta + 1)i_2$$

Solving for i_2 yields

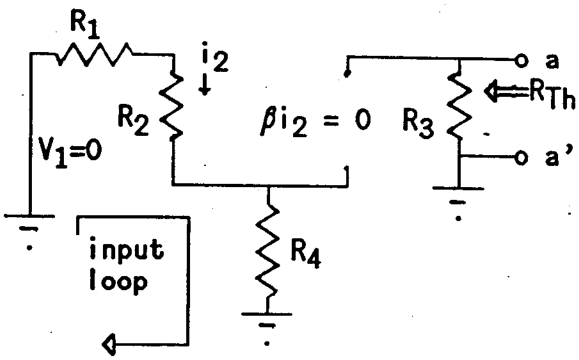
$$i_2 = \frac{V_1}{R_1 + R_2 + (\beta + 1)R_4}$$

so that

$$v_{OC} = -\beta i_2 R_3 = V_1 \frac{-\beta R_3}{R_1 + R_2 + (\beta + 1)R_4}$$

where $v_{Th} \equiv v_{OC}$.

To find R_{Th}, set V_1 to zero (short circuit) and find the resistance seen looking into the terminals a-a'.



From KVL around the input loop,

$$(R_1 + R_2)i_2 + R_4(\beta + 1)i_2 = 0$$

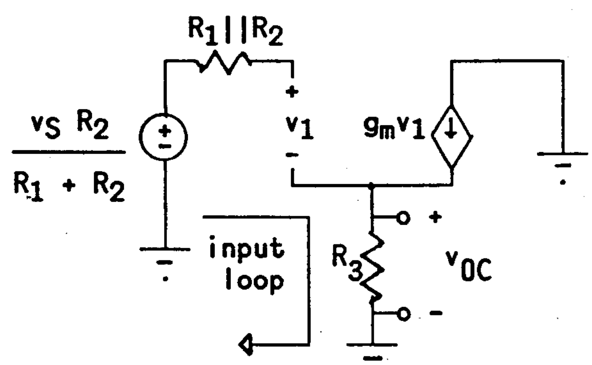
Solving for i_2 yields $i_2 = 0$. Consequently, the dependent source is set to zero as well (open circuit), and R_{Th} in the remaining circuit becomes just R_3.

• Alternative method:

With V_1 on, the current flowing through a short circuit applied to the a-a' terminals becomes $-\beta i_2$, so that

$$R_{Th} = \frac{v_{OC}}{i_{SC}} = \frac{-\beta i_2 R_3}{-\beta i_2} = R_3$$

1.6 Step 1: Represent v_S, R_1, and R_2 by a simpler Thevenin circuit consisting of one resistor and one voltage source:



Next find v_{OC} (i.e., v_{Th} at the terminals b-b') by applying KVL to the "input loop". Note that no current flows through the open circuit at the terminals where v_1 is defined, hence no current flows through $R_1||R_2$, and the voltage drop across the resistance $R_1||R_2$ is zero. Applying KVL yields

$$v_S \frac{R_2}{R_1 + R_2} = v_1 + g_m v_1 R_3$$

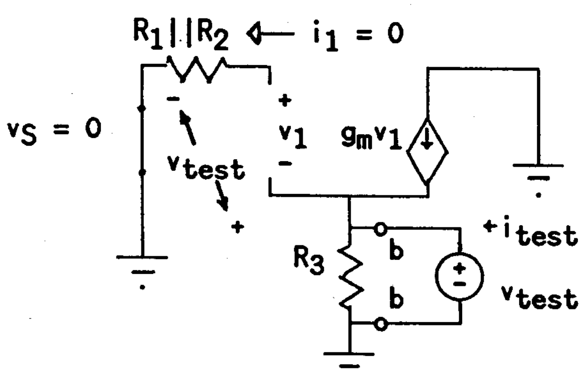
or

$$v_1 = v_S \frac{R_2}{R_1 + R_2} \frac{1}{(1 + g_m R_3)}$$

The open-circuit voltage thus becomes

$$v_{OC} = g_m v_1 R_3 = v_S \frac{R_2}{R_1 + R_2} \frac{g_m R_3}{(1 + g_m R_3)}$$

To find R_{Th}, set v_S to zero and apply a "test" voltage source to b-b'. The value of R_{Th} is determined by v_{test}/i_{test}. Note that a test current source could also be used, but a test voltage source simplifies the computation because it directly fixes v_1 to a known value.



