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Key Points:
- The long-standing issue of the Reynolds number dependence of scalar roughness length and laminar resistance is examined
- Different scaling laws proposed in previous studies can be reconciled in a unified framework
- The scalar transfer mechanism in the viscous sublayer determines the Reynolds number dependence

Supporting Information:
- Supporting Information S1

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Reconciling the Reynolds number dependence of scalar roughness length and laminar resistance

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Abstract The scalar roughness length and laminar resistance are necessary for computing scalar fluxes in numerical simulations and experimental studies. Their dependence on flow properties such as the Reynolds number remains controversial. In particular, two important power laws ("1/4" and "1/2"), both having strong theoretical foundations, have been widely used in various parameterizations and models. Building on a previously proposed phenomenological model for interactions between the viscous sublayer and the turbulent flow, it is shown here that the two scaling laws can be reconciled. The 1/4 power law corresponds to the situation where the vertical diffusion is balanced by the temporal change or advection due to a constant velocity in the viscous sublayer, while the 1/2 scaling corresponds to the situation where the vertical diffusion is balanced by the advection due to a linear velocity profile in the viscous sublayer. In addition, the recently proposed "1" power law scaling is also recovered, which corresponds to the situation where molecular diffusion dominates the scalar budget in the viscous sublayer. The formulation proposed here provides a unified framework for understanding the onset of these different scaling laws and offers a new perspective on how to evaluate them experimentally.

1. Introduction

It is hard to overstate the importance of scalar fluxes from the Earth's surface such as sensible heat and water vapor fluxes. In numerical models and experimental studies, Monin-Obukhov similarity theory (MOST) is widely employed to compute scalar fluxes (as well as momentum fluxes) from the mean differences of state variables between the surface and the air [*Stull*, 1988; *Garratt*, 1994; *Brutsaert*, 2005]. Theoretically, MOST applies to a high-Reynolds number turbulent flow that is stationary, horizontally homogenous, without any subsidence, and not subject to any mean horizontal pressure gradients or Coriolis forces [*Obukhov*, 1946; *Monin and Obukhov*, 1954; *Businger and Yaglom*, 1971]. As a result, MOST only applies to the lowest 5–10% of the atmospheric boundary layer (ABL). This layer is often called the atmospheric surface layer (ASL) and is about 50–100 m above the ground [*Stull*, 1988; *Garratt*, 1994]. However, very close to the surface (~1 mm), the flow is not fully turbulent. This layer is often termed "viscous sublayer" or "diffusive sublayer" to emphasize the importance of viscosity and/or molecular diffusivity [*Brutsaert*, 1982].

To account for the effect of viscous and diffusive sublayers in MOST (but without resolving them), the surface in MOST is often assumed to start from some distance above the "real" ground level even when there is no canopy (i.e., when the displacement height is zero). This distance is called the "roughness length" [*Brutsaert*, 1982; *Garratt*, 1994]. The roughness lengths for momentum and scalars are typically different for two reasons [*Brutsaert*, 1982]. First and foremost, scalar transfer is always dominated by molecular diffusion but momentum transfer is also significantly affected by the pressure force. This is particularly the case for hydraulically rough surfaces where the viscous sublayer depth (δ) is smaller than the characteristic size of roughness elements (k_s). In fact, the momentum roughness length is often larger than the scalar roughness length over rough surfaces because the pressure force is more efficient in generating momentum flux than the viscous force. Second, the molecular diffusivities for scalars are not necessarily equal to the viscosity. The ratio of viscosity (ν) to molecular diffusivity (D) is the Schmidt number (for mass transfer) or the Prandtl number (for heat transfer). This study focuses on the scalar roughness length assuming a prior knowledge of the momentum roughness length, which can be, for example, related to the height and geometry of roughness elements [*Brutsaert*, 1982; *Garratt*, 1994]. This ratio of momentum to scalar roughness lengths is also closely related to the concept of "laminar resistance" [*Brutsaert*, 1982; *Garratt*, 1994] that is widely used in atmospheric chemistry and biogeoscience [*Wesely and Hicks*, 2000].

