Sec 2.6

Existence and Uniqueness Theorem

Let $\frac{d\bar{Y}}{dt} = \bar{F}(t, \bar{Y})$

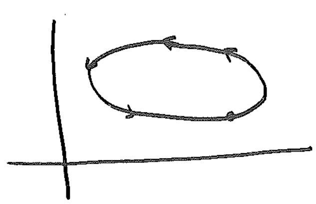
be a system of differential equations. Suppose that t_0 is an initial time and $\bar{Y}_0$ is an initial value. Suppose that the function $\bar{F}$ is continuously differentiable. Then there exists an ε and $\bar{Y}(t)$ defined for $t_0 - \varepsilon < t < t_0 + \varepsilon$ such that

$$\frac{d\bar{Y}}{dt} = \bar{F}(t, \bar{Y}) \quad \text{and} \quad \bar{Y}(t_0) = \bar{Y}_0$$

Moreover, for t in this interval, this solution is unique.

Consequences of Uniqueness for Autonomous Systems

1.) The solution curve for a single solution cannot loop back and intersect itself unless the solution is periodic and the solution curve is a simple closed curve.



2) Solution curves for two different solutions cannot intersect unless they sweep out the same curve.

Example: $\frac{dy}{dt} = v$

$\frac{dv}{dt} = -y$

Example: $\frac{dx}{dt} = x^2 + 1$ $\quad (x(0), y(0)) = (0, 0)$

$\frac{dy}{dt} = 1$