Since $i_1 = 0$, the voltage drop across $R_1||R_2$ is zero, so that $v_1 = -v_{test}$.

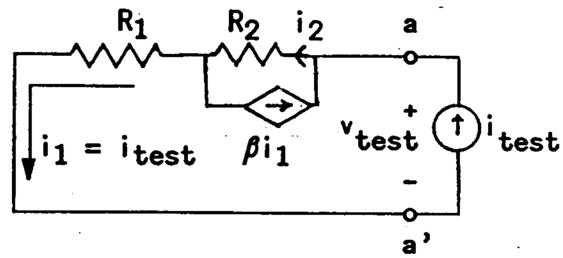
The current i_{test} has two components, one through R_3 and one through the dependent source:

$$i_{test} = \frac{v_{test}}{R_3} - g_m v_1 = \frac{v_{test}}{R_3} + g_m v_{test}$$

The ratio of v_{test} to i_{test} can be computed from this equation, i.e.,

$$R_{Th} = \frac{v_{test}}{i_{test}} = \left[\frac{1}{R_3} + g_m\right]^{-1} \equiv R_3 \Big|\Big| \frac{1}{g_m}$$

1.7 Find R_{Th} by applying a test source to the terminals a-a'. In this case, a test current source works best because it directly fixes i_1 to the value i_{test}.



Applying KCL to node "a" yields

$$i_{test} = i_2 - \beta i_1 = i_2 - \beta i_{test}$$

Solving for i_2 results in

$$i_2 = (\beta + 1) i_{test}$$

Adding up the drops across R_1 and R_2 yields

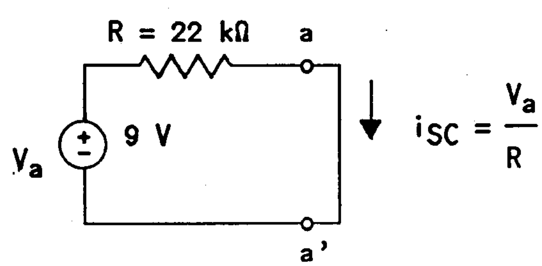
$$v_{test} = i_1 R_1 + i_2 R_2 = i_{test} R_1 + (\beta + 1) i_{test} R_2$$

so that

$$R_{Th} = \frac{v_{test}}{i_{test}} = R_1 + (\beta + 1) R_2$$

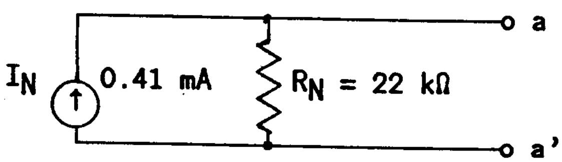
The value of R_2 as seen from a-a' has essentially been multiplied by the factor $(\beta + 1)$ via the action of the dependent source.

1.8 The Norton equivalent consists of a current source and parallel resistance. The value of the source is equal to the short-circuit current measured at terminals a-a':

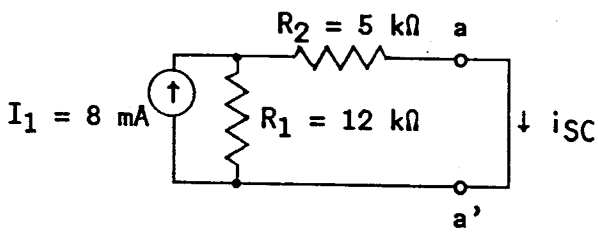


where $i_{SC} = V_a/R = (9 \text{ V})/(22 \text{ k}\Omega) = 0.41$ mA/V. The Norton resistance is found by setting the V_a source to zero (short circuit), yielding $R_N = R = 22$ kΩ.

The complete Norton equivalent circuit is shown below.



