Sec 2.1 Modeling Via Systems

Predator - Prey System Revisited

$$\frac{dR}{dt} = 2R - 1.2RF$$

$$\frac{dF}{dt} = -F + 0.9RF$$

Equilibrium Solutions

Ask $\frac{dR}{dt} = 0$ and $\frac{dF}{dt} = 0$

$$\left.\begin{aligned} R(2-1.2F) &= 0 \\ F(-1+.9F) &= 0 \end{aligned}\right\}$$

Two equilibrium points

$(R,F) = (0,0)$

$(R,F) = \left(\frac{1}{.9}, \frac{2}{1.2}\right)$

$\approx (1.11, 1.67)$

Note: A system of two differential Equations requires two initial conditions, one for each function.

For example, consider the initial condition

$R(0) = 1 \qquad F(0) = .5$

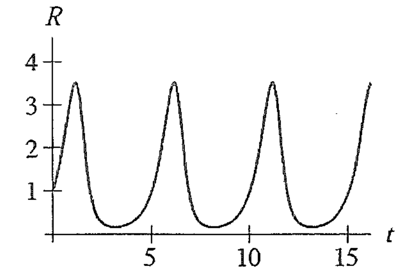


Figure 2.1
The $R(t)$-graph if $R_0 = 1$ and $F_0 = 0.5$.

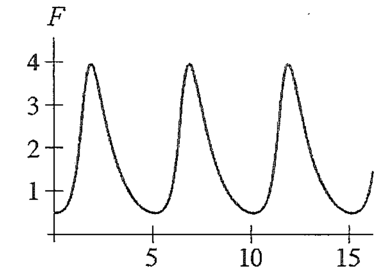
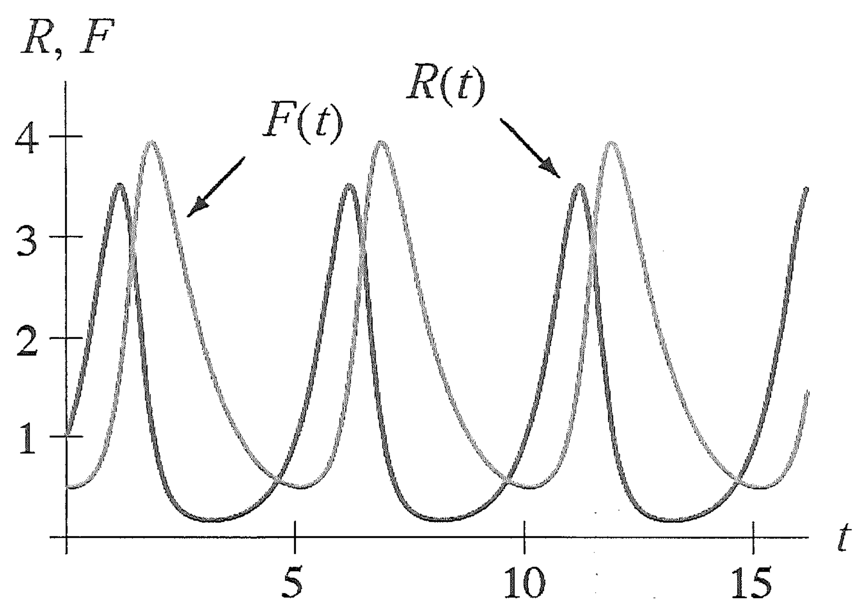
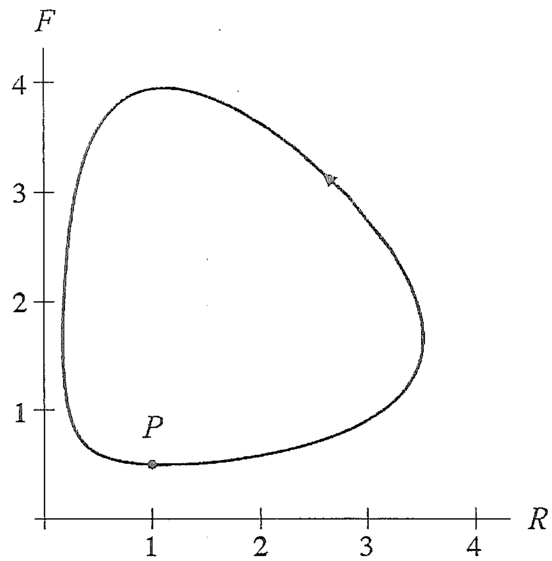
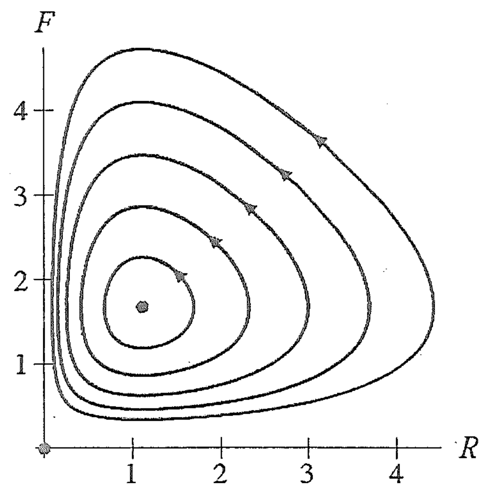


