

Sec 6.3 Homework

$$33.) \frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 9y = 20 u_2(t) \sin(t-2) \quad y(0)=1, \quad y'(0)=2$$

$$\mathcal{L}\left[\frac{d^2 y}{dt^2}\right] + 4 \mathcal{L}\left[\frac{dy}{dt}\right] + 9 \mathcal{L}[y] = 20 \mathcal{L}[u_2(t) \sin(t-2)]$$

$$s^2 \mathcal{L}[y] - s y(0) - y'(0) + 4[s \mathcal{L}[y] - y(0)] + 9 \mathcal{L}[y] =$$

$$20 \frac{e^{-2s}}{s^2+1}$$

$$(s^2+4s+9)\mathcal{L}[y] - s - 2 - 4 = \frac{20 e^{-2s}}{s^2+1}$$

$$(s^2+4s+9)\mathcal{L}[y] - s - 6 = \frac{20 e^{-2s}}{s^2+1}$$

$$\mathcal{L}[y] = \frac{s+6}{s^2+4s+9} + \frac{20 e^{-2s}}{(s^2+1)(s^2+4s+9)}$$

$$\mathcal{L}[y] = \frac{s+6}{(s+2)^2+5} + \frac{(-s+2)e^{-2s}}{s^2+1} + \frac{(s+2)e^{-2s}}{(s+2)^2+5}$$

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page
partial
fractions

$$\mathcal{L}[y] = \frac{s+2}{(s+2)^2+5} + \frac{4}{(s+2)^2+5} - \frac{s e^{-2s}}{s^2+1} + \frac{2 e^{-2s}}{s^2+1} + \frac{(s+2) e^{-2s}}{(s+2)^2+5}$$

$$y(t) = e^{-2t} \cos \sqrt{5}t + \frac{4}{\sqrt{5}} e^{-2t} \sin \sqrt{5}t - u_2(t) \cos(t-2) + 2 u_2(t) \sin(t-2) + u_2(t) e^{-2(t-2)} \cos \sqrt{5}(t-2)$$

$$y(t) = e^{-2t} \left(\cos \sqrt{5}t + \frac{4}{\sqrt{5}} \sin \sqrt{5}t \right) + u_2(t) \left[e^{-2(t-2)} \cos \sqrt{5}(t-2) + 2 \sin(t-2) - \cos(t-2) \right]$$

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33.) (continued)

$$\frac{20}{(s^2+1)(s^2+4s+9)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4s+9}$$

$$20 = (As+B)(s^2+4s+9) + (s^2+1)(Cs+D)$$

$$20 = As^3 + (4A+B)s^2 + (9A+4B)s + 9B + Cs^3 + Ds^2 + Cs + D$$

$$20 = (A+C)s^3 + (4A+B+D)s^2 + (9A+4B+C)s + 9B+D$$

$$A+C = 0 \Rightarrow C = -A$$

$$\begin{aligned} 4A+B+D &= 0 \Rightarrow 4A+B+20-9B=0 \\ 9A+4B+C &= 0 \Rightarrow 9A+4B-A=0 \\ 9B+D &= 20 \Rightarrow D=20-9B \end{aligned} \quad \left\{ \begin{aligned} 4A-8B &= -20 \\ 8A+4B &= 0 \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \begin{aligned} A-2B &= -5 \\ 4A+2B &= 0 \end{aligned}$$

$$5A = -5$$

$$A = -1 \Rightarrow C = 1$$

$$B = 2 \Rightarrow D = 2$$