

Sec 6.3 Homework

$$32.) \quad \frac{d^2 y}{dt^2} + 3y = u_4(t) \cos(5(t-4)) \quad y(0) = 0, y'(0) = -2$$

$$\mathcal{L}\left[\frac{d^2 y}{dt^2}\right] + 3\mathcal{L}[y] = \mathcal{L}[u_4(t) \cos(5(t-4))]$$

$$s^2 \mathcal{L}[y] - sy(0) - y'(0) + 3\mathcal{L}[y] = e^{-4s} \cdot \frac{s}{s^2 + 5^2}$$

$$(s^2 + 3)\mathcal{L}[y] + 2 = \frac{e^{-4s} \cdot s}{s^2 + 5^2}$$

$$\mathcal{L}[y] = \frac{-2}{s^2 + 3} + \frac{s \cdot e^{-4s}}{(s^2 + 3)(s^2 + 5^2)}$$

(see *
on next
page)

$$\mathcal{L}[y] = \frac{-2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{s^2 + 3} + \frac{1}{22} \frac{s e^{-4s}}{s^2 + (\sqrt{3})^2} - \frac{1}{22} \frac{s e^{-4s}}{s^2 + 5^2}$$

$$y(t) = \mathcal{L}^{-1}[\mathcal{L}[y]] = \frac{-2}{\sqrt{3}} \mathcal{L}^{-1}\left[\frac{\sqrt{3}}{s^2 + (\sqrt{3})^2}\right] + \frac{1}{22} \mathcal{L}^{-1}\left[\frac{s e^{-4s}}{s^2 + (\sqrt{3})^2}\right] - \frac{1}{22} \mathcal{L}^{-1}\left[\frac{s e^{-4s}}{s^2 + 5^2}\right]$$

$$= \frac{-2}{\sqrt{3}} \sin(\sqrt{3}t) + \frac{1}{22} u_4(t) \cos(\sqrt{3}(t-4)) - \frac{1}{22} u_4(t) \cos(5(t-4))$$

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32.) (continued)

$$\frac{s}{(s^2+3)(s^2+25)} = \frac{As+B}{s^2+3} + \frac{Cs+D}{s^2+25}$$

$$s = (As+B)(s^2+25) + (Cs+D)(s^2+3)$$

$$s = As^3 + 25As + Bs^2 + 25B + Cs^3 + 3Cs + Ds^2 + 3D$$

$$s = (A+C)s^3 + (B+D)s^2 + (25A+3C)s + (25B+3D)$$

$$\Rightarrow A+C=0 \Rightarrow A=-C$$

$$B+D=0$$

$$25A+3C=1 \Rightarrow -25C+3C=1 \Rightarrow$$

$$25B+3D=0$$

$$\boxed{C = -\frac{1}{22} \quad A = \frac{1}{22}}$$

$$B=0 \quad D=0$$

$$\frac{s}{(s^2+3)(s^2+25)} = \frac{1}{22} \cdot \frac{s}{s^2+3} - \frac{1}{22} \cdot \frac{s}{s^2+25} \quad (*)$$