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NOTES AND COMMENTS

An Update of Non-iterative Solutions for Surface Fluxes Under Unstable Conditions

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Abstract A new non-iterative approach has been developed to calculate surface fluxes. The range $-5 \leq Ri_{\mathrm{B}} < 0, 10 \leq z/z_0 \leq 10^5$ and $-0.5 \leq \ln(z_0/z_{0\mathrm{h}}) \leq 30$ is divided into eight regions, and in each of the regions, multiple linear regression is performed to obtain non-iterative solutions for surface fluxes. As compared to the other two most recent non-iterative schemes, we show that the suggested scheme has the smallest bias. The maximum relative errors of turbulent transfer coefficients for momentum (C_{M}) and sensible heat (C_{H}), as compared to those obtained from the classic iterative method, are always smaller than 2 % from our new non-iterative scheme.

Keywords Multiple linear regression · Non-iterative equation · Surface fluxes · Turbulent transfer coefficient

1 Introduction

In current numerical weather and climate models, algorithms based on the Monin–Obukhov similarity theory (MOST; Monin and Obukhov 1954) are commonly used to calculate surface

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fluxes. In MOST, the calculation of fluxes is represented by surface scaling parameters: friction velocity u_* and scalar temperature θ_* (see Sect. 2). Such calculation involves stability correction functions that depend solely on the stability parameter ($\zeta = z/L$, where z is the height above the displacement height and L is the Obukhov length), and roughness lengths (aerodynamic roughness length, z_0, and thermal roughness length, z_{0h}). However, the definition of the Obukhov length, $L \equiv u_*^2\overline{\theta}/(kg\theta_*)$, involves again u_* and θ_*, where g is the gravitational acceleration (m s^{-2}), k is the von Kármán constant, and $\overline{\theta}$ is the mean potential temperature at the surface. Sorbjan (2010) and Sorbjan and Grachev (2010) referred this self-dependent property to as 'self-correlation', and as a result, iterations are required in the calculation of surface fluxes.

To avoid iterations, non-iterative schemes that directly calculate the stability parameter ζ from the bulk Richardson number (Ri_B) and roughness lengths (z_0 and z_{0h}), have been proposed and widely used (e.g., Launiainen 1995; Kot and Song 1998; Yang et al. 2001; Li et al. 2010, 2014; Wouters et al. 2012; Sharan and Srivastava 2014). For example, the recently proposed non-iterative scheme of Li et al. (2010), which is valid for $-2 \le Ri_B \le 1$, $10 \le z/z_0 \le 10^5$ and $0.5 \le z_0/z_{0h} \le 100$, has been implemented in the Weather Research and Forecasting (WRF) model version 3.4 and later versions (e.g., http://www.nco.ncep.noaa.gov/pmb/codes/nwprod/sorc/wrf_shared.fd/phys/module_sf_mynn.F), and in the Japan Coastal Ocean Predictability Experiment-Tides (JCOPET) model (Miyazawa et al. 2012). To improve the accuracy and to expand the applicable range of the scheme of Li et al. (2010), Wouters et al. (2012, WRL12 hereafter) and Sharan and Srivastava (2014, SS14 hereafter) used semi-analytical methods to obtain new non-iterative equations for ζ. The WRL12 scheme is valid for $-5 \le Ri_B \le 2.5$, $10 \le z/z_0 \le 10^5$ and $-0.5 \le \ln(z_0/z_{0h}) \le 30$, and the SS14 scheme is valid for $-5 \le Ri_B \le 0$, $10 \le z/z_0 \le 10^5$ and $0 \le \ln(z_0/z_{0h}) \le 29$. However, Li et al. (2014, LGL14 hereafter) divided the roughness lengths (z_0 and z_{0h}) into eight regions in which the bulk Richardson number (Ri_B) range was further separated into several (4–7) sections, and then performed regression for each section. The LGL14 scheme is valid for $0 \le Ri_B \le 2.5$, $10 \le z/z_0 \le 10^5$ and $-0.5 \le \ln(z_0/z_{0h}) \le 30$. With the calculation range divided into several sections, the LGL14 scheme produced more coefficients but higher accuracy than did the WRL12 scheme. Nevertheless, LGL14 equations are only applicable in stable conditions. The objective of the present study is to extend the approach of LGL14 to unstable conditions in order to obtain more accurate non-iterative solutions for surface fluxes over a wide range of stability conditions.

