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MA 226 Section B – Exam 2a

Question Number	Possible Points	Student Score
1	15	
2a	5	
2b	5	
2c	5	
2d	5	
2e	5	
3	10	
4	8	
5	12	
6	8	
7	14	
8	8	
Total Points	100	

You must show your work to receive full credit

Discussion Sections:

B2: Tuesday 4:30-5:30

B3: Tues : 3:30-4:30

B4: Weds: 9-10

B5: Weds: 10-11

B6: Weds: 4:30-5:30

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1. Short Answer (15 pts)

a) Characterize the behavior of the harmonic oscillator with mass m = 2, spring constant k = 1 and damping coefficient b = 3, as under damped, over damped or critically damped. Show the work that justifies your answer.

b) Find all the equilibrium solutions of the system of differential equations

$$\frac{dx}{dt} = x(x-y)$$

$$\frac{dy}{dt} = (x^2-4)(y^2-9)$$

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c) Suppose $\frac{d\vec{Y}}{dt} = A\vec{Y}$ is a linear system with two distinct real valued eigenvalues. One eigenvalue $\lambda_1 = -1$ has a corresponding eigenvector $\vec{V_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$. The two functions $\vec{Y_1}(t) = 2e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\vec{Y_2}(t) = 4e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ are both straight line solutions and they trace out the same points for $-\infty < t < \infty$. Why is this not a violation of the Uniqueness Theorem and what is the specific relationship between $\vec{Y_1}(t)$ and $\vec{Y_2}(t)$?

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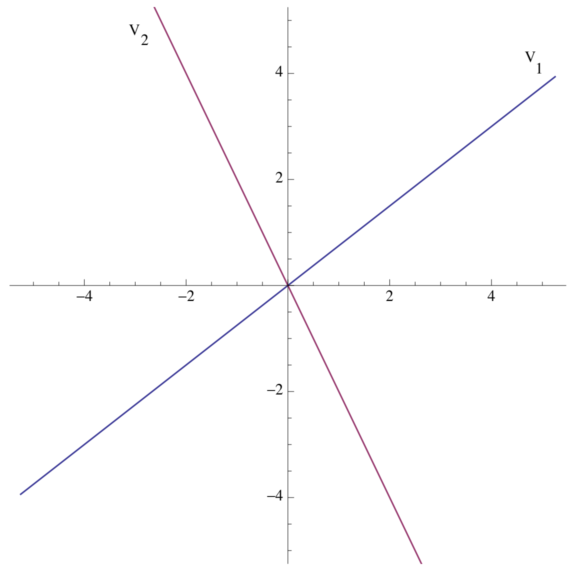
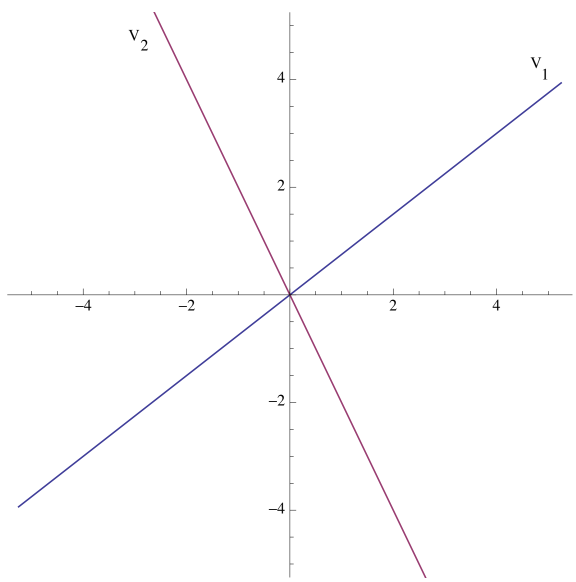
2. (5 pts each)Draw the phase portrait of a linear system given the following information:

a.) Given eigenvalues $\lambda_1 = -3$ and $\lambda_2 = 1$ with corresponding eigenvectors $\vec{V_1} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\vec{V_2} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$



b.) Given eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 1$ with corresponding eigenvectors $\vec{V_1} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\vec{V_2} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$



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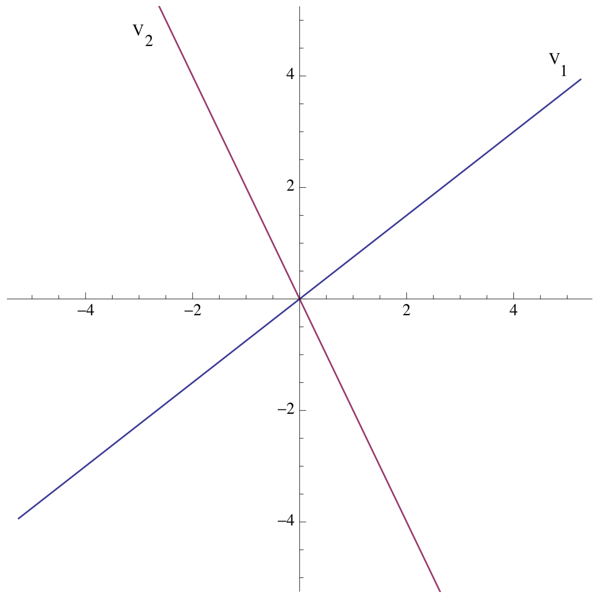
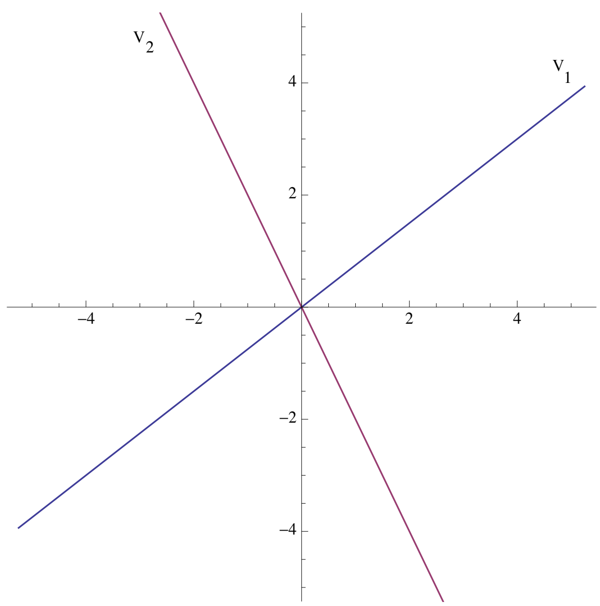
c.) Given eigenvalues $\lambda_1 = 0$ and $\lambda_2 = -2$ with corresponding eigenvectors $\overrightarrow{V_1} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\overrightarrow{V_2} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$



