Supporting Information for "Structural uncertainty in the sensitivity of urban temperatures to anthropogenic heat flux"

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Introduction

In texts S1 to S3, the partial derivatives of T_C, T_S, and T_2 with respect to surface temperatures and T_A are presented, which can assist the interpretation of numerical simulation results.

Text S1. The partial derivatives of T_C with respect to surface temperatures and T_A

In this section, the partial derivatives of T_C with respect to various impervious surface temperatures and T_A are presented.

a) SVs 1 and 3

For SVs 1 and 3, T_C is computed using

$$T_C = \frac{\frac{2H}{r_W}T_W + \frac{G}{r_G}T_G + \frac{G}{r_C}T_A + (R+G)\frac{Q_{AH}}{C_a}\delta_{3i}}{\frac{2H}{r_W} + \frac{G}{r_G} + \frac{G}{r_C}}. \tag{1}$$

From Eq. 1, one can derive

$$\frac{\partial T_C}{\partial T_W} = \frac{\frac{2H}{r_W}}{\frac{2H}{r_W} + \frac{G}{r_G} + \frac{G}{r_C}}, \tag{2}$$

$$\frac{\partial T_C}{\partial T_G} = \frac{\frac{G}{r_G}}{\frac{2H}{r_W} + \frac{G}{r_G} + \frac{G}{r_C}}, \tag{3}$$

and

$$\frac{\partial T_C}{\partial T_A} = \frac{\frac{G}{r_C}}{\frac{2H}{r_W} + \frac{G}{r_G} + \frac{G}{r_C}}. \tag{4}$$

From Eqs. 2, 3, and 4 one can see that $\partial T_C/\partial T_W$ and $\partial T_C/\partial T_G$ decreases with smaller r_C (i.e., larger in SV 1 than in SV 3), while $\partial T_C/\partial T_A$ increases with smaller r_C (i.e., larger in SV 3 than in SV 1).

b) SVs 2 and 4

For SVs 2 and 4, T_C is computed using

$$T_C = \frac{\frac{R}{r_R}T_R + \frac{2H}{r_W}T_W + \frac{G}{r_G}T_G + \frac{R+G}{r_C}T_A + (R+G)\frac{Q_{AH}}{C_a}\delta_{3i}}{\frac{R}{r_R} + \frac{2H}{r_W} + \frac{G}{r_G} + \frac{R+G}{r_C}}, \tag{5}$$

From Eq. 5, one can derive

$$\frac{\partial T_C}{\partial T_R} = \frac{\frac{R}{r_R}}{\frac{R}{r_R} + \frac{2H}{r_W} + \frac{G}{r_G} + \frac{R+G}{r_C}}, \tag{6}$$

$$\frac{\partial T_C}{\partial T_W} = \frac{\frac{2H}{r_W}}{\frac{R}{r_R} + \frac{2H}{r_W} + \frac{G}{r_G} + \frac{R+G}{r_C}}, \tag{7}$$

$$\frac{\partial T_C}{\partial T_G} = \frac{\frac{G}{r_G}}{\frac{R}{r_R} + \frac{2H}{r_W} + \frac{G}{r_G} + \frac{R+G}{r_C}}, \tag{8}$$

and

$$\frac{\partial T_C}{\partial T_A} = \frac{\frac{R+G}{r_C}}{\frac{R}{r_R} + \frac{2H}{r_W} + \frac{G}{r_G} + \frac{R+G}{r_C}} = \frac{\frac{R+G}{r_C}}{\frac{2H}{r_W} + \frac{R+G}{r_G} + \frac{R+G}{r_C}}. \tag{9}$$

Note that Eq. 9 invokes the assumption that $r_R = r_G$ for SVs 2 and 4.

Comparing Eq. 7 to Eq. 2 (and Eq. 8 to Eq. 3) reveals that $\partial T_C/\partial T_W$ and $\partial T_C/\partial T_G$ are always smaller in SV 2 than SV 1. On the other hand, comparing Eq. 9 to Eq. 4 indicates that $\partial T_C/\partial T_A$ is always larger in SV 2 than SV 1.

Text S2. The partial derivatives of T_S with respect to T_G and T_A

The surface temperature T_S is computed following

$$T_S = (1 - f_{urban})T_{GRASS} + f_{urban}\left(\frac{Q_U}{C_a}r_U + T_A\right) \tag{10}$$

for all SVs. Thus, one can show that

$$\frac{\partial T_S}{\partial T_G} = f_{urban}\frac{r_U}{C_a}\frac{\partial Q_U}{\partial T_G}, \tag{11}$$

and

$$\frac{\partial T_S}{\partial T_A} = f_{urban}\left[\frac{r_U}{C_a}\frac{\partial Q_U}{\partial T_A} + 1\right]. \tag{12}$$

However, the computation of Q_U differs across SVs and is discussed below.

a) SVs 1 and 3

For SVs 1 and 3, Q_U is computed as

$$Q_U = Q_R\frac{R}{R+G} + Q_C\frac{G}{R+G}. \tag{13}$$

Hence one can derive

$$\frac{\partial T_S}{\partial T_G} = f_{urban}\frac{r_U}{C_a}\frac{G}{R+G}\frac{\partial Q_C}{\partial T_G}. \tag{14}$$

Given $Q_C = C_a(T_C - T_A)/r_C$, one can further simplify the above equation to

$$\frac{\partial T_S}{\partial T_G} = f_{urban}\frac{r_U}{r_C}\frac{G}{R+G}\frac{\partial T_C}{\partial T_G} = f_{urban}\frac{r_U}{r_C}\frac{\frac{G}{r_G}}{\frac{R+G}{G}\frac{2H}{r_W} + \frac{R+G}{r_G} + \frac{R+G}{r_C}}. \tag{15}$$

It is clear that $\partial T_S/\partial T_G$ increases with smaller r_C (i.e., larger in SV 3 than in SV 1).