The paper is organized as follows: Sect. 2 describes the classic iterative schemes with coefficients proposed by Paulson (1970), Businger et al. (1971), Dyer (1974) and Högström (1996), and briefly introduces non-iterative schemes proposed by WRL12 and SS14. Section 3 proposes the new non-iterative scheme and Sect. 4 intercompares these schemes. A summary and conclusions are presented in Sect. 5.

2 Revisiting the Classic Iterative Scheme and Non-iterative Schemes (the WRL12 and SS14 Scheme)

2.1 The Classic Iterative Scheme

The momentum flux τ and sensible heat flux H are defined as

$$\tau \equiv \rho u_*^2, \tag{1}$$

$$H \equiv -\rho c_p u_* \theta_*, \tag{2}$$

where ρ is the air density (kg m^{-3}) and c_p is the specific heat capacity at constant pressure (J kg^{-1}). Based on MOST, u_* and θ_* can be calculated from

$$u_* = uk \Big/ \left[\ln\left(\frac{z}{z_0}\right) - \psi_m\left(\frac{z}{L}\right) + \psi_m\left(\frac{z_0}{L}\right) + \psi_m^*\left(\frac{z}{L}, \frac{z}{z_*}\right) \right], \tag{3}$$

$$\theta_* = (\theta - \theta_0)k \Big/ R\left[\ln\left(\frac{z}{z_{0h}}\right) - \psi_h\left(\frac{z}{L}\right) + \psi_h\left(\frac{z_{0h}}{L}\right) + \psi_h^*\left(\frac{z}{L}, \frac{z}{z_*}\right) \right], \tag{4}$$

where k is the von Kármán constant, u and θ are the wind speed and potential temperature at the reference height, θ_0 is the potential temperature at z_{0h}, and z_* is the roughness sublayer height. Following Sarkar and Ridder (2010) and WRL12, $z_*/z_0 = 16.7$ is adopted in this study. Also, R is the turbulent Prandtl number under neutral conditions, $\psi_m$and ψ_h are the integrated stability functions for momentum and heat, respectively, and ψ_m^* and ψ_h^* are the integrated correction functions that also account for the roughness sublayer effect,

$$\psi_m(z/L) = \int_0^{z/L} [1 - \phi_m(\zeta)]\frac{d\zeta}{\zeta}, \tag{5}$$

$$\psi_h(z/L) = \int_0^{z/L} [1 - \phi_h(\zeta)/R]\frac{d\zeta}{\zeta}, \tag{6}$$

$$\psi_{m,h}^*\left(\frac{z}{L}, \frac{z}{z_*}\right) = \phi_{m,h}\left[\left(1 + \frac{\nu}{\mu_{m,h} z/z_*}\right)\frac{z}{L}\right]\frac{1}{\lambda}\ln\left(1 + \frac{\lambda}{\mu_{m,h} z/z_*}\right)\exp(-\mu_{m,h} z/z_*). \tag{7}$$

De Ridder (2010) used $\lambda = 1.5$, $\mu = \mu_m = 2.59$, $\mu = \mu_h = 0.95$ and $\nu = 0.5$, which are also used in our study. Note that $\phi_{m,h}$ are the stability functions for momentum and heat, and under unstable conditions,

$$\phi_m(z/L) = (1 - A_m z/L)^{-\frac{1}{4}}, \tag{8}$$

$$\phi_h(z/L) = R(1 - A_h z/L)^{-\frac{1}{2}}. \tag{9}$$

As such,

$$\psi_m(z/L) = \ln\left[\left(\frac{1+x^2}{2}\right)\left(\frac{1+x}{2}\right)^2\right] - 2\arctan x + \frac{\pi}{2}, \tag{10}$$

$$\psi_h(z/L) = 2\ln\frac{1+y}{2}, \tag{11}$$

$$x = (1 - A_m z/L)^{\frac{1}{4}}, \tag{12}$$

$$y = (1 - A_h z/L)^{\frac{1}{2}}, \tag{13}$$

where A_m and A_h are empirical constants derived from observational datasets. Paulson (1970) used $k = 0.40$, $R = 1$, $A_m = A_h = 16$, and in Businger et al. (1971), $k = 0.35$, $R = 0.74$, $A_m = 15$ and $A_h = 9$, in Dyer (1974) $k = 0.41$, $R = 1$, $A_m = A_h = 16$, and in Högström (1996) $k = 0.4$, $R = 0.95$, $A_m = 19$ and $A_h = 11.6$.