d.) Given eigenvalues $\lambda_1 = -2$ and $\lambda_2 = -1$ with corresponding eigenvectors $\overrightarrow{V_1} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\overrightarrow{V_2} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$



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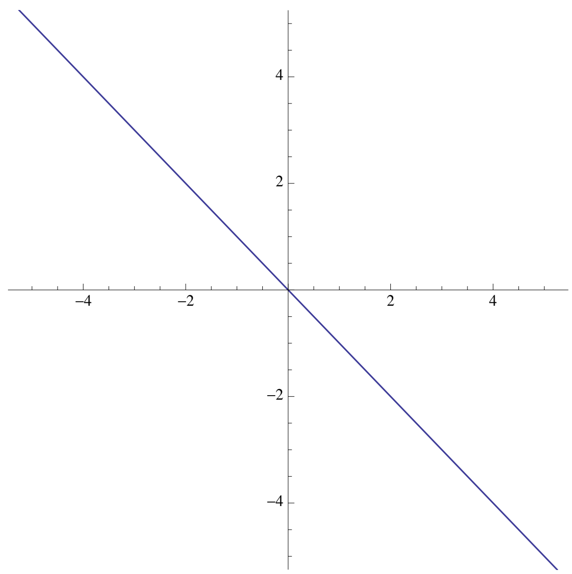
e.) Given the linear system $\frac{d\vec{Y}}{dt} = \begin{pmatrix} -6 & -5 \\ 5 & 4 \end{pmatrix}\vec{Y}$ with repeated eigenvalue $\lambda = -1$ and corresponding eigenvector $\vec{V} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ draw the phase portrait. Please show how you determined the correct orientation to use.



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3. (a) (7 pts) Find the general solution to the partially coupled system **without** using matrix techniques.

$$\frac{dx}{dt} = -3x$$
$$\frac{dy}{dt} = 2x - 3y$$

(b) Find the solution with the initial value $(x_0, y_0) = (4,3)$

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4. (8 pts) Use Euler's Method with a step size of .25 to approximate the solution of the initial value problem at time t = .75.

$$\frac{dx}{dt} = -3x$$
$$\frac{dy}{dt} = 2x - 3y$$

where $(x_0, y_0) = (4, 3)$

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5. (12 pts) Solve the initial value problem: $\frac{d\vec{Y}}{dt} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \vec{Y}$ with initial condition $\vec{Y}(0) = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$. Be sure to show the general solution too. What type of equilibrium point do we have at the origin?

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6. (8 pts) The linear system $\frac{d\vec{Y}}{dt} = \begin{pmatrix} 3 & -13 \\ 5 & 1 \end{pmatrix} \vec{Y}$ has complex eigenvalues.

a.) Compute the eigenvalues

b.) Compute one eigenvector

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7. The linear system $\frac{d\vec{Y}}{dt} = \begin{pmatrix} 2 & 2 \\ -4 & 6 \end{pmatrix}\vec{Y}$ has complex eigenvalues. One eigenvalue is $\lambda = 4 + 2i$ and the corresponding eigenvector is $\vec{V} = \begin{pmatrix} 1-i \\ 2 \end{pmatrix}$.

a.) (8 pts) Find two linearly independent real valued functions that solve the system and write the general solution for the system.

b.) (4 pts.)Find the particular solution that satisfies the initial condition $\vec{Y}(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

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Problem # 7 Continued: $\frac{d\vec{Y}}{dt} = \begin{pmatrix} 2 & 2 \\ -4 & 6 \end{pmatrix} \vec{Y}$

c.) (2 pts) Classify the type of equilibrium point we find at the origin. What is it's orientation? Clockwise or counter clockwise?

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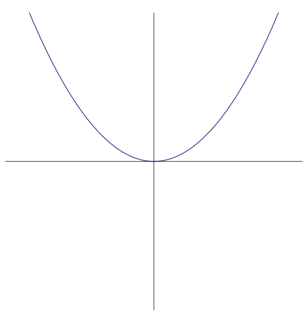
8. Trace determinant plane. Using the definitions of Trace and Determinant we can express the eigenvalues of a 2x2 linear system as $\lambda = \dfrac{T \pm \sqrt{T^2 - 4D}}{2}$

For each of the linear systems given below: (i) Compute the Trace and the Determinant (ii) Shade in the region of the Trace determinant plane where the specified matrix lives. (iii) State the behavior at the equilibrium point of systems in this region.

a.) $\dfrac{d\vec{Y}}{dt} = \begin{pmatrix} -2 & -3 \\ 3 & -2 \end{pmatrix} \vec{Y}$



b.) $\dfrac{d\vec{Y}}{dt} = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} \vec{Y}$