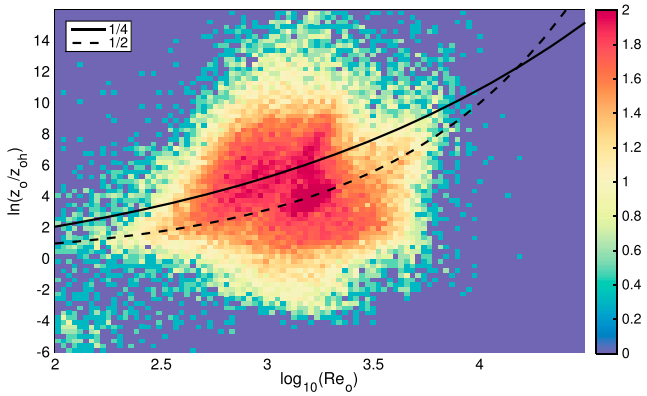

Figure 1. The dependence of $\ln(z_o/z_{oh})$ on the roughness Reynolds number ($Re_o = u_* z_o/\nu$). The color indicates the 2-D histogram in the logarithmic scale. The line ($\ln(z_o/z_{oh}) = 1.29\,Re_o^{1/4} - 2$) and dashed line ($\ln(z_o/z_{oh}) = 0.1\,Re_o^{1/2}$) show the 1/4 and 1/2 scaling, respectively, and are taken from two widely used parameterizations [*Kanda et al.*, 2007; *Zilitinkevich*, 1995]. The data are taken from half-hourly eddy covariance measurements over 17 AmeriFlux grassland sites. Only data in the summer season and only the eight data points in the daytime when the sensible heat flux is the largest and larger than 20 W m^{-2} are used.

Due to the different transfer mechanisms for momentum and scalars, it is expected that the ratio of momentum roughness length (z_o) to scalar roughness length (z_{oh}) and the laminar resistance are flow dependent and, in particular, Reynolds number (Re) dependent. They should be also Schmidt (or Prandtl) number dependent across fluids, but for a given fluid and a given scalar the Schmidt (or Prandtl) number is a constant. As such, this study only focuses on the Reynolds number dependence over rough surfaces. Previous studies have diverging views on the Reynolds number dependence of the scalar roughness length and laminar resistance. In particular, two different scaling results have been proposed and both seem to have strong theoretical foundations. *Zilitinkevich* [1995] [see also *Zilitinkevich et al.*, 2001] and *Yaglom and Kader* [1974] argued that the natural logarithm of the ratio of momentum to scalar roughness lengths $\ln(z_o/z_{oh})$ and the laminar resistance, respectively, should scale as $Re^{1/2}$. On the other hand, Brutsaert's theoretical work demonstrated that they should scale as $Re^{1/4}$ [*Brutsaert*, 1965, 1975a, 1975b]. This important difference in the scaling is reflected in the most widely used parameterizations of scalar roughness length to date [*Yang et al.*, 2008; *Chen and Zhang*, 2009; *Chen et al.*, 2010, 2011; *Zheng et al.*, 2012; *Li and Bou-Zeid*, 2014]. Besides these two scaling laws, recent studies suggested a Re^1 scaling [*Haghighi et al.*, 2013; *Haghighi and Or*, 2013] and other empirically based scaling laws also exist [*Owen and Thomson*, 1963; *Zeng and Dickinson*, 1998].

The scatter in field data is often too large to allow a rigorous rebuttal of one scaling or another. Figure 1 shows an example where $\ln(z_o/z_{oh})$ inferred from half-hourly eddy covariance measurements over 17 AmeriFlux sites (see Supporting Information S1 for details) is plotted as a function of the roughness Reynolds number (Re_o) defined later. The solid line and the dashed line, representing the "1/4" and "1/2" scaling, respectively, are derived from two widely used parameterizations [*Kanda et al.*, 2007; *Zilitinkevich*, 1995]. As a data quality control measure, only data collected over grassland sites in the summer season were used. In addition, we only selected the eight data points in the daytime when the sensible heat flux was the largest. Despite these stringent data quality control measures to avoid the effects of land cover types, seasonality, and diurnal cycle, it is still difficult to rigorously determine which scaling is better supported by field data. At higher Reynolds numbers, the scatter in observational data is even larger [*Kanda et al.*, 2007].