One can also derive

$$\begin{aligned}\frac{\partial T_S}{\partial T_A} &= f_{urban}\left[\frac{r_U}{C_a}\left(-\frac{R}{R+G}\frac{C_a}{r_R} + \frac{G}{R+G}\frac{C_a}{r_C}\left(\frac{\partial T_C}{\partial T_A} - 1\right)\right) + 1\right] \\ &= f_{urban}\left[\frac{r_U}{C_a}\left(-\frac{R}{R+G}\frac{C_a}{r_R} - \frac{G}{R+G}\frac{C_a}{r_C}\frac{\frac{2H}{r_W} + \frac{G}{r_G}}{\frac{2H}{r_W} + \frac{G}{r_G} + \frac{G}{r_C}}\right) + 1\right],\end{aligned} \tag{16}$$

which implies that $\partial T_S/\partial T_A$ decreases with smaller r_C.

b) SVs 2 and 4

For SVs 2 and 4, Q_U is computed as

$$Q_U = Q_C. \tag{17}$$

Given $Q_C = C_a(T_C - T_A)/r_C$, one can further simplify the above equation to

$$\frac{\partial T_S}{\partial T_G} = f_{urban}\frac{r_U}{r_C}\frac{\partial T_C}{\partial T_G} = f_{urban}\frac{r_U}{r_C}\frac{\frac{G}{r_G}}{\frac{2H}{r_W}+\frac{R+G}{r_G}+\frac{R+G}{r_C}}. \tag{18}$$

Comparing Eq. 18 to Eq. 15 reveals that $\partial T_S/\partial T_G$ is larger in SV 2 than SV 1.

The partial derivative of T_S to T_A can be derived as

$$\begin{aligned}\frac{\partial T_S}{\partial T_A} &= f_{urban}\left[\frac{r_U}{C_a}\frac{C_a}{r_C}\left(\frac{\partial T_C}{\partial T_A}-1\right)+1\right]\\ &= f_{urban}\left[\frac{r_U}{r_C}\left(-\frac{\frac{2H}{r_W}+\frac{R+G}{r_G}}{\frac{2H}{r_W}+\frac{R+G}{r_G}+\frac{R+G}{r_C}}\right)+1\right].\end{aligned} \tag{19}$$

It is unclear whether $\partial T_S/\partial T_A$ is larger in SV 2 than SV 1 by simply comparing Eq. 19 to Eq. 16.

Text S3. The partial derivatives of T_2 with respect to T_G and T_A

For all SVs, T_2 is computed using

$$\begin{aligned}T_2 &= T_S - \frac{(1-f_{urban})Q_{GRASS}+f_{urban}Q_U}{C_a}r_2\\ &= (1-f_{urban})T_{GRASS}+f_{urban}\left(\frac{Q_U}{C_a}r_U+T_A\right)-\frac{(1-f_{urban})Q_{GRASS}+f_{urban}Q_U}{C_a}r_2\\ &= (1-f_{urban})T_{GRASS}+f_{urban}\frac{Q_U}{C_a}(r_U-r_2)+f_{urban}T_A-(1-f_{urban})\frac{Q_{GRASS}}{C_a}r_2\end{aligned} \tag{20}$$

Hence, one can show that for all SVs

$$\frac{\partial T_2}{\partial T_G} = f_{urban}\frac{1}{C_a}\frac{\partial Q_U}{\partial T_G}(r_U-r_2) = \left(\frac{r_U-r_2}{r_U}\right)\frac{\partial T_S}{\partial T_G}. \tag{21}$$

The above equation indicates that $\partial T_2/\partial T_G$ scales with but is smaller than $\partial T_S/\partial T_G$. Hence, $\partial T_2/\partial T_G$ increases with smaller r_C (i.e., is larger in SV 3 than SV 1) and is larger in SV 2 than SV 1.

One can further show that

$$\frac{\partial T_2}{\partial T_A} = f_{urban}\frac{1}{C_a}\frac{\partial Q_U}{\partial T_A}(r_U - r_2) + f_{urban} + (1 - f_{urban})\frac{r_2}{r_{GRASS}}. \tag{22}$$

Since $\partial Q_U/\partial T_A < 0$, $0 < r_U - r_2 < r_U$, and $f_{urban} < 1$, one can prove that $\partial T_2/\partial T_A > \partial T_S/\partial T_A$. Moreover, the above equation can be formulated as

$$\frac{\partial T_2}{\partial T_A} = \left(\frac{r_U - r_2}{r_U}\right)\left(\frac{\partial T_S}{\partial T_A} - f_{urban}\right) + f_{urban} + (1 - f_{urban})\frac{r_2}{r_{GRASS}}. \tag{23}$$

Given that $\partial T_S/\partial T_A$ decreases with smaller r_C, $\partial T_2/\partial T_A$ decreases with smaller r_C (i.e., is smaller in SV 3 than SV 1).