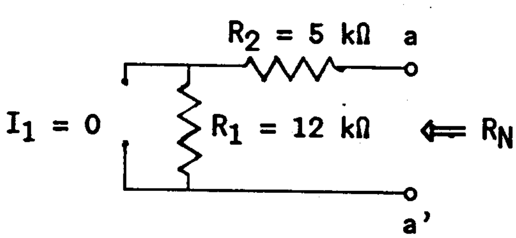
1.9 The Norton current is found by applying a short-circuit to the a-a' terminals:



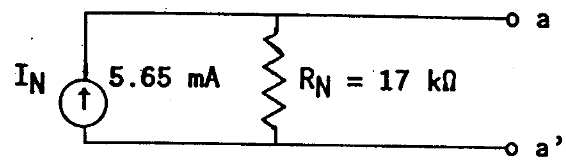
From current division:

$$i_{SC} = \frac{R_1}{R_1 + R_2} I_1 = \frac{12 \text{ k}\Omega}{17 \text{ k}\Omega} 8 \text{ mA} = 5.65 \text{ mA}$$

The Norton resistance is found by setting I_1 to zero (open circuit) and observing the net resistance at the terminals a-a':



$R_N = R_1 + R_2 = 5$ kΩ + 12 kΩ = 17 kΩ. The complete Norton equivalent of the circuit is shown below.



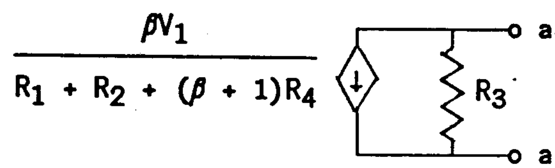
1.10 The open-circuit voltage of this circuit was found in Prob. 1.5:

$$v_{OC} = -\beta i_2 R_3 = V_1 \frac{-\beta R_3}{R_1 + R_2 + (\beta + 1)R_4}$$

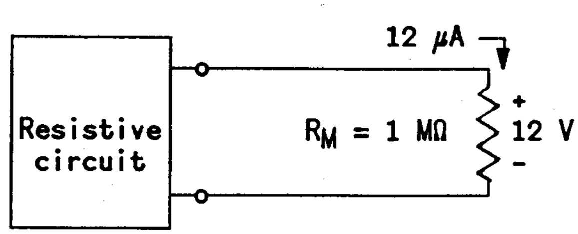
Similarly, the Thevenin resistance, computed with V_1 set to zero, was found to be $R_{Th} = R_3$. These elements of the Thevenin equivalent circuit can be used to find the element values of the Norton equivalent circuit. Specifically, $R_N = R_3$, and

$$I_N = \frac{v_{OC}}{R_3} = \frac{-\beta V_1}{R_1 + R_2 + (\beta + 1)R_4}$$

The Norton equivalent of the original circuit can be represented in the following form:

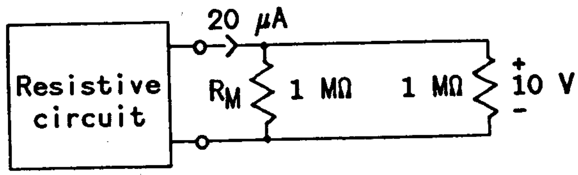


1.11 Here is a summary of the measurements. With the meter alone, the current out of the resistive circuit is (12 V)/(1 MΩ) = 12 μA.:

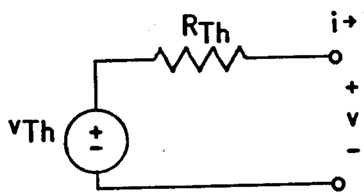


With an additional 1 MΩ resistor connected, the current out of the resistive circuit becomes

$$\frac{10\ \text{V}}{(1\ \text{M}\Omega)\,||\,(1\ \text{M}\Omega)} = 20\ \mu\text{A}$$



For the general Thevenin equivalent shown below, $v = v_{Th} - iR_{Th}$.



Applying the known data results in:

$12\ \text{V} = v_{Th} - (12\ \mu\text{A})R_{Th}$, and

$10\ \text{V} = v_{Th} - (20\ \mu\text{A})R_{Th}$.

Simultaneous solution of these equations yields $v_{Th} = 15$ V; $R_{Th} = 250$ kΩ

1.12 The power extracted from the supply is equal to (10 V)(2 mA) = 20 mW. The power dissipated in the load is $(5\ \text{V})^2/(10\ \text{k}\Omega) = 2.5$ mW. (Alternatively, (5 V)(0.5 mA) = 2.5 mW). The power dissipated in the circuit is equal to the difference: 20 mW - 2.5 mW = 17.5 mW