Motivated by this, our work aims to unravel the origin of the aforementioned differences in the Reynolds number dependence of scalar roughness length and laminar resistance using a theoretical approach. By reconciling these scaling laws in a unified framework, conditions under which each scaling applies can be gained. Additional benefits of such a unified framework pertain to the design of (laboratory, field or numerical) experiments so as to evaluate the onset of these scaling results and any intermediate transition. Nonetheless, it is outside the scope of this work to generate better parameterizations for the scalar roughness length or laminar resistance. As a logical starting point, we make the following assumptions throughout the paper:

1. The Schmidt (or Prandtl) number ν/D is unity. As a result, the viscous sublayer and diffusive sublayer will be used interchangeably. Note that ν/D is the molecular Schmidt (or Prandtl) number in contrast to the turbulent Schmidt (or Prandtl) number. This is a reasonable assumption since the molecular Schmidt (or Prandtl) number of most scalars (e.g., temperature, water vapor, and CO_2) ranges from 0.5 to 1.2 in the air.
2. All scalars considered in this study are passive scalars, which do not affect the flow dynamics. In particular, the buoyancy effect is not considered and the flow is neutrally stratified.
3. There is no canopy (vegetation or urban) on the surface (i.e., the displacement height is zero).

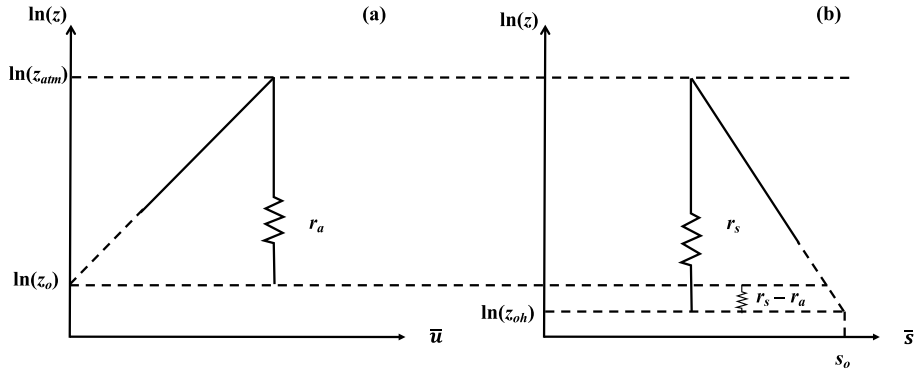

Figure 2. A schematic of the logarithmic (a) mean velocity and (b) scalar profiles under neutral conditions. When the logarithmic profiles are interpolated to their surface values, the intercepts are $\ln(z_o)$ and $\ln(z_{oh})$ for velocity and scalars, respectively. The dashed lines here are interpolations or idealizations and do not reflect the profiles in reality.

2. Background

2.1. MOST

The assumptions made by MOST, when applied to the mean velocity and scalar concentration budgets, yield that momentum and scalar fluxes are constant in the vertical direction, so the ASL is also called the "constant-flux" layer [*Stull*, 1988; *Garratt*, 1994; *Brutsaert*, 2005]. Therefore, the surface fluxes are equal to the turbulent fluxes at any height z above the ground surface (or above the displacement height when it is not zero) in the ASL and are commonly parameterized as $\tau = \rho u_*^2 = -\rho C_m \overline{u}(0 - \overline{u})$ and $F_s = C_s \overline{u}(\overline{s_o} - \overline{s})$, where τ denotes the momentum flux or the turbulent stress; F_s denotes the scalar flux; ρ is the air density; u_* is the friction velocity; C_m and C_s are the turbulent transfer coefficients for momentum and scalars, respectively; u denotes the horizontal or streamwise velocity at height z; s is the scalar concentration at z, and s_o is the scalar concentration at the surface ($z = 0$). Note here that z has to be well outside of the viscous sublayer and the diffusive sublayer. The overline denotes Reynolds averaging. It is clear that the above parameterizations relate turbulent fluxes to the vertical gradients of the mean variables (e.g., mean velocity and mean scalar concentration), which can be measured in experimental studies or resolved in numerical models.