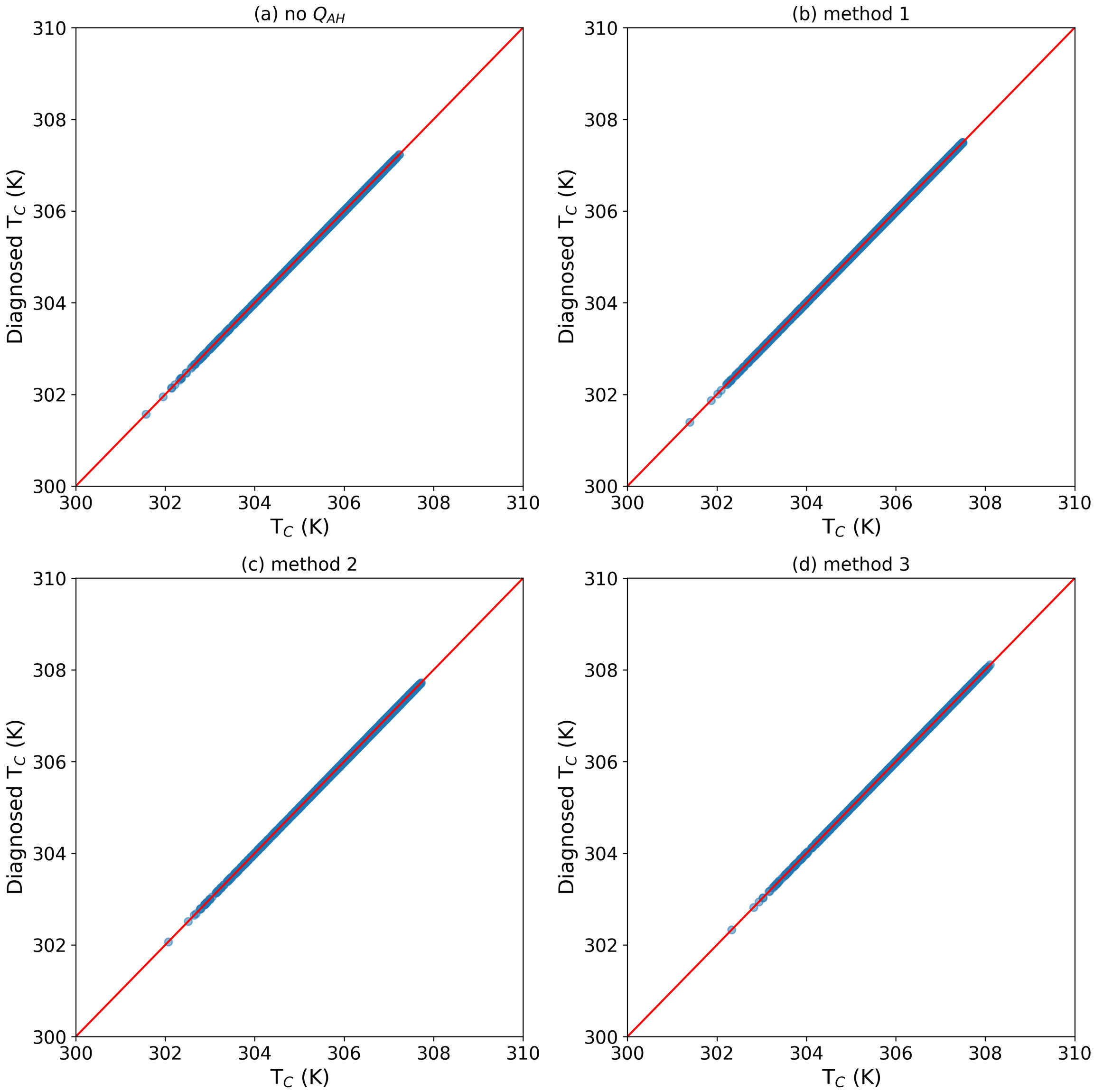

Figure S1. (a-d) Comparison between WRF outputted T_C and diagnosed T_C. These results are for SV 1 (a) with no Q_{AH} and (b-d) with $\Delta Q_{AH} = 10$ W m^{-2}.

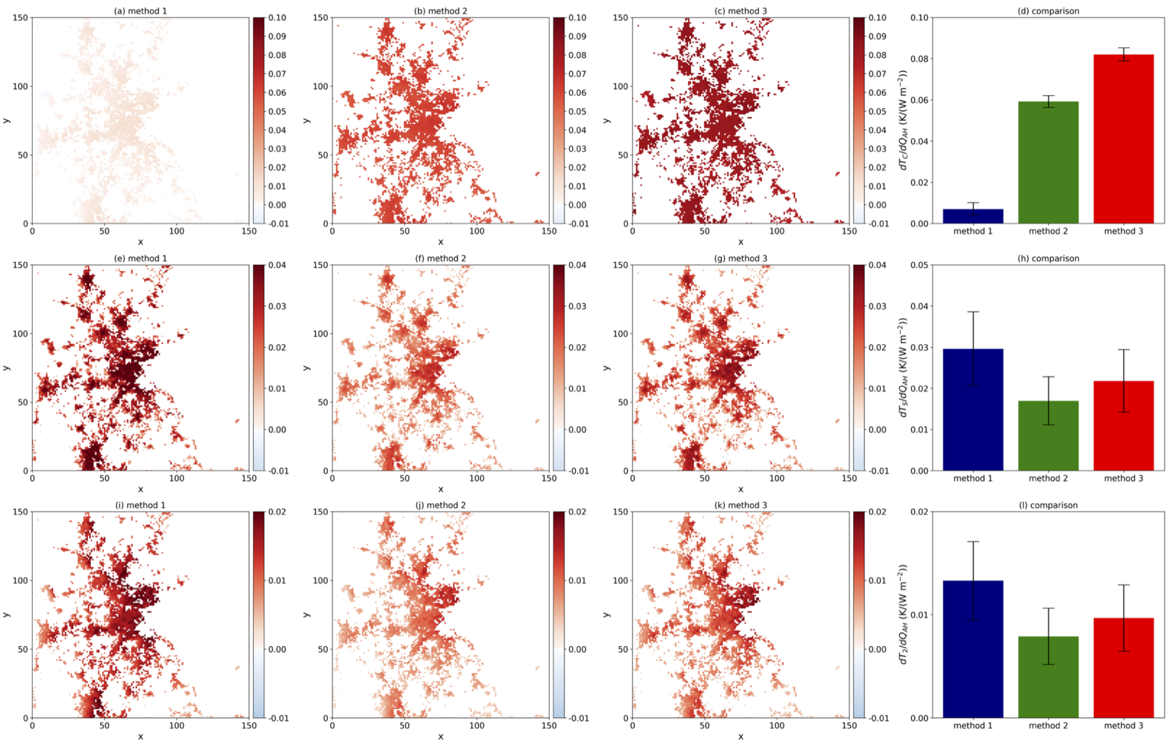

Figure S2. (a-d) Spatial patterns of dT_C/dQ_{AH} (unit: K/(W m^{-2})) estimated with three different Q_{AH} release methods and their spatial mean values and variabilities; (e-h) spatial patterns of dT_S/dQ_{AH} (unit: K/(W m^{-2})) estimated with three different Q_{AH} release methods and their spatial mean values and variabilities; (i-l) spatial patterns of dT_2/dQ_{AH} (unit: K/(W m^{-2})) estimated with three different Q_{AH} release methods and their spatial mean values and variabilities. These results are for $\Delta Q_{AH} = 100$ W m^{-2} and SV 1.