Combining the definition of the Obukhov length and Eqs. 3 and 4, the stability parameter ζ can be obtained through

$$\zeta = \frac{z}{L} = \frac{Ri_B}{R}\frac{\left(1 - \frac{z_{0h}}{z}\right)}{\left(1 - \frac{z_0}{z}\right)^2}\frac{\left(\ln\left(\frac{z}{z_0}\right) - \psi_m\left(\frac{z}{L}\right) + \psi_m\left(\frac{z_0}{L}\right) + \psi_m^*\left(\frac{z}{L}, \frac{z}{z_*}\right)\right)^2}{\left(\ln\left(\frac{z}{z_{0h}}\right) - \psi_h\left(\frac{z}{L}\right) + \psi_h\left(\frac{z_{0h}}{L}\right) + \psi_h^*\left(\frac{z}{L}, \frac{z}{z_*}\right)\right)}. \tag{14}$$

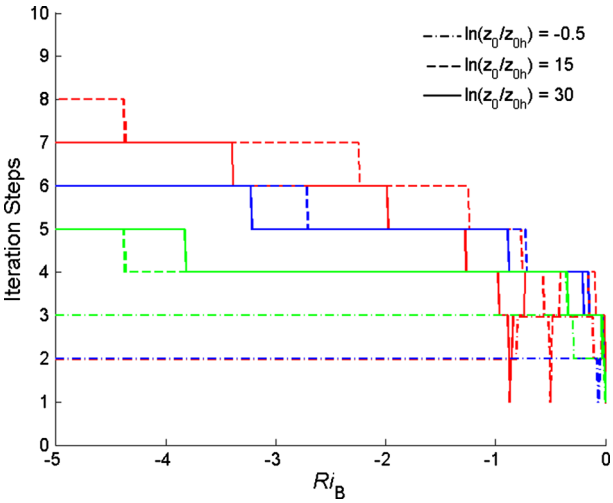

Fig. 1 Iteration steps needed to converge into $\left|(\zeta_n - \zeta_{n-1})/\zeta_{n-1}\right| \leq 0.1$ % with similarity constants from Paulson (1970) or Dyer (1974) under different z_0 and z_{0h} conditions. *Red*, *blue* and *green lines* indicate $z/z_{0m} = 10,\ 10^3$ and 10^5, respectively

The bulk Richardson number (Ri_B) is defined as

$$Ri_B = \frac{g}{\bar{\theta}} \frac{(\theta - \theta_0)(z - z_0)^2}{u^2(z - z_{0h})}, \tag{15}$$

which can be determined from observations. With Eq. 14, it can be found that for a given value of Ri_B, z_0 and z_{0h}, ζ can be obtained through iterations. Over the range $-5 \leq Ri_B < 0$, $10 \leq z/z_0 \leq 10^5$ and $-0.5 \leq \ln(z_0/z_{0h}) \leq 30$, the iteration step number (n) for $|(\zeta_n - \zeta_{n-1})/\zeta_{n-1}| \leq 0.1$ % is generally <5, but is sometimes up to 8. The average values of n are 5.0 for similarity constants from Paulson (1970) and Dyer (1974), 4.6 for similarity constants from Businger et al. (1971), and 5.2 for similarity constants from Högström (1996), respectively. The iteration steps needed for $|(\zeta_n - \zeta_{n-1})/\zeta_{n-1}| \leq 0.1$ % with similarity constants from Paulson (1970) or Dyer (1974) under different z_0 and z_{0h} conditions are shown in Fig. 1. For similarity constants from Businger et al. (1971) and Högström (1996), the iteration steps needed are similar to those for similarity constants from Paulson (1970) and Dyer (1974) and are not shown.

