Sec 5.1 HW

5.) a) $\frac{dx}{dt} = -x$ $\boxed{x(t) = k_1 e^{-t}}$

b.) $\frac{dy}{dt} = -4x^3 + y = -4k_1^3 e^{-3t} + y$ (*)

$y_h(t) = k_2 e^{t}$ $y_p(t) = A e^{-3t}$

$y_p'(t) = -3Ae^{-3t}$

plug into (*) $-3Ae^{-3t} = Ae^{-3t} - 4k_1^3 e^{-3t}$

$-4Ae^{-3t} = -4k_1^3 e^{-3t} \Rightarrow A = k_1^3$

$$\boxed{y(t) = k_2 e^{t} + k_1^3 e^{-3t}}$$

c) General Solution $x(t) = k_1 e^{-t}$

$y(t) = k_1^3 e^{-3t} + k_2 e^{t}$

If we express the results interms of the initial condition (x_0, y_0) we have

$x(0) = x_0 = k_1 \Rightarrow \boxed{k_1 = x_0}$

$y(0) = y_0 = k_1^3 + k_2$ $\boxed{k_2 = (y_0 - k_1^3) = (y_0 - x_0^3)}$

General Solution

$$\boxed{\begin{aligned} x(t) &= x_0 e^{-t} \\ y(t) &= x_0^3 e^{-3t} + (y_0 - x_0^3) e^{t} \end{aligned}}$$

Sec 5.1 HW

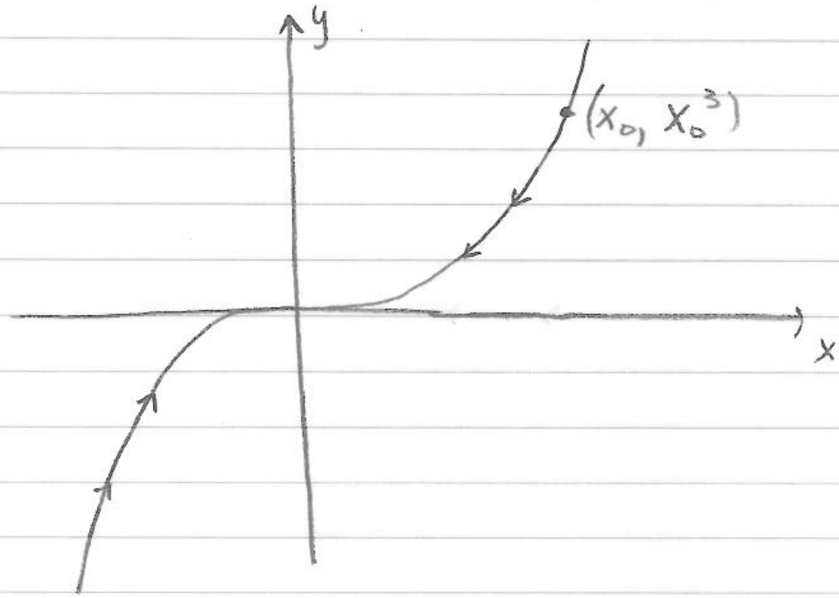
d.) Find solution curves of the system that tend toward (0,0) as $t \to \infty$

$$x(t) = x_0 e^{-t} \to 0 \quad \text{as } t \to \infty \quad \text{for all } x_0$$

