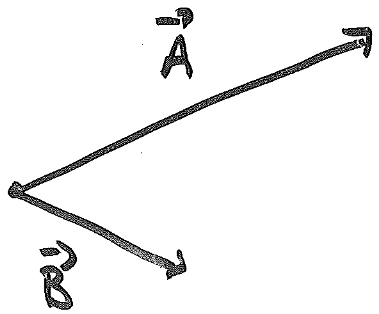
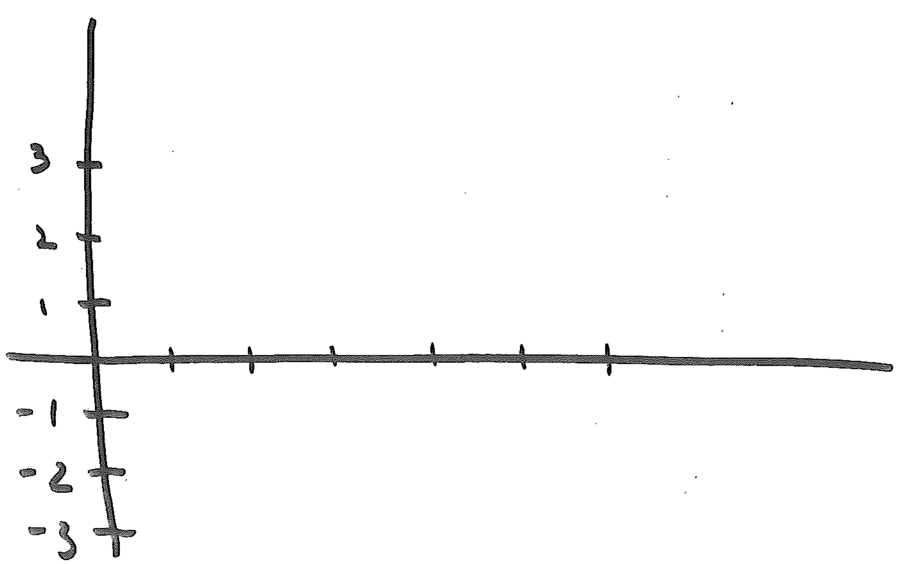
Sec 2.5 Euler's Method for Systems

Recall vector addition

$$\vec{A} + \vec{B} = \vec{C}$$

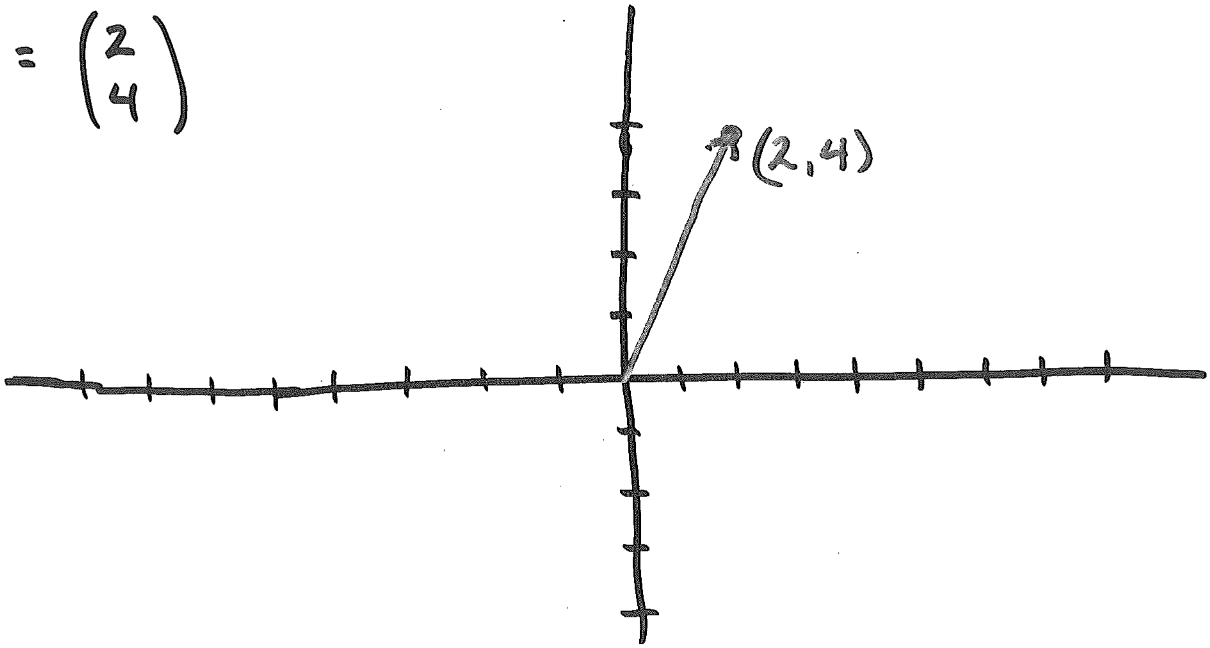


Example: $\vec{A} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ $\vec{B} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$