2.2 WRL12 Scheme and SS14 Scheme

To avoid iterations, WRL12 proposed a group of non-iterative equations to calculate ζ directly from Ri_B, while the SS14 scheme involves the gradient Richardson number (Ri), which is obtained from Ri_B, z_0 and z_{0h} by a regression equation, and ζ is calculated from Ri though an analytical relationship. The SS14 scheme does not consider the roughness sublayer effect and is valid for $-5 \leq Ri_B \leq 0$, $10 \leq z/z_0 \leq 10^5$ and $0 \leq \ln(z_0/z_{0h}) \leq 29$. As such, the SS14 scheme is not valid when z_0 becomes smaller than z_{0h}, which sometimes occurs over tall forests (e.g., Chen and Zhang 2009) and over oceans (e.g., Zeng et al. 1998; Li et al. 2015).

Table 1 The eight regions for regression

Region	Ri_B	z_0	z_{0h}
1	$-2 < Ri_B < 0$	$10 \le z/z_0 \le 80$	$0.607 \le z_0/z_{0h} \le 10$
2	$-2 < Ri_B < 0$	$10 \le z/z_0 \le 80$	$10 < z_0/z_{0h} \le 1.069 \times 10^{13}$
3	$-2 < Ri_B < 0$	$80 < z/z_0 \le 10^5$	$0.607 \le z_0/z_{0h} \le 10$
4	$-2 < Ri_B < 0$	$80 < z/z_0 \le 10^5$	$10 < z_0/z_{0h} \le 1.069 \times 10^{13}$
5	$-5 < Ri_B \le -2$	$10 \le z/z_0 \le 80$	$0.607 \le z_0/z_{0h} \le 10$
6	$-5 < Ri_B \le -2$	$10 \le z/z_0 \le 80$	$10 < z_0/z_{0h} \le 1.069 \times 10^{13}$
7	$-5 < Ri_B \le -2$	$80 < z/z_0 \le 10^5$	$0.607 \le z_0/z_{0h} \le 10$
8	$-5 < Ri_B \le -2$	$80 < z/z_0 \le 10^5$	$10 < z_0/z_{0h} \le 1.069 \times 10^{13}$

3 The New Non-iterative Scheme

Because WRL12 scheme considers the roughness sublayer effect and covers a wider range of z_0/z_{0h} than SS14 scheme, the derivation of our new solution is based on the range defined in WRL12 (i.e., $-5 \le Ri_B < 0, 10 \le z/z_0 \le 10^5$ and $-0.5 \le \ln(z_0/z_{0h}) \le 30$), which is further divided into eight regions following the method of LGL14 (see Table 1). Then, for each of the regions, multiple linear regression similar to Yang et al. (2001) is carried out using the following,

$$\zeta = Ri_B \frac{L_{0M}^2}{L_{0H}} \sum C_{ijk} \left(\frac{-Ri_B}{1 - Ri_B} \right)^i L_{0M}^{(-j)} L_{0H}^{(-k)}, \tag{16}$$

where $i = 0$ or 1; j and $k = 0, 1, 2$, or 3, and $i + j + k \le 4$. During the regression, the significance of each term in Eq. 16 is tested, and terms with little influence on the regression results are removed. The retrieved coefficients C_{ijk} can be found in Tables 2, 3, and 4 for similarity constants proposed by Paulson (1970), Businger et al. (1971) and Högström (1996), respectively. For each region, although the number of terms evaluated is always 23, there are zero terms that can be neglected, and the number of non-zero terms in Eq. 16 varies between 7 and 14, as compared to the 12 coefficients in WRL12 and 10 coefficients in SS14. Note that Paulson (1970) and Dyer (1974) have the same similarity constants in the stability correction functions ϕ_m and ϕ_h (see Sect. 2), so that they also share the same coefficients in (16) (see Table 2).