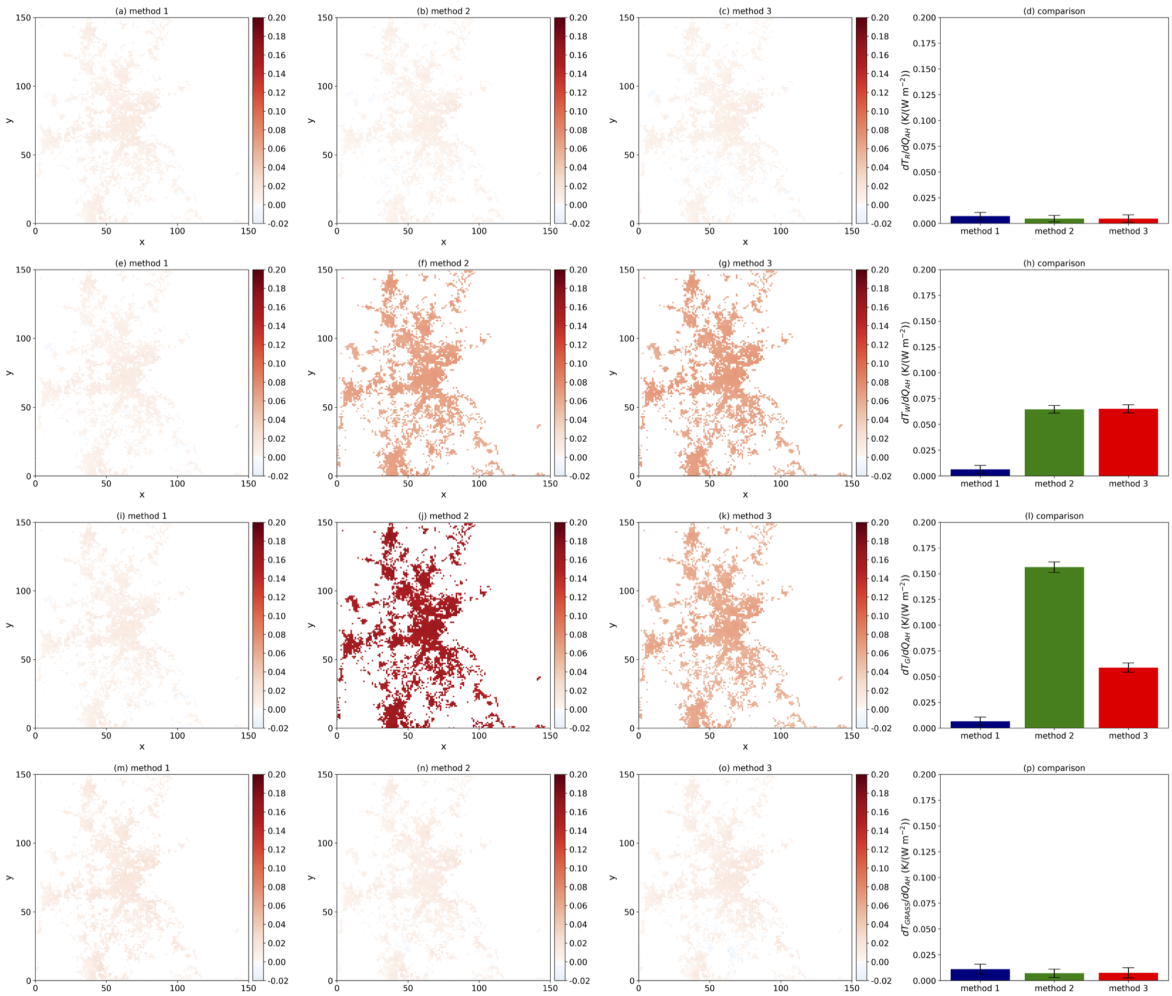

Figure S3. Similar to Figure S2 but for (a-d) dT_R/dQ_{AH}, (e-h) dT_W/dQ_{AH}, (i-l) dT_G/dQ_{AH}, (o-r) dT_{GRASS}/dQ_{AH}. These results are for $\Delta Q_{AH} = 100$ W m^{-2} and SV 1.