MOST then provides a way to compute C_m and C_s, which under neutral conditions are $C_m = \kappa^2 / \left[ln(\frac{z}{z_o}) \right]^2$ and $C_s = \kappa^2 / \left[ln(\frac{z}{z_o}) ln(\frac{z}{z_{oh}}) \right]$, where $\kappa \approx 0.4$ is the von Kármán constant. This is based on the fact that under neutral conditions, the profiles of mean velocity and mean concentration are logarithmic (see Figure 2), and the assumption that the turbulent Schmidt (or Prandtl) number is also unity [*Kays*, 1994; *Katul et al.*, 2013, 2014, 2016; *Li et al.*, 2012, 2015a, 2015b; *Li*, 2016]. Note that the effect of a nonunity turbulent Schmidt (or Prandtl) number can be easily incorporated. It is evident that momentum (z_o) and scalar (z_{oh}) roughness lengths are needed for calculating C_m and C_s and thus τ and F_s. The difference between z_{oh} and z_o largely controls the ratio C_m/C_s. When $z_{oh} < z_o$, $C_s < C_m$, which means that the turbulent transfer of momentum is more efficient than that of scalars.

2.2. The Extra and Laminar Resistance

In many disciplines the resistance concept is used instead of the roughness length [*Brutsaert*, 1982; *Garratt*, 1994]. The momentum and scalar fluxes are defined as $u_*^2 = \overline{u}/r_a$ and $F_s = (\overline{s_o} - \overline{s})/r_s$, respectively, where r_a and r_s are the total aerodynamic resistance (including both laminar and turbulent parts) for momentum and scalars, respectively (see Figure 2). Their difference $(r_s - r_a)$ is again related to the fact that very close to the surface, scalar transfer is always dominated by molecular diffusion while the stress is dominated by the form drag for rough surfaces at high Reynolds number. This leads to the notion that scalar transfer experiences an extra resistance $(r_s - r_a)$ due to the molecular diffusion process. It is straightforward to show that the extra resistance is $r_s - r_a = (\kappa u_*)^{-1} ln(z_o/z_{oh})$. Hence, $\ln(z_o/z_{oh})$ has the same scaling as $u_*(r_s - r_a)$.

To understand the extra resistance further, the scalar flux formulation can be evaluated between the surface and the top of the viscous/diffusive sublayer (δ), $F_s = \left[s_o - s(z = \delta) \right] / r_{sl}$. Note for laminar flows $F_s = -Dds/dz$. Assuming $ds/dz = [s(z = \delta) - s_o]/\delta$ yields $r_{sl} = \delta/D$. We drop the overbar here since the flow is laminar thus the Reynolds average is meaningless. Here the resistance r_{sl} is no longer the total resistance or a turbulent resistance. It is a resistance for scalars in the viscous sublayer and is a 'laminar resistance'. In order to

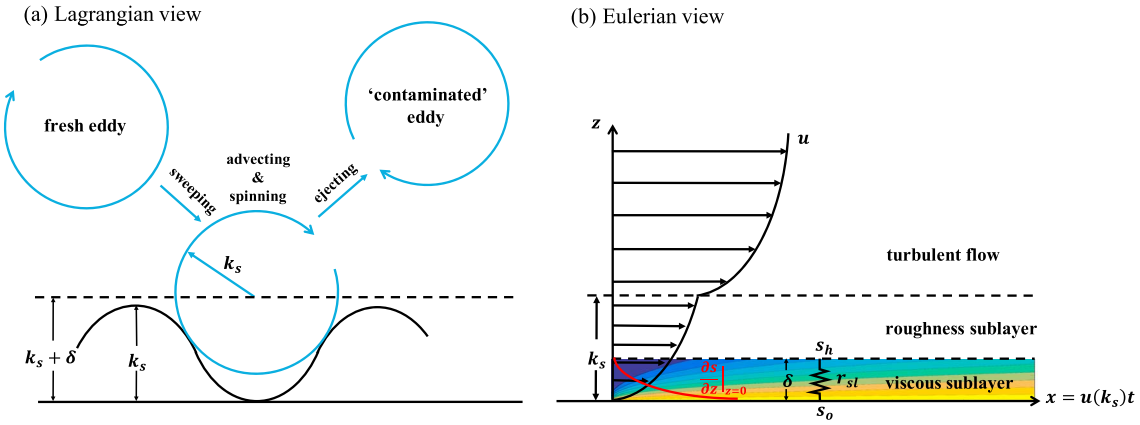

Figure 3. A schematic of the phenomenological model from the (a) Lagrangian view and the (b) Eulerian view. In Figure 3a, turbulent eddies carry away the flux through the process of sweeping, advecting, spinning, and ejecting. Here only the largest eddies that are constrained by the characteristic length of roughness elements (k_s) are shown. In Figure 3b, the velocity profile is assumed to follow a general power law below k_s and the logarithmic profile in the turbulent flow regime. Scalar transfer within the viscous sublayer (δ) is dominated by advection and diffusion. The color contour map is derived from the solution equation (3) using typical values and the red solid line shows the $\partial s/\partial z$ at the surface as a function of x, which is also the advection distance by a turbulent eddy ($= u(k_s)t$).

