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# MA 226 Section B – Exam 3a

# Spring 2014

| Question Number | Possible Points | Student Score |
|---|---|---|
| 1 | 16 | |
| 2 | 12 | |
| 3 | 14 | |
| 4 | 12 | |
| 5 | 10 | |
| 6 | 12 | |
| 7 | 10 | |
| 8 | 14 | |
| Total Points | 100 | |

## You must show your work to receive full credit

Discussion Sections:

B2: Tuesday 4:30-5:30

B3: Tues : 3:30-4:30

B4: Weds: 9-10

B5: Weds: 10-11

B6: Weds: 4:30-5:30

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1.) (4 pts each) Find the Inverse Laplace Transform of the following expressions:

a) $\mathcal{L}^{-1}\left[\dfrac{3}{4s-2}\right]=$

b) $\mathcal{L}^{-1}\left[\dfrac{s+3}{s\left(s^2+3\right)}\right]=$

c) $\mathcal{L}^{-1}\left[\dfrac{e^{-s}}{s^3}\right]=$

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1.) (continued) Find the inverse Laplace transform:

d) $\mathcal{L}^{-1}\left[\dfrac{s\cdot e^{-3s}}{s^2-4s+6}\right]=$

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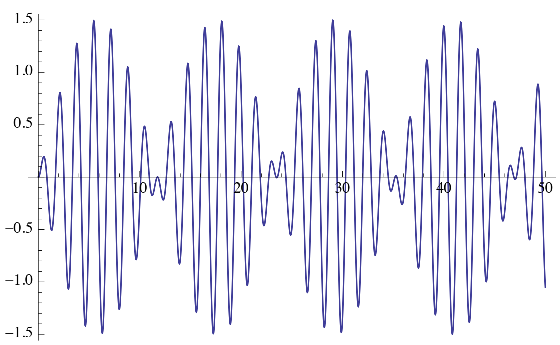
2. (12 pts) Six second order equations and four $y(t)$ - graphs are given below. For each $y(t)$ - graph, determine the second-order equation for which $y(t)$ is a solution, and state briefly how you know your choice is correct.

(i) $\dfrac{d^2y}{dt^2}+18y=3\cos 4t$ (ii) $\dfrac{d^2y}{dt^2}+16y=8$ (iii) $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}+16y=3\cos 4t$

(iv) $\dfrac{d^2y}{dt^2}+16y=3\cos 4t$ (v) $\dfrac{d^2y}{dt^2}+16y=-8$ (vi) $\dfrac{d^2y}{dt^2}+12y=3\cos 4t$

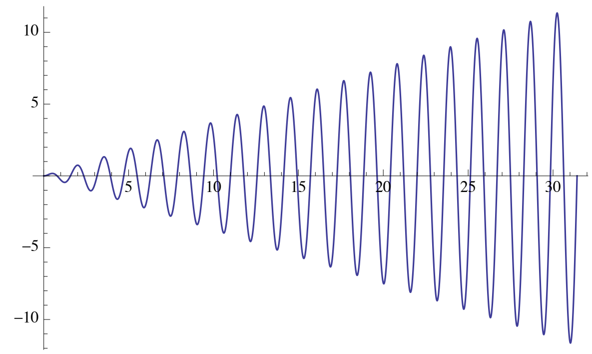
A




equation: _______

reason:

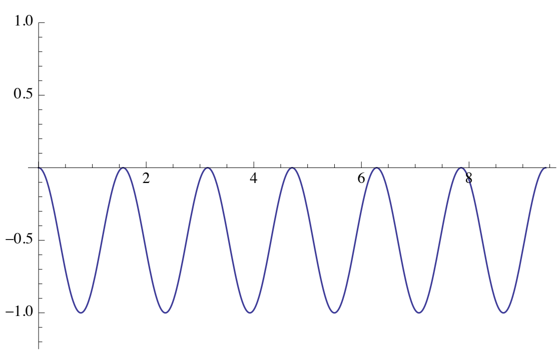
B




equation:_______

reason:

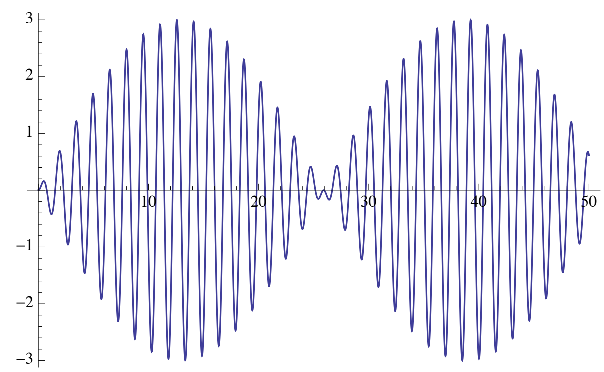
C




equation: _______

reason:

D




equation: _______

reason:

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3. (14 pts) Find the general solution of : $\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} + 8y = 16t + e^{-2t}$

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4. Short answer (4 pts each)

a) What is the beating frequency and the rapid response frequency of the solution to the undamped equation: $\frac{d^2y}{dt^2} + 9y = 2\cos(4t)$

b) Given the 2nd order linear non-homogeneous differential equation $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = 3\cos(2t)$ has general solution $y(t) = k_1e^{-2t}\cos 3t + k_2e^{-2t}\sin 3t + \frac{27}{145}\cos 2t + \frac{24}{145}\sin 2t$. What is the general solution of $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = 15\cos(2t)$

c) What is the amplitude of the steady state behavior of behavior of a 2nd order differential equation whose general solution is given by : $y(t) = k_1e^{-t} + k_2e^{-3t} + 3sin2t + 5\cos 2t$

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5. (10 pts) Compute the Laplace Transform of the function $w(t)$ using the definition of the Laplace Transform:

$$w(t) = \begin{cases} 2t, & \text{if } 0 \le t < 1 \\ 2, & \text{if } t \ge 1 \end{cases}$$

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6. (12 pts) Find a particular solution of $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 5\cos 3t$ using complexification.

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7. (10 pts) The nonlinear autonomous system

$$\frac{dx}{dt} = x(2-x-y)$$
$$\frac{dy}{dt} = y(y-x^2)$$

has an equilibrium point at $(-2,4)$. Use Linearization to classify they type of equilibrium point. If the equilibrium point is a spiral sink or a spiral source give the direction of the spiral.

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8. (14 pts) Solve the 2nd order linear non-homogeneous initial value problem:

(you can use the next page too if you need more space)

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 13y = 13u_4(t) \quad \text{with } y(0) = 1 \text{ and } y'(0) = 2$$