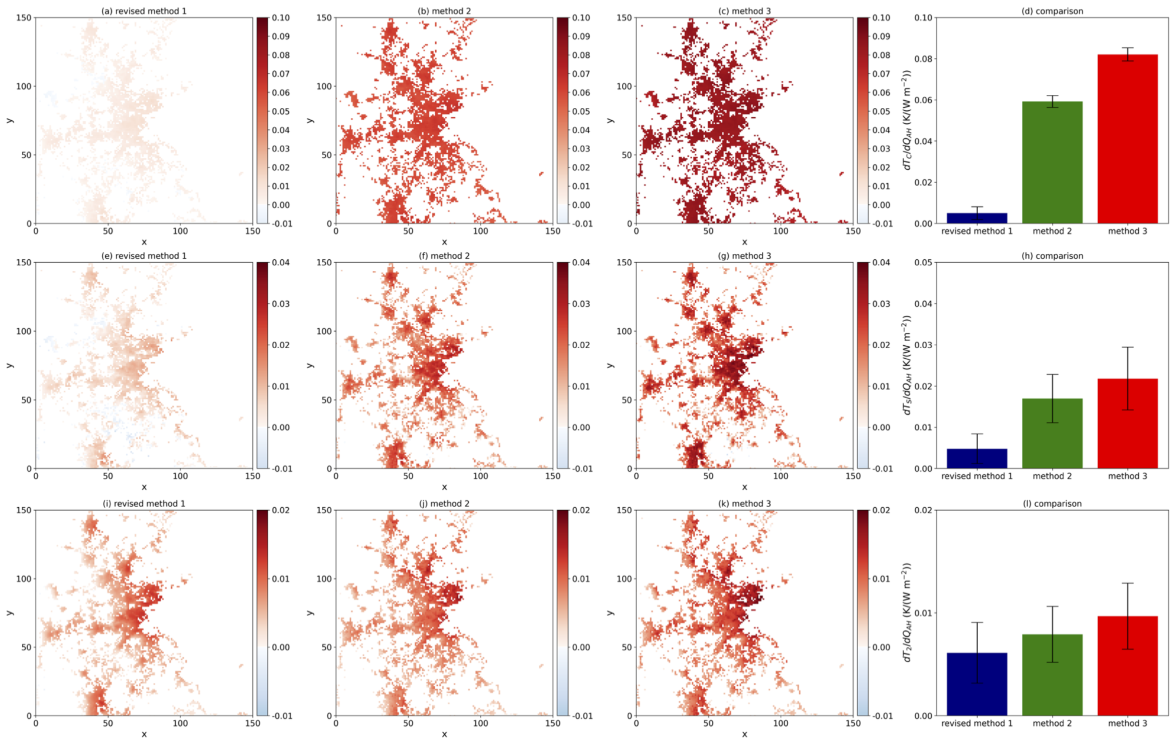

Figure S4. Similar to Figure S2 but with revised method 1 replacing method 1.

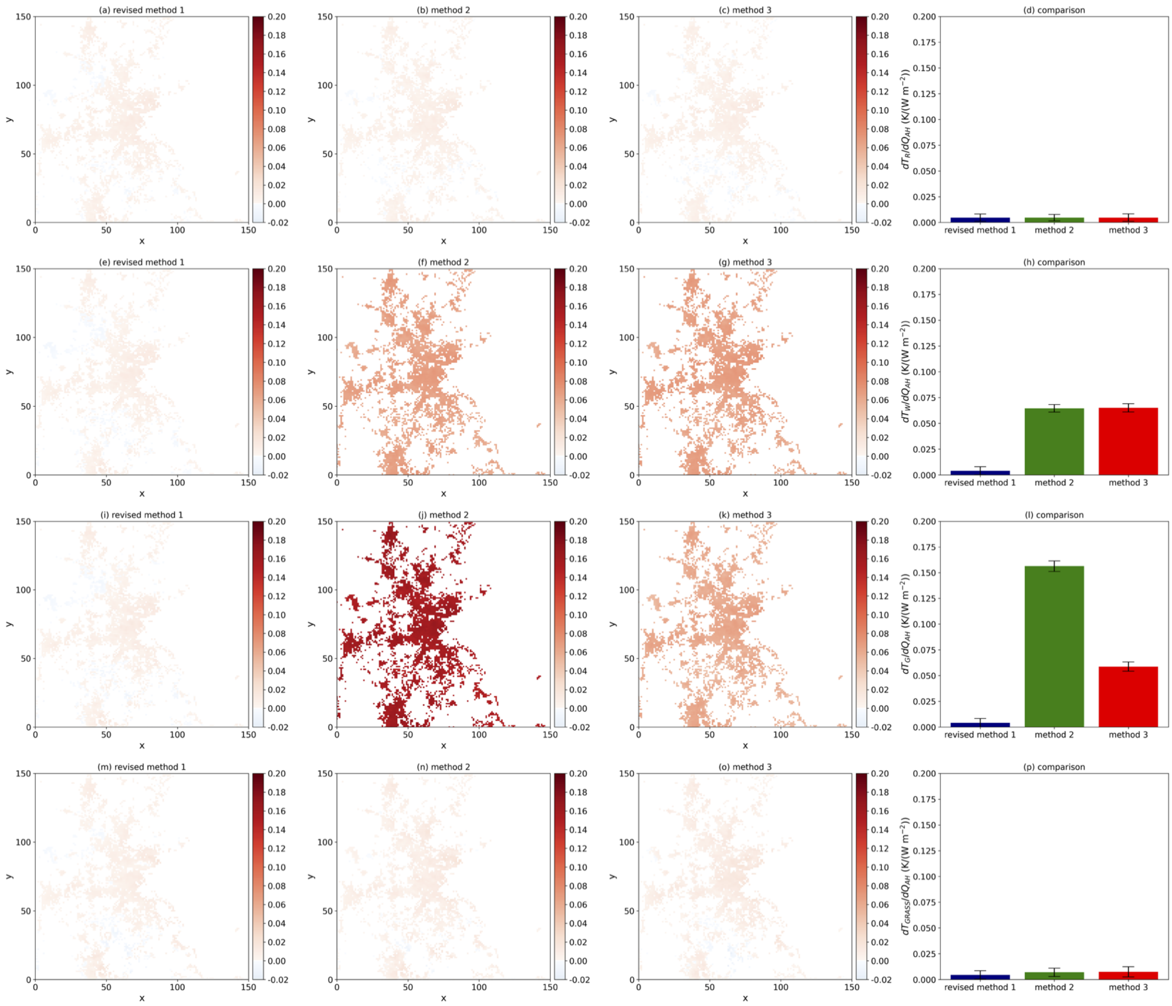

Figure S5. Similar to Figure S3 but with revised method 1 replacing method 1.