distinguish the two, we use r_{sl} with the subscript 'l' denoting laminar. In the following, r_{sl} will be called the laminar resistance and $r_s - r_a$ will be called the extra resistance. The laminar resistance r_{sl} is closely related to, but not exactly equal to, the extra resistance $r_s - r_a$. If one assumes that the mean velocity and scalar concentration profiles in the turbulent flow follow MOST and are continuous at the interface between the turbulent flow and the viscous sublayer, it can be shown that the two are related through $r_{sl} = r_s - r_a + u(z = \delta)/u_*^2$ [*Brutsaert*, 1982], where $u(z = \delta)/u_*{}^2$ can be viewed as the laminar resistance for momentum, which is rather small compared to r_a and is often treated as a constant for both hydraulically smooth and rough surfaces [*Brutsaert*, 1982]. As such, the scaling of $\ln(z_o/z_{oh})$, $u_*(r_s - r_a)$, and $u_* r_{sl}$ with respect to the Reynolds number is the same.

2.3. The Roughness Reynolds Number

Given the focus of this study, an unambiguous definition of the Reynolds number is needed. The velocity scale in the definition of Reynolds number is clear since u_* is the best choice. The length scale, on the other hand, varies among studies. For practical purposes, especially in numerical modeling, the roughness Reynolds number $Re_o = u_* z_o/\nu$ is often used (Figure 1). This is because the momentum roughness length (z_o) is often an input to numerical models. However, z_o is not a physical length scale but rather the intercept of the logarithmic mean velocity profile from MOST (see Figure 2). In this study, the characteristic length of roughness elements (k_s) is used as it is more physically based (see Figure 3) and the resulting Reynolds number is denoted as Re_k. However, the two are often related to each other through simple geometric relations (e.g., $z_o = 0.1k_s$) [*Brutsaert*, 1982].

2.4. The 1/2 Scaling

The "1/2" scaling argument is rooted in laminar boundary layer theory of Prandtl and the exact solutions are obtained by Blasius and Pohlhausen for velocity and temperature profiles, respectively [*Schlichting and Gersten*, 2003]. These exact solutions were initially developed for smooth surfaces and validated at the leaf scale [*Parlange et al.*, 1971; *Parlange and Waggoner*, 1972]. Later, *Yaglom and Kader* [1974] applied the same scaling to rough surfaces. They argued that the laminar boundary layer or the viscous sublayer depth $\delta \sim Re^{-1/2}L$, where $Re = UL/\nu$, U is the characteristic velocity scale ($\sim u_*$), and L is the characteristic length scale ($\sim k_s$) of the outer flow. Note that this choice of outer flow length scale (i.e., the vertical length scale of roughness elements) is different from many other studies over smooth surfaces where the outer flow length scale is characterized by the streamwise distance [*Schlichting and Gersten*, 2003; *Parlange et al.*, 1971; *Parlange and Waggoner*, 1972]. Given $r_{sl} = \delta/D$, $u_* r_{sl} \sim Re_k^{1/2}$. A different argument was provided by *Zilitinkevich et al.* [2001] using momentum and scalar roughness lengths, which nonetheless yielded the same scaling.

2.5. The 1/4 Scaling

In the 1/4 scaling argument, which is now well accepted in air-water interactions [*Lorke and Peeters*, 2006; *Zappa et al.*, 2007; *Katul and Liu*, 2017], it is assumed that δ scales as the Kolmogorov scale $\eta = (\nu^3/\epsilon)^{1/4}$

[*Kolmogorov*, 1941] or the smallest scale in the turbulent flow, where ϵ is the mean dissipation rate of turbulent kinetic energy (TKE) and is equal to the production rate $u_*^3/(\kappa k_s)$ under the assumption of local equilibrium. As a result, $r_{sl} = \delta/D \sim \eta/\nu \sim (\nu^3 k_s/u_*^3)^{1/4}/\nu$ and thus $u_* r_{sl} \sim Re_k^{1/4}$. It is worth noting that the 1/4 scaling does not require considerations of the scalar budget in the viscous sublayer as it can be derived from turbulent quantifies such as the Kolmogorov scaling and the TKE budget.