Figure 2.2
The $F(t)$-graph if $R_0 = 1$ and $F_0 = 0.5$.







A Modified Predator Prey Model

$$\frac{dR}{dt} = 2R\left(1-\frac{R}{2}\right) - 1.2RF$$

$$\frac{dF}{dt} = -F + 0.9RF$$

Equilibrium Values

Ask $\frac{dR}{dt} = 0$ and $\frac{dF}{dt} = 0$

$$R(2 - R - 1.2F) = 0$$

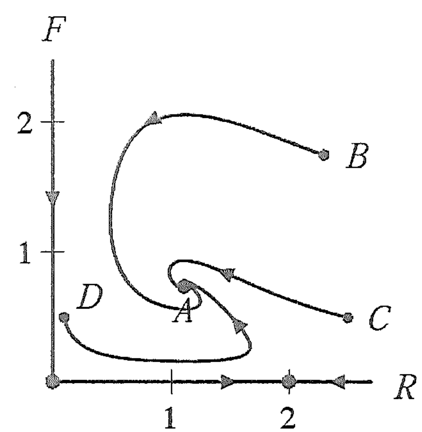
$$F(-1 + 0.9R) = 0$$

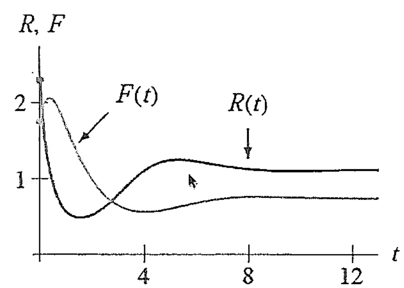
Three equilibrium points:

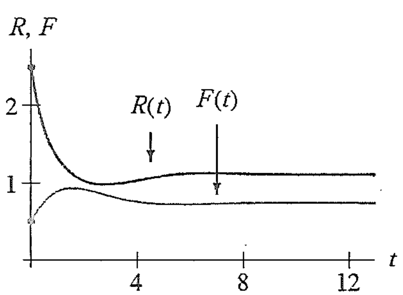
$(R,F) = (0,0)$

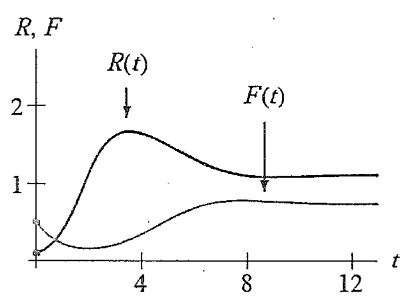
$(R,F) = (2,0)$

$(R,F) \cong (1.11, 0.74)$









Motion of a mass attached to a spring

Hooke's Law: The restoring force exerted by a spring is linearly proportional to the spring's displacement from its rest position and is directed toward the rest position.