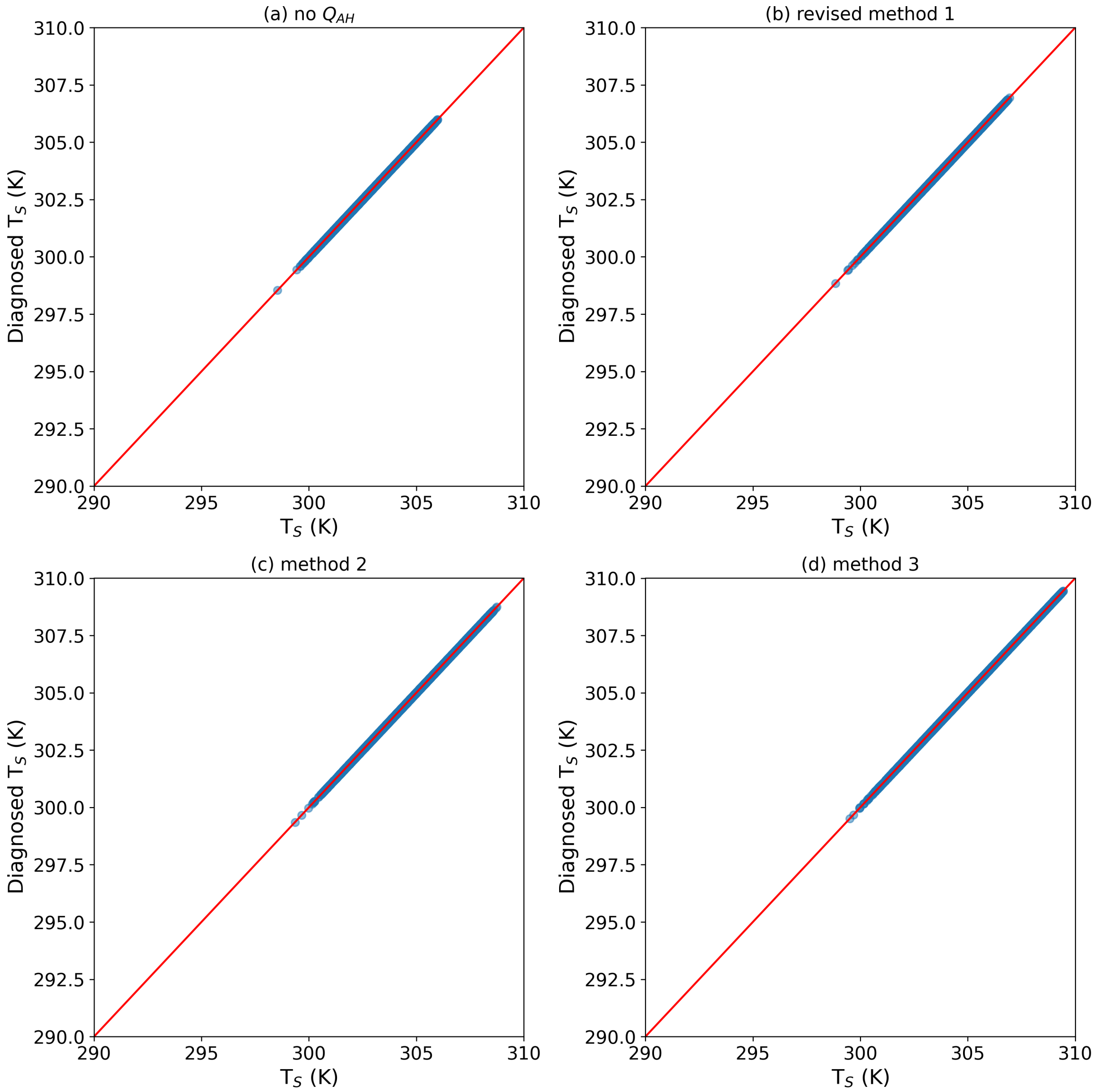

Figure S6. (a-d) Comparison between WRF outputted T_S and diagnosed T_S. These results are for SV 1 with (a) $\Delta Q_{AH} = 0$ W m^{-2} and (b-d) $\Delta Q_{AH} = 100$ W m^{-2}.

