Name: Prof. D Date: _____

MA 226 Quiz 11 – B

Please show your work.

1. (5 pts) Find the solution of the given initial value problem:

(*) $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=4t+e^{3t}$ with $y(0)=y'(0)=0$

① $y_h(t)$ try $y(t)=e^{st} \Rightarrow s^2+3s+2=0$

$(s+2)(s+1)=0$

$s=-1$ or $s=-2$

$$y_h(t)=K_1e^{-t}+K_2e^{-2t}$$

② $y_{P_1}(t)=At+B$, $y'_{P_1}(t)=A$, $y''_{P_1}(t)=0$ } plug into *

$0+3A+2(At+B)=4t$

$2At+3A+2B=4t$ $\Rightarrow$ $2A=4 \Rightarrow A=2$, $3A+2B=0 \Rightarrow B=-3$

$$y_{P_1}(t)=2t-3$$

$y_{P_2}(t)=Ae^{3t}$, $y'_{P_2}(t)=3Ae^{3t}$, $y''_{P_2}(t)=9Ae^{3t}$ } plug into *

$(9A+3\cdot 3A+2A)e^{3t}=e^{3t}$

$20A=1 \Rightarrow A=\frac{1}{20}$

$y_{P_2}(t)=\frac{1}{20}e^{3t}$

③ General solution $y(t)=y_h(t)+y_{P_1}(t)+y_{P_2}(t)$

$$y(t)=K_1e^{-t}+K_2e^{-2t}+2t-3+\frac{1}{20}e^{3t}$$

④ Solve IVP

$y'(t)=-K_1e^{-t}-2K_2e^{-2t}+2+\frac{3}{20}e^{3t}$

$y(0)=0=K_1+K_2-3+\frac{1}{20}$

$y'(0)=0=-K_1-2K_2+2+\frac{3}{20}$

$K_1+K_2=3-\frac{1}{20}$, $-K_1-2K_2=-2-\frac{3}{20}$ }

$-K_2=1-\frac{4}{20}$

$K_2=-\frac{16}{20}=-\frac{4}{5}$

$K_1=\frac{4}{5}+3-\frac{1}{20}=\frac{16}{20}+\frac{60}{20}-\frac{1}{20}=\frac{75}{20}=\frac{15}{4}$

$$y(t)=\frac{15}{4}e^{-t}-\frac{4}{5}e^{-2t}+2t-3+\frac{1}{20}e^{3t}$$

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2. (5 pts) For the equation given below find the frequency of the beats and the frequency of the rapid oscillation.

$$\frac{d^2y}{dt^2} + 6y = 4\cos(2t)$$

$F_N = \frac{\sqrt{6}}{2\pi}$ $F_f = \frac{2}{2\pi}$

Beating Freq $\left| \frac{F_N - F_f}{2} \right| = \frac{\frac{\sqrt{6}}{2\pi} - \frac{2}{2\pi}}{2} = \frac{\sqrt{6}-2}{4\pi}$

Rapid Oscillation $\left| \frac{F_N + F_f}{2} \right| = \frac{\frac{\sqrt{6}}{2\pi} + \frac{2}{2\pi}}{2} = \frac{\sqrt{6}+2}{2\pi}$