Sec 5.1 # 3 Given the nonlinear system

$$\frac{dx}{dt} = -2x + y$$

$$\frac{dy}{dt} = -y + x^2$$

with equilibrium values at $(0,0)$ and $(2,4)$

(a) Find linearized system at $(0,0)$

$$J(x,y) = \begin{bmatrix} -2 & 1 \\ 2x & -1 \end{bmatrix} \qquad J(0,0) = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\frac{d\bar{Y}}{dt} = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} \bar{Y}$$

(b) classify the behavior at $(0,0)$ Trace $= -3$ $D = 2$

since $D = 2 < \frac{T^2}{4} = \frac{9}{4} = 2.5 \Rightarrow$ $\boxed{\text{SINK}}$

or using eigenvalues $\det(A - \lambda I) = 0$

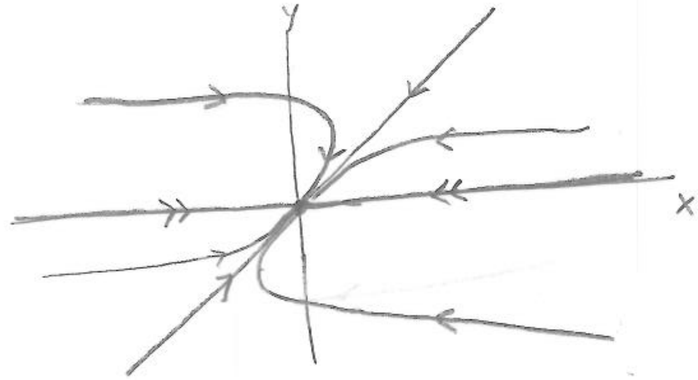
$$\det\begin{pmatrix} -2-\lambda & 1 \\ 0 & -1-\lambda \end{pmatrix} = 0 \Rightarrow \begin{array}{l} (-2-\lambda)(-1-\lambda) = 0 \\ (2+\lambda)(1+\lambda) = 0 \\ \boxed{\lambda = -1 \text{ and } \lambda = -2} \end{array}$$

2 negative eigenvalues
$\Rightarrow$ sink

(c) sketch the phase portrait at $(0,0)$

$$\lambda_1 = -1 \qquad \left.\begin{array}{l} -2x + y = -x \\ 0x - y = -y \end{array}\right\} \Rightarrow \begin{array}{l} y = x \\ y = y \end{array} \Rightarrow \bar{V}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda_2 = -2 \qquad \left.\begin{array}{l} -2x + y = -2x \\ 0x - y = -2y \end{array}\right\} \Rightarrow \begin{array}{l} y = 0 \\ y = 0 \end{array} \Rightarrow \bar{V}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



Sec 5.1 #3 Continued

(a.) Linearized system at $(2,4)$ $\qquad J(2,4) = \begin{bmatrix} -2 & 1 \\ 4 & -1 \end{bmatrix}$

$$\frac{d\bar{Y}}{dt} = \begin{bmatrix} -2 & 1 \\ 4 & -1 \end{bmatrix} \bar{Y}$$

(b.) Classify The system at $(2,4)$ $\qquad T = -3 \quad D = 2 - 4 = -2 \Rightarrow \boxed{\text{saddle}}$

Using eigenvalues $\quad \left.\begin{aligned} \lambda_1 &= -\frac{3}{2} + \frac{\sqrt{17}}{2} > 0 \\ \lambda_2 &= -\frac{3}{2} - \frac{\sqrt{17}}{2} < 0 \end{aligned}\right\} \Rightarrow$ saddle

c.) Sketch the phase portrait at $(2,4)$. First sketch at $(0,0)$ for Linearized system

$$\lambda_1 = -\frac{3}{2} + \frac{\sqrt{17}}{2} \Rightarrow \left.\begin{aligned} -2x + y &= -\frac{3}{2}x + \frac{\sqrt{17}}{2}x \\ 4x - y &= \left(-\frac{3}{2} + \frac{\sqrt{17}}{2}\right) y \end{aligned}\right\} \Rightarrow \left.\begin{aligned} -4x + 2y &= (-3 + \sqrt{17})x \\ 8x - 2y &= (-3 + \sqrt{17}) y \end{aligned}\right\} \Rightarrow$$