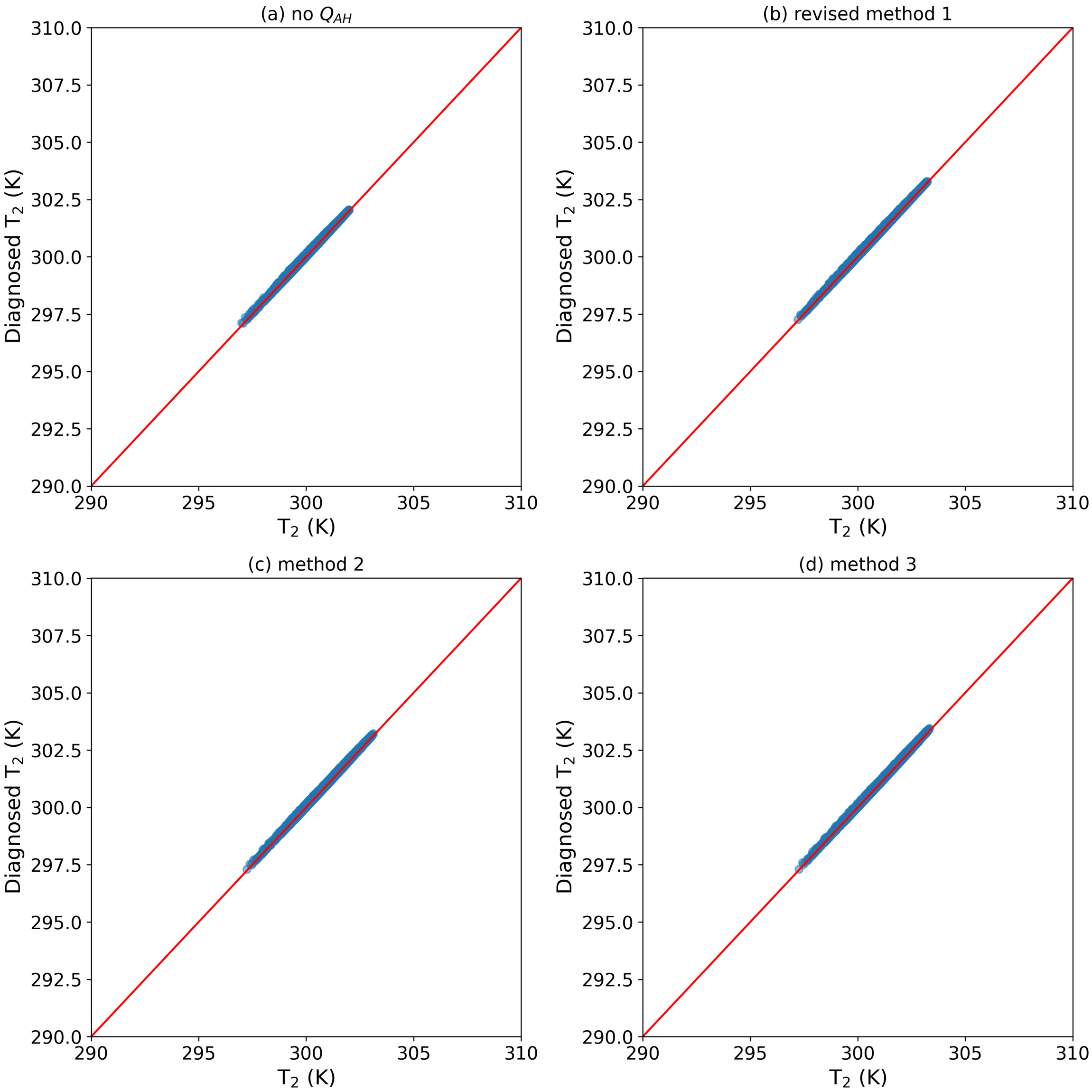

Figure S7. (a-d) Comparison between WRF outputted T_2 and diagnosed T_2. These results are for SV 1 with (a) $\Delta Q_{AH} = 0$ W m^{-2} and (b-d) $\Delta Q_{AH} = 100$ W m^{-2}.

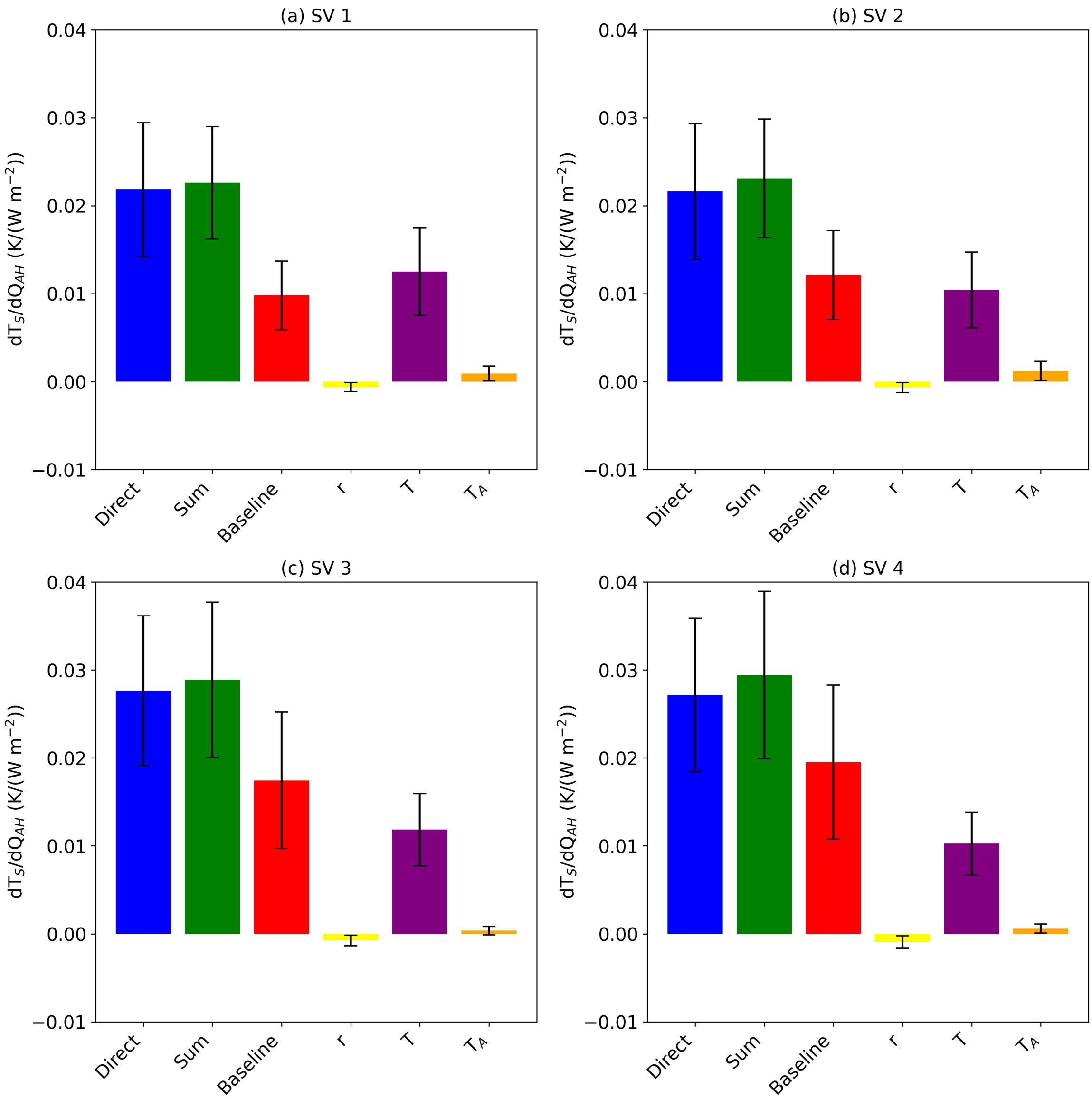

Figure S8. dT_S/dQ_{AH} across 4 SVs. These results are for method 3, $\Delta Q_{AH} = 100$ W m^{-2}, and all urban types.

