Sec 6.3 Homework

31.) $\frac{d^2y}{dt^2} + 4y = \cos(2t) \qquad y(0) = -2 \quad y'(0) = 0$

Recall $\mathcal{L}[t \sin \omega t] = \frac{2\omega s}{(s^2+\omega^2)^2}$ $\qquad \mathcal{L}[t\cos\omega t] = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$

$$\mathcal{L}\left[\frac{d^2y}{dt^2}\right] + 4\mathcal{L}[y] = \mathcal{L}[\cos 2t]$$

$$s^2\mathcal{L}[y] - s\,y(0) - y'(0) + 4\mathcal{L}[y] = \frac{s}{s^2+4}$$

$$(s^2+4)\mathcal{L}[y] + 2s = \frac{s}{s^2+4}$$

$$\mathcal{L}[y] = \frac{-2s}{s^2+4} + \frac{s}{(s^2+4)^2}$$

$$\mathcal{L}[y] = -2\cdot\frac{s}{s^2+4} + \frac{1}{4}\cdot\frac{2\cdot 2\cdot s}{(s^2+4)^2}$$

$$y(t) = \mathcal{L}^{-1}[\mathcal{L}[y]] = -2\,\mathcal{L}^{-1}\left[\frac{s}{s^2+2^2}\right] + \frac{1}{4}\,\mathcal{L}^{-1}\left[\frac{2\cdot 2\cdot s}{(s^2+2^2)^2}\right]$$

$$\boxed{y(t) = -2\cos(2t) + \frac{1}{4}t\sin(2t)}$$