$$y(t) = x_0^3 e^{-t} + (y_0 - x_0^3) e^{t} \to (y_0 - x_0^3) e^{t}$$

$$\Rightarrow y(t) \to 0 \text{ only if } y_0 - x_0^3 = 0$$

$$\text{or } \quad y_0 = x_0^3$$



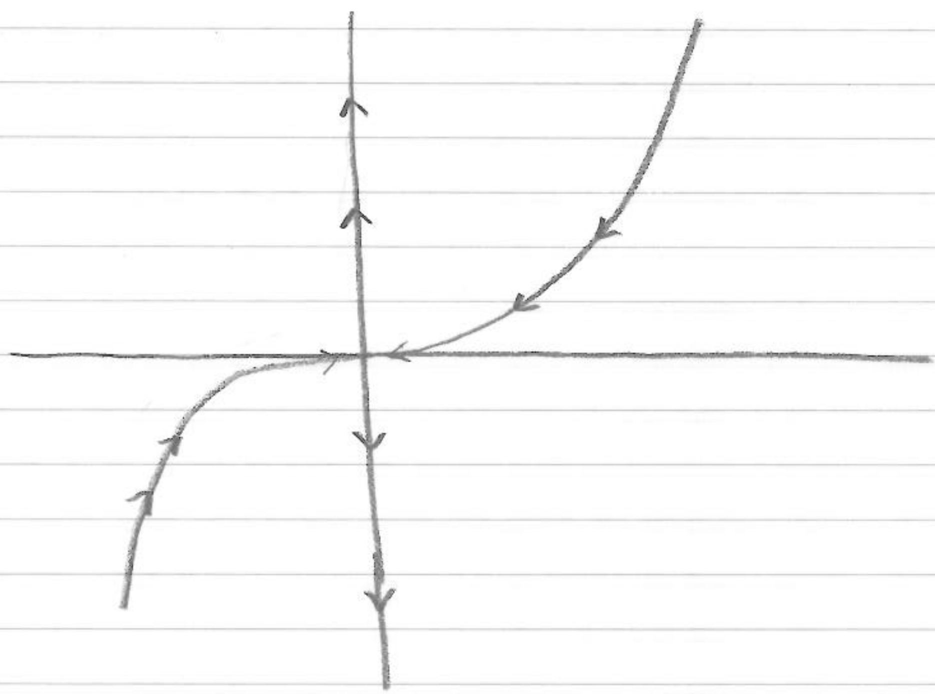
Sec 5.1 HW

e) Find solution curves that tend toward the origin as $t \to -\infty$

since $x(t) = x_0 e^{-t} = 0$ as $t \to -\infty$ only if $x_0 = 0$

In that case $y(t) = y_0 e^{t} \to 0$ as $t \to -\infty$

$\therefore$ Along the positive and negative y-axis $(x_0 = 0)$ solutions tend to $(0,0)$ as $t \to -\infty$



Sec 5.1 HW

f.) The Linearized System for

$$\frac{dx}{dt} = -x$$

$$\frac{dy}{dt} = -4x^3 + y$$

at $(0,0)$ is

$$\frac{dx}{dt} = -x$$

$$\frac{dy}{dt} = y$$

$$\Rightarrow \frac{d\bar{y}}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \bar{y}$$

Find eigenvalues : $\det \begin{bmatrix} -1-\lambda & 0 \\ 0 & 1-\lambda \end{bmatrix} = 0$

$$(1+\lambda)(1-\lambda) = 0$$

$$-\lambda^2 + 1 = 0$$

$$\lambda^2 - 1 = 0$$

$$\boxed{\lambda_1 = +1 \quad \text{or} \quad \lambda_2 = -1}$$

eigenvector for $\lambda_1 = 1$

$-x = x \Rightarrow x = 0$

$y = y \Rightarrow y = \text{any value}$

$$\bar{V}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

eigenvector for $\lambda_2 = -1$

$-x = -x \Rightarrow x = \text{any value}$

$y = -y \Rightarrow y = 0$

$$\bar{V}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

See 5.1

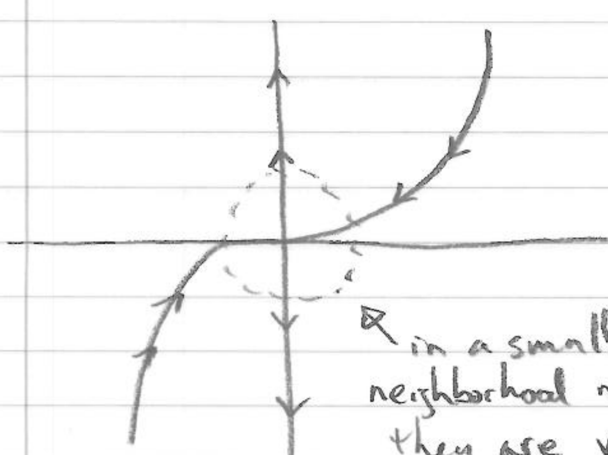
#5 continued

f.)

Very close to (0,0) the two pictures look the *same* in that solutions approach (0,0) along the positive and negative x-axis and solutions leave the origin along the positive and negative y-axis

side by side comparison

Non-Linear



in a small neighborhood near (0,0) they are very *similar*

Linear