Recall, multiplying a vector by a scalar

$\vec{A} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

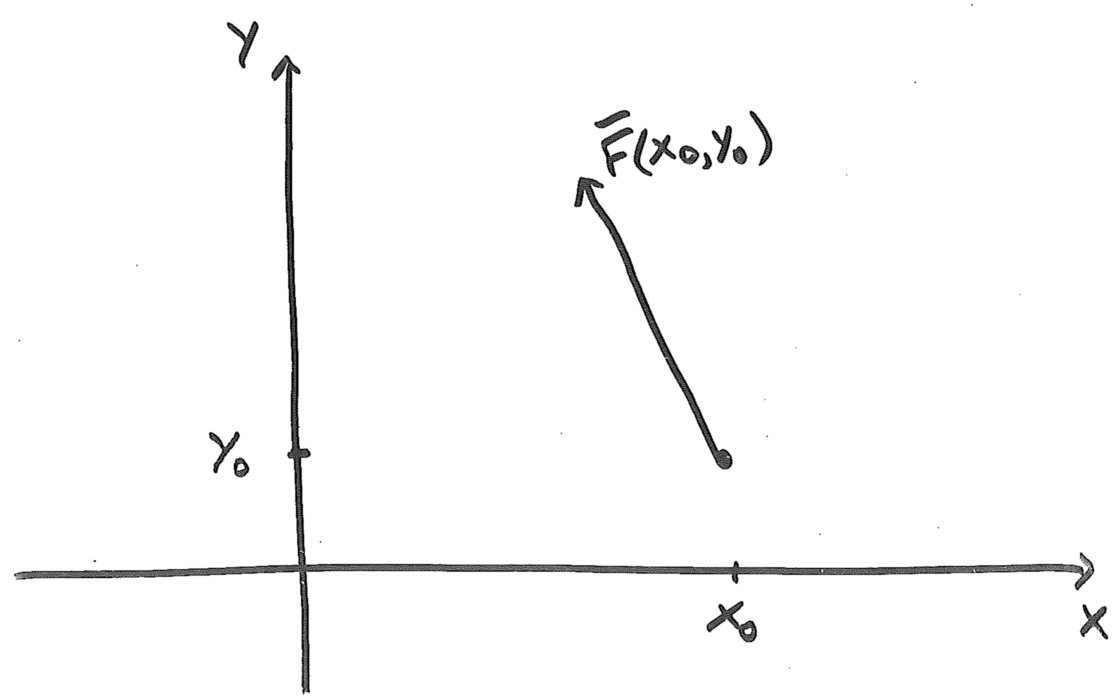


Given the autonomous system

$$\boxed{\frac{dx}{dt} = f(x,y) \qquad \frac{dy}{dt} = g(x,y) \qquad \text{with initial condition } (x(t_0), y(t_0)) = (x_0, y_0)}$$

Let $\bar{Y}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ $\qquad \dfrac{d\bar{Y}}{dt} = \begin{bmatrix} \dfrac{dx}{dt} \\ \dfrac{dy}{dt} \end{bmatrix} = \begin{bmatrix} f(x,y) \\ g(x,y) \end{bmatrix} = \bar{F}\begin{pmatrix} x \\ y \end{pmatrix} = \bar{F}(\bar{Y}$

$$\boxed{\frac{d\bar{Y}}{dt} = \bar{F}(\bar{Y}) \qquad \bar{Y}(0) = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \bar{Y}_0}$$



$$\frac{dx}{dt} = f(x,y)$$

$$\frac{dy}{dt} = g(x,y)$$

$$\frac{d\bar{y}}{dt} = \bar{F}(\bar{y}) = \begin{pmatrix} f(x,y) \\ g(x,y) \end{pmatrix}$$