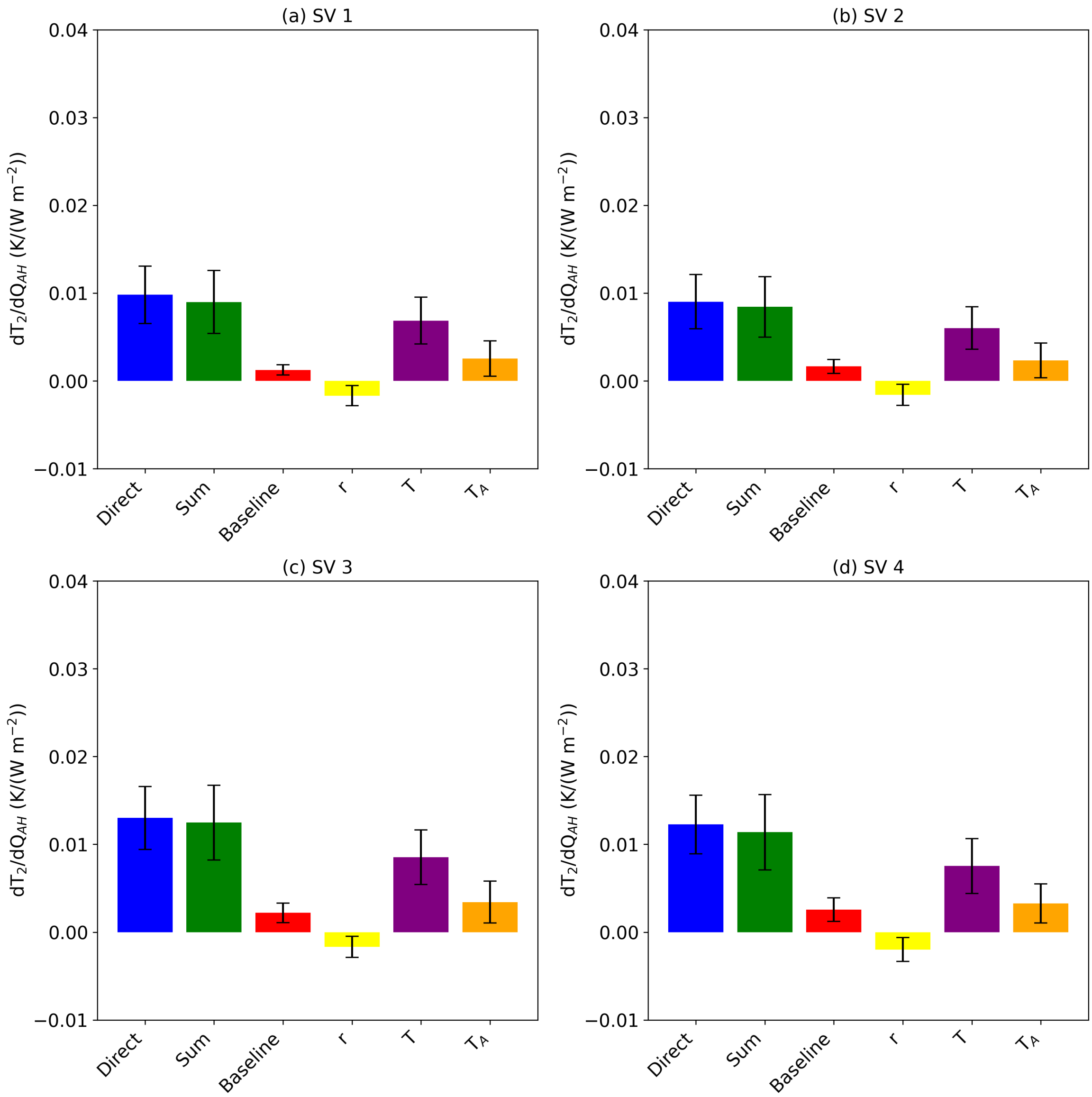

Figure S9. dT_2/dQ_{AH} across 4 SVs. These results are for method 3, $\Delta Q_{AH} = 100$ W m^{-2}, and all urban types.

Table S1. Land use classification for NLCD40 in WRF

21	Open Water
22	Perennial Ice/Snow
23	Developed Open Space
24	Developed Low Intensity
25	Developed Medium Intensity
26	Developed High Intensity
27	Barren Land
28	Deciduous Forest
29	Evergreen Forest
30	Mixed Forest
31	Dwarf Scrub
32	Shrub/Scrub
33	Grassland/Herbaceous
34	Sedge/Herbaceous
35	Lichens
36	Moss
37	Pasture/Hay
38	Cultivated Crops
39	Woody Wetland
40	Emergent Herbaceous Wetland

Table S2. Summary of morphological/thermal properties in SLUCM for 3 urban types

Parameters	1	2	3
Mean building height (m)	5.0	7.5	10.0
Roof width (m)	8.3	9.4	10.0
Road width (m)	8.3	9.4	10.0
Standard deviation of roof height (m)	1.0	3.0	4.0
Heat capacity of roof (MJ m^{-3} K^{-1})	1.0	1.0	1.0
Heat capacity of wall (MJ m^{-3} K^{-1})	1.0	1.0	1.0
Heat capacity of road (MJ m^{-3} K^{-1})	1.4	1.4	1.4
Thermal conductivity of roof (W m^{-1} K^{-1})	0.67	0.67	0.67
Thermal conductivity of wall (W m^{-1} K^{-1})	0.67	0.67	0.67
Thermal conductivity of road (W m^{-1} K^{-1})	0.4004	0.4004	0.4004
Albedo of roof	0.2	0.2	0.2
Albedo of wall	0.2	0.2	0.2
Albedo of road	0.2	0.2	0.2
Emissivity of roof	0.9	0.9	0.9
Emissivity of wall	0.9	0.9	0.9
Emissivity of road	0.95	0.95	0.95