4 Comparison of the Results from WRL12 Scheme, SS14 Scheme and the New Scheme

The turbulent transfer coefficients for momentum (C_M) and heat (C_H) are calculated from

$$C_M = \frac{k^2}{\left[\ln(\frac{z}{z_0}) - \psi_m(\frac{z}{L}) + \psi_m(\frac{z_0}{L}) + \psi_m^*(\frac{z}{L}, \frac{z}{z_*}) \right]^2}, \tag{17}$$

$$C_H = \frac{k^2}{R \left[\ln(\frac{z}{z_0}) - \psi_m(\frac{z}{L}) + \psi_m(\frac{z_0}{L}) + \psi_m^*(\frac{z}{L}, \frac{z}{z_*}) \right] \left[\ln(\frac{z}{z_{0h}}) - \psi_h(\frac{z}{L}) + \psi_h(\frac{z_{0h}}{L}) + \psi_h^*(\frac{z}{L}, \frac{z}{z_*}) \right]}. \tag{18}$$

Table 2 The coefficients of C_{ijk} in Eq. 16 for Paulson (1970) and Dyer (1974)

	Region 1	Region 2	Region 3	Region 4	Region 5	Region 6	Region 7	Region 8
C031	−28.813	0	0	0	0	0	0	0
C013	0	0	0	0	0	0	0	0
C103	0	0	0	0	0	0	0	0
C130	0	0	0	0	0	0	0	0
C022	171.37	21.868	0	0	0	71.863	0	0
C112	0	0	0	0	0	0	0	0
C121	0	0	0	0	98.068	0	0	0
C003	−4.4659	−2193.5	0	0	0	−1490	0	0
C030	0	0	0	0	−30.825	0	−333.57	0
C012	−145.38	24.905	63.562	0	−26.723	−48.521	−625.04	−35.527
C021	0	0	−133.56	0	0	0	914.04	0
C102	−2.9284	6.6752	0	0	−11.479	0	0	0
C120	11.867	0	0	0	−1.2311	0	0	0
C111	−7.1901	1.9789	0	0	−67.415	2.8259	0	0
C002	31.481	5.7057	0	4.5019	16.059	27.861	149.79	32.682
C020	0	3.2231	34.817	5.4319	23.452	2.9699	83.588	10.73
C011	19.158	−14.009	0	0	5.5104	−7.3367	−209.17	−19.373
C101	7.1458	0	5.337	0.65443	20.643	0	1.7803	0
C110	−7.8929	−1.2357	−4.8263	−1.6828	0	0	0	0
C001	−6.2982	0	1.5325	−0.7537	−5.5493	−2.1701	0	0
C010	0	0	−7.0107	−1.4565	−4.764	−1.0425	−4.9982	−3.344
C100	0	−0.43163	−0.21693	−0.20131	−1.6828	−0.79024	−0.51948	−0.53418
C000	1.1233	1.0384	1.2577	1.0603	1.6062	1.3643	1.4222	1.3386

The relative error of C_{M} (or C_{H}) is defined as the relative difference between the calculated C_{M} (or C_{H}) from the non-iterative scheme and that from the classic iterative scheme,

$$\Delta C_{\mathrm{M,H}} = 100 \frac{\left| C_{\mathrm{M,H(noniterative)}} - C_{\mathrm{M,H(iterative)}} \right|}{C_{\mathrm{M,H(iterative)}}}, \tag{19}$$

where $C_{\mathrm{M,H(noniterative)}}$ is calculated with the non-iterative scheme, and $C_{\mathrm{M,H(iterative)}}$ is calculated with the classic iterative method.

The obtained ζ, C_{M} and C_{H} values using our non-iterative scheme are compared against the results from the classic iterative method with empirical constants from Paulson (1970), as shown in Fig. 2. It is evident that the results from (16) are very close to those from the classic iterative method using constants from Paulson (1970). For the similarity constants from Businger et al. (1971), Dyer (1974) and Högström (1996), the comparisons are similar and therefore not shown.