3. Interaction Between the Viscous/Diffusive Sublayer and the Turbulent Flow: A Phenomenological Model

The overarching target of this work is to reconcile the different scaling laws under the same framework. To do so, we adopt the phenomenological model proposed by *Brutsaert* [1965, 1975a, 1975b], which originally yielded the 1/4 scaling. We will show that the same framework can also produce the 1/2 scaling and the 1 scaling. More importantly, it illustrates that the difference between these scalings lies in how the scalar transfer in the viscous/diffusive sublayer, rather than in the turbulent flow, is represented.

The framework starts by assuming that the flux from the top of the viscous/diffusive layer (δ), covering roughness elements of characteristic size k_s, is continuously "renewed" or carried away by turbulent eddies (Figure 3a). For rough surfaces, $\delta \ll k_s$. Such idealizations of the viscous sublayer and roughness elements have been also proposed elsewhere [*Gioia and Bombardelli*, 2002; *Gioia and Chakraborty*, 2006]. The model does not explicitly account for quiescent periods, and thus, the conceptualization of continuous movement and replacement of turbulent eddies is more justified if an ensemble view is taken. Following *Brutsaert* [1975b], it is further assumed that these eddies come in sequence following an exponential distribution with the Kolmogorov time scale $t_m = (\nu/\epsilon)^{1/2}$ being the mean time scale. The Kolmogorov scales are turbulent scales at which viscosity still has a significant influence. As a result, they are used here to characterize the turbulent eddies interacting with the viscous sublayer where viscosity and molecular diffusivity dominate the transfer. Given these assumptions, the scalar flux is defined as

$$F_s = -D \int_0^\infty a e^{-at} \left(\frac{\partial s}{\partial z}\right)_{z=0} \mathrm{d}t \tag{1}$$

where a is the average renewal rate ($= c_1/t_m$) and c_1 is a constant. Despite that the exponential distribution is used here, it can be shown that the Reynolds number dependence is not dependent on the assumed distribution [*Katul and Liu*, 2017]. The dissipation rate of TKE (ϵ) in the Kolmogorov time scale is assumed to be equal to the local production rate of TKE by the largest eddies interacting with roughness elements ($= u_*^3/(\kappa k_s)$), which thus are constrained by the size of roughness elements.

Inside the viscous sublayer (δ, see Figure 3b), the balance between the temporal change or local storage, advection and vertical diffusion is considered:

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} = D\frac{\partial^2 s}{\partial z^2}, \tag{2}$$

with the boundary conditions $s = s_h(z > 0, x = 0)$, $s = s_h(z \to \infty, x > 0)$, and $s = s_o(z = 0, x > 0)$. The first boundary condition is the inflow condition and assumes that the scalar concentration is uniform in the z direction. The second is the top boundary condition, which is imposed at $z \to \infty$ instead of $z = \delta$ because the solution of this problem is relatively insensitive to the exact location of the top boundary [*Brutsaert*, 1975b]. The third is the surface boundary condition, which implicitly assumes that s_o is not affected by the eddy renewal process. The diffusion in the x direction is assumed to be sufficiently smaller than the advection term and the vertical diffusion term and is thus neglected. We further neglect the temporal change term, but it shall be seen later that the balance between the temporal change and the vertical diffusion is in fact a special case of our model. This is accomplished by assuming that the velocity profile follows a general power law $u = u_*^{1+\alpha} z^\alpha/\nu^\alpha$, where α is the exponent. When $\alpha = 0$, $u = u_*$, and the advection term ($u\partial s/\partial x$) can be readily changed to the temporal change term ($\partial s/\partial t$) through the transformation of $x = u_* t$. Besides this, the use of a power law profile for u is motivated by a few other reasons: First, over smooth surfaces it is well accepted that the velocity profile is linear ($\alpha = 1$) inside the viscous sublayer and should gradually transition to a logarithmic profile in the turbulent flow. However, for rough surfaces the velocity profile remains unknown and deviations of α from unity should be expected given the influence of roughness elements on the flow. Second, the use of a general power law gives more flexibility to the model. As shall be seen later, the velocity profile will be

