

Name: _____

Date: _____

MA 226 Quiz 10 - A**Please show your work.**

1. (5 pts) Find the solution of the given initial value problem:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 5y = 0 \text{ with } y(0) = 11 \text{ and } y'(0) = -7$$

$$\text{try } y = e^{st} \Rightarrow s^2 + 4s + 5 = 0$$

$$s = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = -2 \pm i$$

$$y = e^{(-2 \pm i)t} = \underbrace{e^{-2t} \cos t}_{y_1(t)} + i \underbrace{e^{-2t} \sin t}_{y_2(t)}$$

$$y(t) = k_1 e^{-2t} \cos t + k_2 e^{-2t} \sin t$$

$$y'(t) = k_1 (-2e^{-2t} \cos t - e^{-2t} \sin t) + k_2 (-2e^{-2t} \sin t + e^{-2t} \cos t)$$

$$y(0) = 11 = k_1$$

$$y'(0) = -7 = -2k_1 + k_2 \Rightarrow -7 + 22 = k_2$$

$$k_2 = 15$$

$$y(t) = 11e^{-2t} \cos t + 15e^{-2t} \sin t$$

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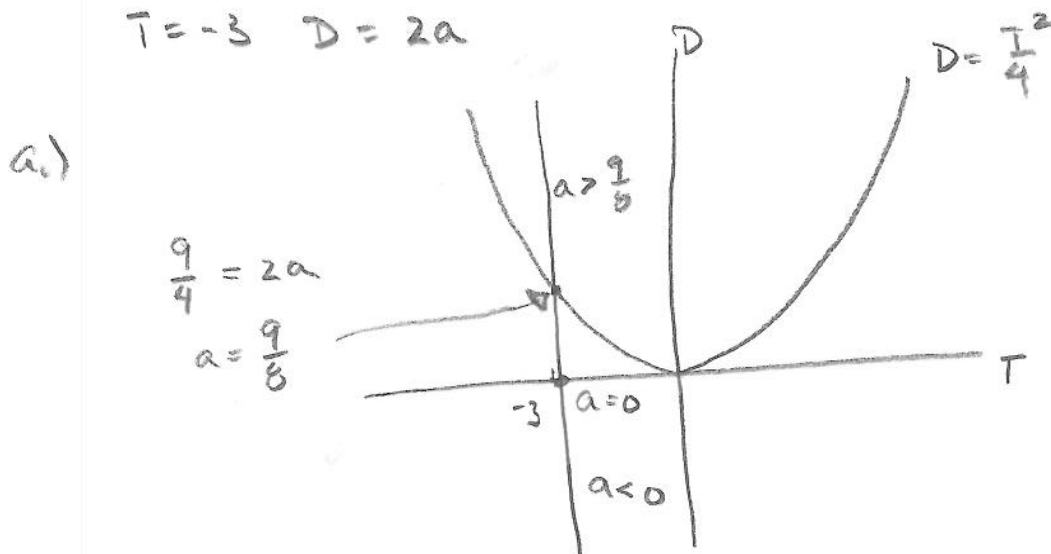
2. (5 pts) The one parameter family shown below yields a unique curve in the trace determinant plane.

a.) Sketch the corresponding curve in the trace-determinant plane.

b.) Make a list of the type of behaviors exhibited by the system as the parameter varies from +infinity to -infinity.

c.) Identify the values of the parameter that correspond to bifurcation values.

$$\frac{d\bar{Y}}{dt} = \begin{pmatrix} 0 & a \\ -2 & -3 \end{pmatrix}$$



b.)

- $a < 0$ saddle
- $a = 0$ $\lambda_1 = 0$ $\lambda_2 < 0$
- $0 < a < \frac{9}{8}$ sink
- $a > \frac{9}{8}$ spiral sink

c.) bifurcations at $a = 0$ and $a = \frac{9}{8}$