Because WRL12 only proposed a non-iterative solution based on the empirical constants of Paulson (1970), the intercomparison among three non-iterative methods is thus carried out when all are based on the empirical constants of Paulson (1970). The maximum relative errors of C_{M} and C_{H} calculated from the WRL12 scheme, the SS14 scheme and the new scheme at different values of Ri_{B} are shown in Fig. 3. For the WRL12 scheme and the new scheme, the

Table 3 The coefficients of C_{ijk} in Eq. 16 for Businger et al. (1971)

	Region 1	Region 2	Region 3	Region 4	Region 5	Region 6	Region 7	Region 8
C031	−79.401	0	0	0	0	0	0	0
C013	0	12,231	0	0	0	0	0	0
C103	0	0	0	0	0	0	0	0
C130	0	0	0	0	0	0	0	0
C022	0	0	0	0	0	153.13	0	0
C112	0	0	0	0	0	0	0	0
C121	0	0	0	0	101.86	0	0	0
C003	0	−7527.6	0	0	0	−2344.6	0	0
C030	0	0	0	0	−31.849	0	−333.37	0
C012	−30.466	0	0	0	−31.746	−92.813	−618.41	−2.809
C021	144.57	0	0	0	0	−16.274	906.59	0
C102	0	0	0	0	−11.401	0	0	0
C120	6.1292	0	0	0	−1.2814	0	0	0
C111	−9.045	7.8544	0	0	−70.981	3.9345	0	0
C002	11.339	27.268	16.461	27.466	18.849	38.241	148.05	31.683
C020	−9.0222	4.1355	28.103	10.314	24.316	4.4467	90.444	14.437
C011	−58.586	−13.292	−39.576	−23.79	8.755	0	−209.81	−27.772
C101	6.2526	0	5.1197	−0.2583	22.004	0	1.9597	0.51391
C110	−4.0189	−2.0023	−5.5072	0	0	0	0	0
C001	5.4761	−3.1214	2.4464	0	−7.9117	−4.3295	0	0
C010	6.0029	0	−4.3644	−2.7888	−5.1386	−1.7488	−7.4383	−4.5577
C100	−0.68962	−0.50721	−0.25583	−0.55259	−1.9844	−1.0413	−0.70861	−0.74036
C000	0.40684	1.3992	1.4187	1.5029	2.1217	1.8321	1.943	1.7877

classic iteration results considering the roughness sublayer effect are used as a benchmark to calculate the relative errors, and the roughness range considered is $10 \leq z/z_0 \leq 10^5$ and $-0.5 \leq \ln(z_0/z_{0h}) \leq 30$. For the SS14 scheme since its derivation did not consider roughness sublayer effects, the biases are calculated also without considering roughness sublayer effects, and the roughness range considered is $10 \leq z/z_0 \leq 10^5$ and $0 \leq \ln(z_0/z_{0h}) \leq 29$. Range separation intervals of 0.01 for Ri_B, 0.035 for $\ln(z/z_0)$ and 0.1 for $\ln(z_0/z_{0h})$ are used to calculate the relative errors. Figure 3 show that the new scheme yields the smallest maximum relative errors for both C_M and C_H, and the WRL12 scheme yields the largest. The maximum relative errors of C_M and C_H from the new scheme are always smaller than 2 %, while they exceed 10 % for the WRL12 scheme and 7 % for the SS14 scheme.

The calculation time each method needs for computing ζ, C_M and C_H from Ri_B, z_0 and z_{0h} in the range $-5 \leq Ri_B < 0$, $10 \leq z/z_0 \leq 10^5$ and $-0.5 \leq \ln(z_0/z_{0h}) \leq 30$ with an interval of 0.01 for Ri_B, 0.035 for $\ln(z/z_0)$ and 0.1 for $\ln(z_0/z_{0h})$ and by using similarity constants proposed by Paulson (1970) is 287.6, 81.9, 56.3 and 66.8 s, for the classic iterative scheme, the SS14 scheme, the WRL12 scheme and the new scheme, respectively. This clearly shows that non-iterative schemes use much less calculation time than the classic iterative scheme. Here, the calculation is performed on a laptop computer with an Intel Core i7 processor, and note that the calculation time can vary with computer.

