MA226 Section B – Exam 3 Study Guide

This exam will cover the material presented in Chapter 4, 5, and 6. Questions will be based primarily on the homework problem and also the examples discussed in lecture which can all be found in the text. You will be permitted the use of a calculator on the exam but no cell phones, ipads or laptops can be used.

Sec 4.1 – Forced Harmonic Oscillators

- Homework problems: 11, 15, 21,27, 34, 37, 39
- All of the homework problems are relevant to what I would ask on this exam.
- Important in this section is the extended linearity principle for 2nd order linear non-homogeneous differential equations. In this theorem we are given that structure of all solutions have the form $y(t) = y_h(t) + y_p(t)$ where $y_h(t)$ is the solution of the associated homogeneous problem and $y_p(t)$ is any particular solution of the non-homogeneous problems.
- Also, key to your understanding of non-homogeneous problems is that the solution of the associated homogeneous problem corresponds to the natural response of the system. If there is any damping at all then the long term behavior of the natural response must return to zero. Therefore, over time the solution of the equation will represent only the effect of the forcing function that we often refer to as the steady state solution.
- The method of undetermined coefficients is introduced for non-homogeneous terms of the form $a \cdot e^{bt}$ as well as polynomials.
- Problems #37 and #39 are of particular interest because the non-homogeneous term (forcing term) is a sum of both an exponential and a polynomial.

Sec 4.2 – Sinusoidal Forcing

- Homework problems: 9, 11, 13, 15, **17**, 19
- All of the homework problems are relevant to what I would ask on this exam
- The method of Undetermined Coefficients was extended to include forcing terms of the form $a\sin(\omega \cdot t)$ and $b\sin(\omega \cdot t)$. The method requires that we try a particular solution of the form $A\cos(\omega \cdot t) + B\sin(\omega \cdot t)$.
- A new method called Complexification was also introduced which rewrites the right hand side as a complex exponential function. The advantage being that the particular solution has the form $A \cdot e^{i\omega \cdot t}$. I will definitely require you to use the method of Complexification on one of the problems.
- We also made use of a trigonometric identity which allows us to rewrite an expression of the form $a \cdot \cos(\omega \cdot t) + b \cdot \sin(\omega \cdot t)$ as a single function $A \cdot \cos(\omega \cdot t + \varphi)$ where $A = \sqrt{a^2 + b^2}$ and φ is a phase shift. The advantage of this identity is that we are able to see the amplitude of the result. The

calculation of φ is tricky in that you need to pay attention to which quadrant φ lives in.

Sec 4.3 – Undamped Forcing and Resonance

- Homework problems: 5,6,13,14,15, 17, **21**
- All homework problems are relevant to what I would ask on this exam
- In the undamped case when the frequency of the forcing function approaches the frequency of the natural response we observe the phenomenon of resonance.
- If the forcing function is exactly equal to the frequency of the natural response then the solutions will grow without bound. A solution of the form $t \cdot \sin(\omega \cdot t)$ would be an example of resonance. The method of Complexification will require solutions of the form $A \cdot t \cdot e^{i\omega \cdot t}$
- If the frequency F_f of the forcing function is relatively close to the frequency F_n of the natural response then we observe the phenomenon called beating. The beating frequency is given by $\left|\frac{F_f - F_n}{2}\right|$ and the frequency of the rapid oscillations is given by $\frac{F_f + F_n}{2}$.

Sec 5.1 – Equilibrium Point Analysis

- Homework problems: 1, 3, 5, 7 ,9
- You are not responsible for questions 5, 17, 21, and 23
- The key fact in this section is the construction and use of the Jacobian matrix.
- Given an autonomous system for the form : $\begin{matrix} \frac{dx}{dt} = f(x,y) \\ \frac{dy}{dt} = g(x,y) \end{matrix}$ with and equilibrium solution at (x_0, y_0) then the Jacobian for this system centered at (x_0, y_0) has the form $J = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix}_{(x_0, y_0)}$. Using the Jacobian we create a linear system of the form $\frac{d\vec{Y}}{dt} = J \cdot \vec{Y}$. The behavior of this linear system at $(0,0)$ approximates the behavior of the non-linear system at it's equilibrium value of (x_0, y_0).
- Using the Jacobian you will be asked to explain the behavior of a non-linear system at one of it's equilibrium points.

Sec 6.1 Laplace Transforms

- Homework problems: 3,9,11,13,15,17,21, 23
- All homework problems are relevant to what I would ask on this exam
- Please know the definition of the Laplace Transform on P. 567
- Using the definition of the Laplace transform you should be able to derive the Laplace Transform of : 1 , t , t^2 , and e^{at}
- It is important for students to properly handle the improper integrals that are introduced through Laplace Transforms. Also, students should be comfortable with the integration method called Integration by Parts.
- Key properties of Laplace Transforms introduced in this section include the Laplace Transform of a derivative (P. 569) , the linearity properties of the Laplace Transform (P.570), the linearity properties of the Inverse Laplace Transform (P. 573).
- The process finding inverse Laplace Transforms regularly require students to use partial fractions to rewrite an expression containing multiple terms in a denominator into a sum of fractions with single terms in the denominators.

Sec 6.2 Discontinuous Functions

- Homework problems: 3,4,7,9,13
- All homework problems are relevant to what I would ask on the exam.
- Using the definition of the Laplace Transform students should be able to derive the Laplace transform of the Heavyside Function (P. 579)
- Problems where the forcing function involves a Heavyside function will also involve a Heavyside function when finding the inverse Laplace Transform. Most problems will require the rule : $\mathrm{L}[u_a(t)f(t-a)] = e^{-as}F(s)$

Sec 6.3 – Second Order Equations

- Homework Problems : 5,11,13,15,17, 29,31,32,**33**,34
- All homework problems are relevant to what I would ask on the exam.
- Here we introduce the Laplace Transform of 2nd derivatives as well as the Laplace Transform for $\sin(\omega \cdot t)$ and $\cos(\omega \cdot t)$
- The process of finding inverse Laplace Transforms will require the algebraic technique of completing the square and the new inverse Laplace Transform rule $L\left[e^{a \cdot t} f(t)\right] = F(s-a)$

Sec 6.4 – Delta Functions and Impulse Forcing

- Homework Problems: 1,3,4,5,6
- All homework problems are relevant to what I would ask on the exam.
- The Laplace Transform for the Dirac Delta Function is the last Laplace Transform that students are responsible for.