Let $\bar{y}(t_0) = \bar{y}_0$

$$t_1 = t_0 + \Delta t$$

Then $\bar{y}(t_1) = \bar{y}_1$

Let $t_{k+1} = t_k + \Delta t$

Then $\bar{y}(t_k) = \bar{y}_k$

Euler's Method for Systems says

$$\bar{y}_{k+1} = \bar{y}_k + \bar{F}(\bar{y}_k)\Delta t$$

$$\begin{pmatrix} x(t_{k+1}) \\ y(t_{k+1}) \end{pmatrix} = \begin{pmatrix} x_{k+1} \\ y_{k+1} \end{pmatrix} = \begin{pmatrix} x_k \\ y_k \end{pmatrix} + \begin{pmatrix} f(x_k, y_k) \\ g(x_k, y_k) \end{pmatrix} \Delta t$$

$$\begin{pmatrix} x_{k+1} \\ y_{k+1} \end{pmatrix} = \begin{pmatrix} x_k \\ y_k \end{pmatrix} + \begin{pmatrix} f(x_k, y_k)\Delta t \\ g(x_k, y_k)\Delta t \end{pmatrix}$$

component wise we write

$$x_{k+1} = x_k + f(x_k, y_k)\Delta t$$

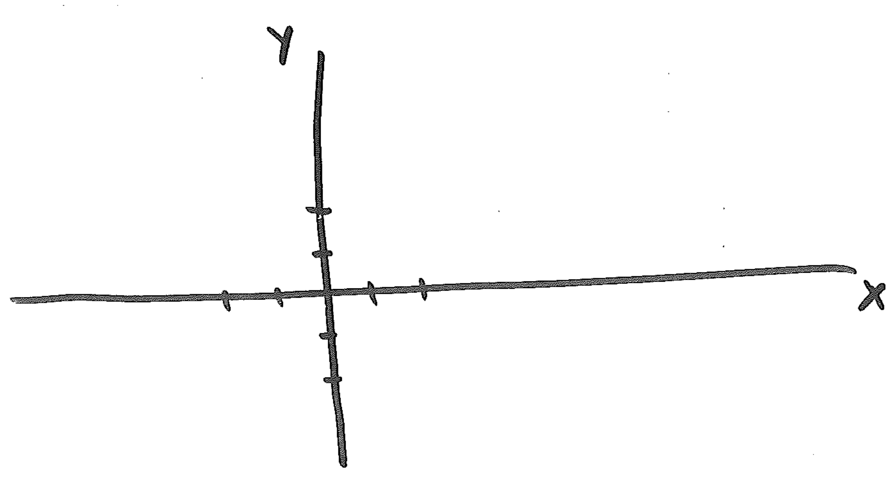
$$y_{k+1} = y_k + g(x_k, y_k)\Delta t$$

<u>Example 1</u>

$$\frac{dx}{dt} = y \qquad x(0) = 2$$

$$\frac{dy}{dt} = -x \qquad y(0) = 0$$

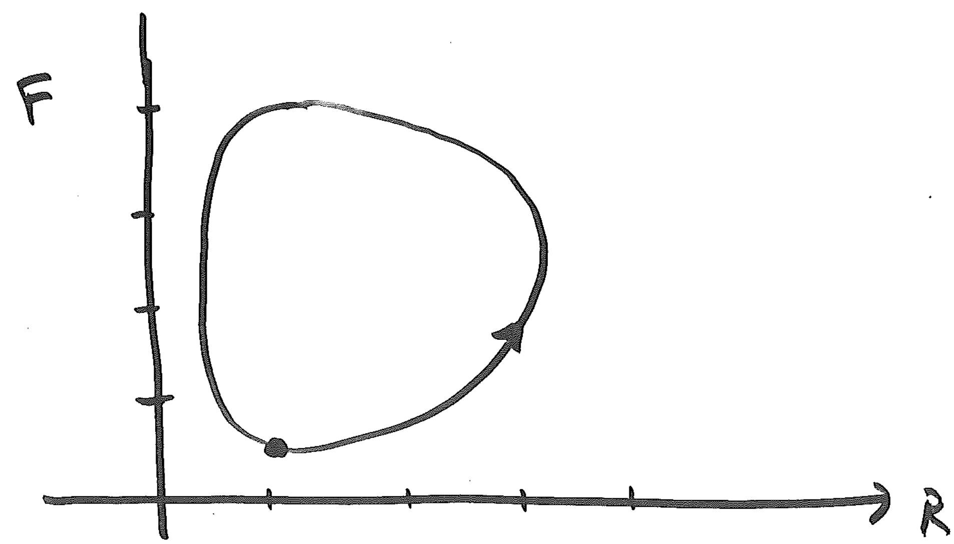
Solution: $x(t) = 2\cos t \qquad y(t) = -2\sin t$



Example 2 Predator Prey (P 153)

$$\frac{dR}{dt} = 2R - 1.2RF$$

$$\frac{dF}{dt} = -F + 0.9RF$$



$$\frac{dx}{dt} = 2x - 1.2x \cdot y = f(x, y)$$

$$\frac{dy}{dt} = -y + 0.9x \cdot y = g(x, y)$$