Table 4 The coefficients of C_{ijk} in Eq. 16 for Högström (1996)

	Region 1	Region 2	Region 3	Region 4	Region 5	Region 6	Region 7	Region 8
C031	−60.131	0	0	0	0	0	0	0
C013	0	9410.7	0	0	0	0	0	0
C103	0	0	0	0	0	0	0	0
C130	0	0	0	0	0	0	0	0
C022	0	0	0	0	0	119.55	0	0
C112	0	0	0	0	0	0	0	0
C121	0	0	0	0	78.471	0	0	0
C003	0	−5825	0	0	0	−1894.6	0	0
C030	0	0	0	0	−24.499	0	−270.83	0
C012	−24.25	0	0	0	−24.639	−72.488	−499.16	−17.137
C021	111.09	0	0	0	0	−12.858	734.22	0
C102	0	0	0	0	−9.3125	0	0	0
C120	5.31	0	0	0	−1.0099	0	0	0
C111	−7.3001	6.1429	0	0	−54.614	3.181	0	0
C002	8.9171	21.294	13.047	21.564	14.791	29.936	118.89	27.58
C020	−7.448	3.2131	22.135	8.0087	18.738	3.4593	72.214	11.262
C011	−45.043	−10.437	−31.252	18.556	6.5197	0	−168.54	−19.995
C101	5.004	0	4.0778	−0.21796	17.281	0	1.422	0
C110	−3.3575	−1.5492	−4.3839	0	0	0	0	0
C001	4.2072	−2.4174	1.8993	0	−6.1451	−3.3916	0	0
C010	4.872	0	−3.3812	−2.1567	−3.9554	−1.3589	−5.7142	−3.5897
C100	−0.53086	−0.39915	−0.19875	−0.43142	−1.5781	−0.81451	−0.53715	−0.55815
C000	0.29652	1.0934	1.1059	1.1715	1.6579	1.4311	1.5035	1.3869

For all the four groups of empirical constants, namely from Paulson (1970), Businger et al. (1971), Dyer (1974) and Högström (1996), the overall maximum relative error of C_{M} and C_{H} from SS14 and the new scheme in their applicable range are shown in Table 5. It can be seen that for any of the four classic iteration schemes, the new scheme obtains C_{M} and C_{H} within a 2 % bias with non-iterative equations.

5 Summary and Conclusions

Through dividing the unstable range $-5 \leq Ri_{\mathrm{B}} < 0, 10 \leq z/z_0 \leq 10^5$ and $-0.5 \leq \ln(z_0/z_{0\mathrm{h}}) \leq 30$ into eight regions, new non-iterative equations have been developed to approximate the results of classic iterative schemes. The comparisons show that the new scheme produces small differences compared to classic iterative results. The maximum relative errors of C_{M} and C_{H} from the new scheme are always smaller than 2 %. Besides, the new scheme is computationally more efficient than classic iterative schemes. Therefore, the new non-iterative scheme is practical, and recommended for use in weather and climate numerical models.

