Name: Prof D Date: _____

MA 226 Quiz 8 – A

Please show your work.

1. (5 pts) Solve the initial value problem $\frac{d\vec{Y}}{dt} = \begin{pmatrix} -2 & -2 \\ -2 & 1 \end{pmatrix}\vec{Y}$ with $\vec{Y}(0) = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$

a.) Find the eignvalues

b.) Find the corresponding eigenvectors

c.) Find the solution of the initial value problem.

a) $\det(A-\lambda I) = \det\begin{pmatrix} -2-\lambda & -2 \\ -2 & 1-\lambda \end{pmatrix} = 0$

$(-2-\lambda)(1-\lambda) - 4 = 0$

$\lambda^2 + \lambda - 6 = 0$

$(\lambda+3)(\lambda-2) = 0$

$\lambda_1 = -3$ $\lambda_2 = 2$

b.) $\lambda_1 = -3$ $A\vec{v} = -3\vec{v}$

$-2x - 2y = -3x$ and $-2x + y = -3y$ $\Rightarrow$ $x - 2y = 0$, $-2x + 4y = 0$ $\Rightarrow$ $x = 2y$ $\vec{v} = \begin{pmatrix} 2y \\ y \end{pmatrix}$ $\vec{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\lambda_2 = 2$

$-2x - 2y = 2x$ and $-2x + y = 2y$ $\Rightarrow$ $-4x - 2y = 0$, $-2x - y = 0$ $\Rightarrow$ $y = -2x$ $\vec{v} = \begin{pmatrix} x \\ -2x \end{pmatrix}$ $\vec{v}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

c.) $\vec{y}(t) = k_1 e^{-3t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + k_2 e^{2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\vec{Y}(0) = \begin{pmatrix} 0 \\ -10 \end{pmatrix} = \begin{pmatrix} 2k_1 + k_2 \\ k_1 - 2k_2 \end{pmatrix}$

$2(2k_1 + k_2 = 0)$

$k_1 - 2k_2 = -10$

$5k_1 = -10$ $k_1 = -2$ $k_1 = 4$

$\vec{y}(t) = -2e^{-3t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + 4e^{2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

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2. (5 pts) The linear system: $\frac{d\vec{Y}}{dt} = A\vec{Y}$ where matrix has a general solution given by $\vec{Y}(t) = k_1 e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + k_2 e^{-4t}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$. Make a sketch of the phase portrait for this system.