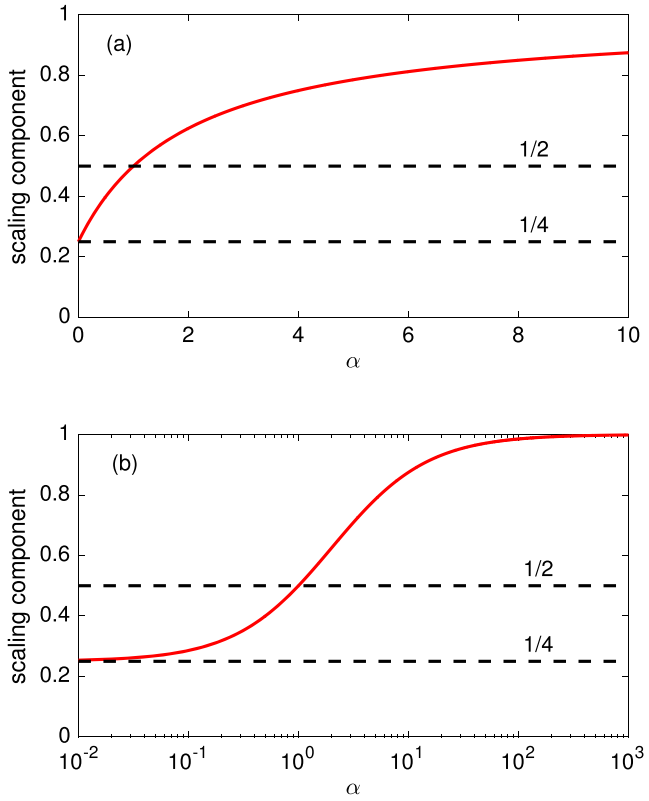

Figure 4. The Reynolds number dependence of (a) laminar resistance and (b) scalar roughness length as a function of the velocity profile power law component (α). The difference between Figures 4a and 4b is that Figure 4b uses the logarithmic scale for the x axis.

interpolated to $z = k_s$ in order to provide the "renewal" or advective velocity of turbulent eddies. A general power law profile justifies such interpolations as the solution is not tied to any particular values of α.

Solving the above equation with the aforementioned assumptions and with the prescribed boundary conditions yields

$$\frac{s - s_h}{s_o - s_h} = 1 - P\left[\frac{1}{2+\alpha}, \frac{u_*^{1+\alpha} z^{2+\alpha}}{(2+\alpha)^2 D \nu^\alpha x}\right], \tag{3}$$

where P is the incomplete gamma function. Since the scalar flux depends on $(\partial s/\partial z)_{z=0}$, which is x dependent, the average $(\partial s/\partial z)_{z=0}$ from 0 to x_o is calculated:

$$\left(\frac{\partial s}{\partial z}\right)_{z=0} = c_3 \left(\frac{u_*^{1+\alpha}}{\nu^\alpha D x_o}\right)^{1/(2+\alpha)} (s_o - s_h), \tag{4}$$

where $c_3 = -\frac{(2+\alpha)^{\frac{2+2\alpha}{2+\alpha}}}{(1+\alpha)\Gamma(\frac{1}{2+\alpha})}$ is a constant, x_o is the distance swept by an eddy. It is further assumed that $x_o = u\,(z = k_s)t$. This is because x_o is the advection distance and thus should be related to the advective velocity instead of the turbulent velocity scale u_*. In addition, the advective velocity at $z = k_s$ is used because below this height the turbulent eddies would be trapped by the roughness elements and thus could not perform the renewal process. Substituting equation (4) into equation (1) yields

$$r_{sl} u_* = \frac{(s_o - s_h) u_*}{F_s} \sim Re_k^{\frac{1+2\alpha}{4+2\alpha}}. \tag{5}$$

The scaling component $(1+2\alpha)/(4+2\alpha)$ as a function of the velocity profile power law component α is shown in Figure 4. Three special cases are identified as follows:

3.1. $\alpha = 0$

When $\alpha = 0$, $r_{sl} u_* \sim Re_k^{1/4}$ and the 1/4 scaling is recovered. Under such conditions, $u = u_*$ and $x = u_* t$. Thus, mathematically, the advection term $(u\partial s/\partial x)$ is equivalent to the temporal change term $(\partial s/\partial t)$. This is also why it recovers the results of *Brutsaert* [1975b], where a balance between the temporal change and the vertical diffusion was assumed.