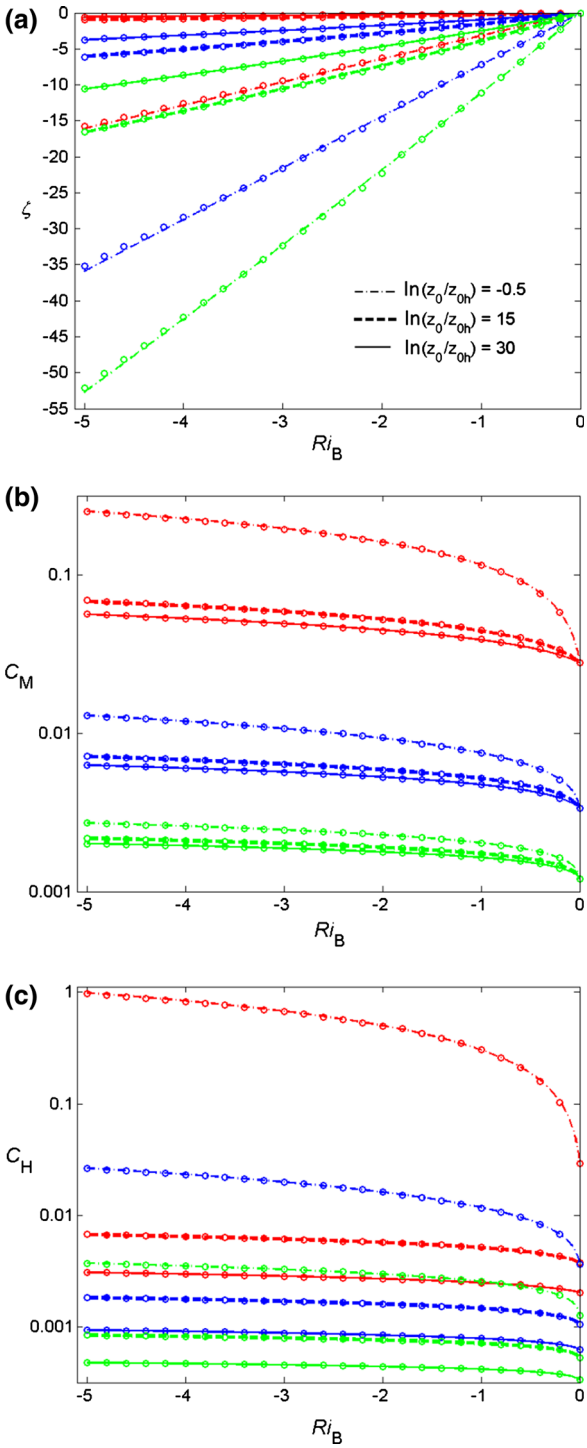

Fig. 2 The computed **a** ζ, **b** C_{M}, and **c** C_{H} from the classic iteration method (Paulson 1970; represented by *lines*) and the new non-iterative scheme (*circles*) with different Ri_{B}, z/z_0 and $z_0/z_{0\mathrm{h}}$. *Red*, *blue* and *green lines* indicate $z/z_0 = 10$, 1000 and 10^5, respectively

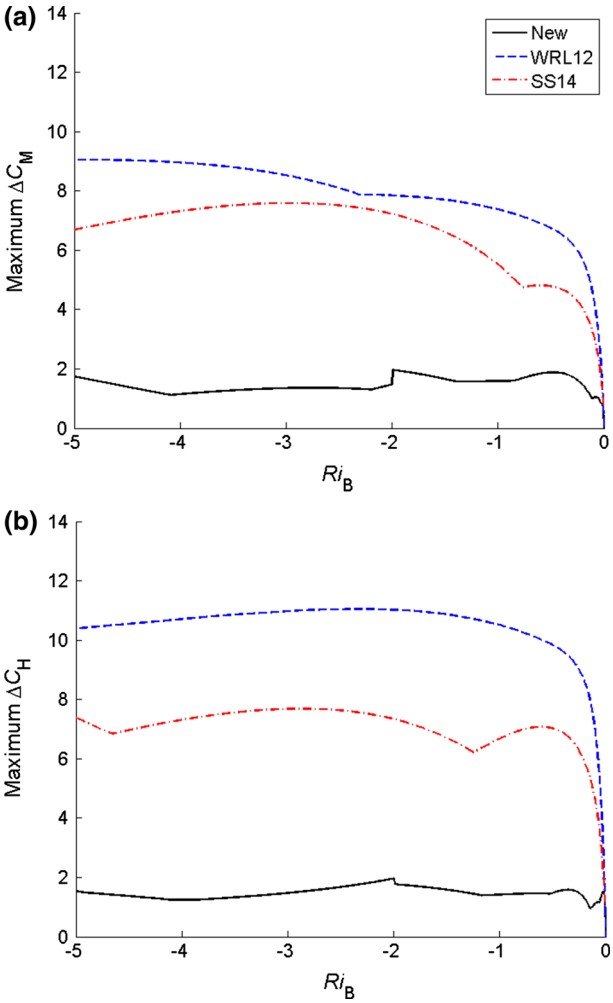

Fig. 3 Maximum relative errors of **a** C_M and **b** C_H calculated from the WRL12 scheme, the SS14 scheme and the new scheme

Table 5 Maximum relative errors of C_M and C_H from SS14 and the new scheme

	New (%)	SS14 (%)
C_M		
Paulson (1970), Dyer (1974)	2.0	7.0
Businger et al. (1971)	2.3	7.0
Högström (1996)	2.1	6.5
C_H		
Paulson (1970), Dyer (1974)	2.0	7.2
Businger et al. (1971)	2.1	7.0
Högström (1996)	2.0	7.0

Values for the SS14 method are taken from SS14

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