① $2y = (1 + \sqrt{17})x$

② $8x = (-1 + \sqrt{17})y$

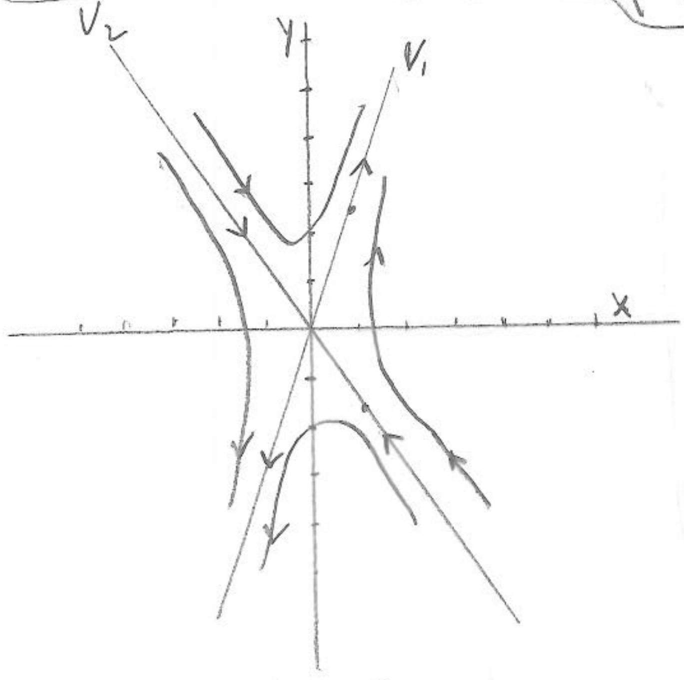
② $\cdot (-1 - \sqrt{17}) \Rightarrow$ ① $2y = (1 + \sqrt{17})x$

$8(-1 - \sqrt{17})x = (1 - 17)y$

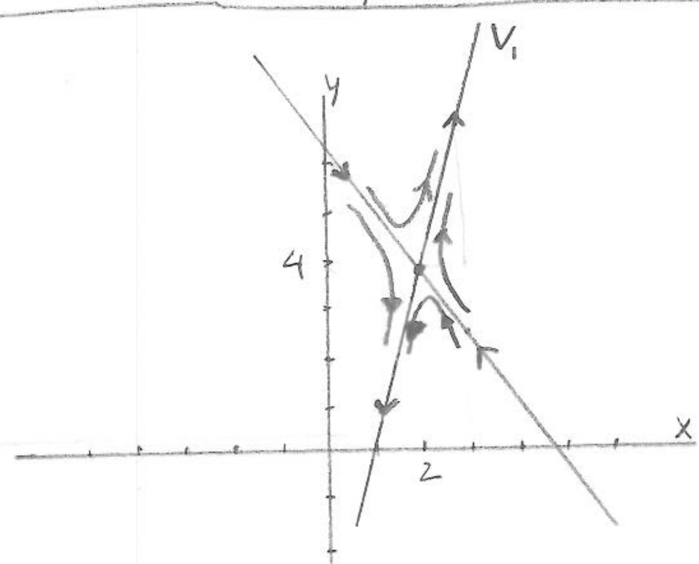
$y = \left(\frac{1 + \sqrt{17}}{2}\right)x$

$V_1 = \begin{pmatrix} 1 \\ 2.56 \end{pmatrix}$ $\qquad y \approx 2.56x$

$\lambda_2 = -\frac{3}{2} - \frac{\sqrt{17}}{2} \qquad V_2 = \begin{pmatrix} 1 \\ -1.56 \end{pmatrix}$



Linearized system



Linearized system viewed at $(2,4)$