3.2. $\alpha = 1$

When $\alpha = 1$, $r_{sl} u_* \sim Re_k^{1/2}$ and the 1/2 scaling is recovered. Under such conditions, $u = u_*^2 z/\nu$ and $x = (u_*^2 k_s/\nu)t$. The linear velocity profile ($\alpha = 1$) in the viscous sublayer is well known for smooth surfaces. We note that a similar linear profile was assumed in the study of *Brutsaert* [1975b], which however yielded a "1/6" scaling instead of 1/2 due to the use of $x_o = u_* t$ in equation (4).

3.3. $\alpha \to \infty$

When $\alpha \to \infty$ (see Figure 4b), $r_{sl} u_* \sim Re_k$ and the 1 scaling is recovered, as suggested by recent work [*Haghighi et al.*, 2013; *Haghighi and Or*, 2013] that assumed a balance between the vertical and horizontal diffusion. In our model, $\alpha \to \infty$ can only occur when the advection term does not play a role in the scalar budget $(\partial s/\partial x = 0)$ and thus molecular diffusion dominates. Hence, our result is broadly consistent with previous work [*Haghighi et al.*, 2013; *Haghighi and Or*, 2013].

4. Discussions and Summary

From the above analysis it is clear that the difference in the Reynolds number dependence of laminar resistance and scalar roughness length, at least within the phenomenological model, arises from the different mechanisms of scalar transfer in the viscous sublayer, rather than in the turbulent flow. When the vertical diffusion is balanced by the advection due to a constant velocity or by the temporal change, the 1/4 scaling is obtained. On the other hand, when the vertical diffusion is balanced by the advection due to a linear velocity profile, the 1/2 scaling prevails. When only molecular diffusion dominates, the 1 scaling appears. Our model not only recovers all the scaling laws previously proposed but also reveals the unique role of the velocity profile in shaping the Reynolds number dependence of laminar resistance and scalar roughness length.

Specifically, the relative importance of the temporal change term and the advection term with a linear velocity profile determines the ranges of applicability of the 1/2 and 1/4 scaling laws. For example, when the temporal change is much smaller than the advection term, $\frac{\partial s}{\partial t} \ll u\frac{\partial s}{\partial x}$, namely, $X \ll u(z = k_s)t_m \sim Re_k^{1/2}k_s$ where X is the characteristic length scale in the streamwise direction, the 1/2 scaling dominates. This requirement might be satisfied, at least marginally, for leaves on trees where X is simply constrained by the size of the leaf. However, for other flat surfaces in nature without discontinuities in terms of scalar sources, X is likely to be much larger than $Re_k^{1/2}k_s$. Under such conditions, the 1/2 scaling should not apply and more broadly the advection should not play an important role in the viscous sublayer. If the temporal change or local storage term remains important as compared to the vertical diffusion term, then the 1/4 scaling should apply.

It should be pointed out that our study is only concerned about rough surfaces as the difference between scalar and momentum transfer is not Reynolds number dependent over smooth surfaces, where their difference is caused by a nonunity Schmidt (or Prandtl) number. Although the linear velocity profile is well accepted for smooth surfaces, the 1/2 scaling should still not be confused with the 1/2 scaling often seen in studies of boundary layer flows over smooth surfaces where the advection distance is typically used to define the Reynolds number [*Schlichting and Gersten*, 2003; *Parlange et al.*, 1971; *Parlange and Waggoner*, 1972]. Over rough surfaces the velocity profile below the characteristic length of roughness elements (k_s) remains elusive and probably depends on the geometry of roughness elements and other factors. Deviations of α from 1 and thus deviations of the Reynolds number dependence from the 1/2 scaling should be expected over rough surfaces [*Owen and Thomson*, 1963] even when advection plays an important role. Figure 4 provides a means to connect the velocity profile to the Reynolds number dependence of laminar resistance and scalar roughness length. Future work involves testing this connection with simulations or observations and extending the analysis to consider the Schmidt (or Prandtl) number dependence, the active role of certain scalars (such as temperature), and the effect of vegetation and urban canopy.

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