Let $y(t)$ = displacement $\Rightarrow$ $\frac{d^2y}{dt^2}$ = acceleration

Force = $m\frac{d^2y}{dt^2} = -ky$ $\quad$ m = mass, k = spring constant.

$$m\frac{d^2y}{dt^2} = -ky$$

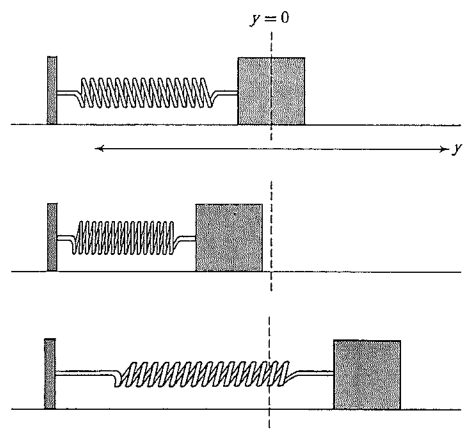
$$m\frac{d^2y}{dt^2} + ky = 0$$

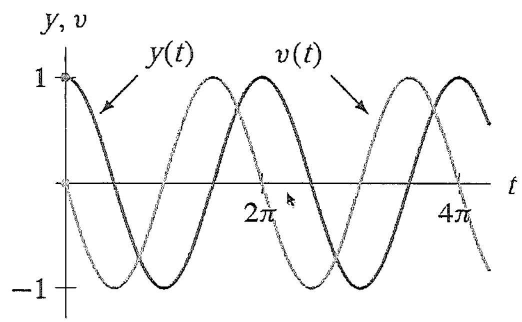
$$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$

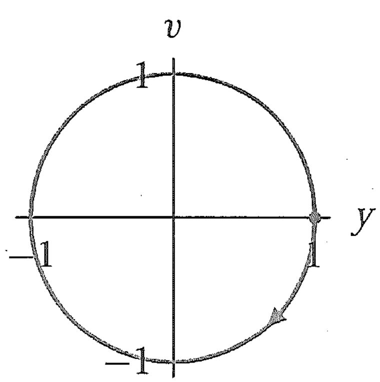
Note: $\frac{dy}{dt} = v$ and $\frac{dv}{dt} = \frac{d^2y}{dt^2} = -\frac{k}{m}y$

Equivalent System

$$\frac{dy}{dt} = v$$
$$\frac{dv}{dt} = -\frac{k}{m}y$$







An <u>example</u> of the motion of a mass attached to a spring when viewed as a system

Suppose $\frac{k}{m} = 1$ and $y(0) = 1$ and $v(0) = 0$

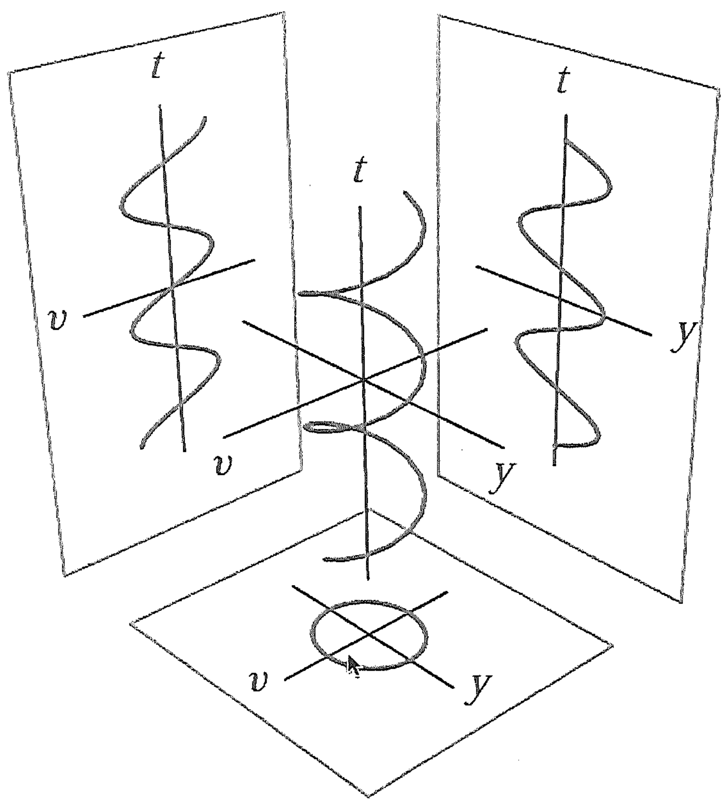
The 2^{nd} order equation

$$\text{I.V.P.} \begin{cases} \frac{d^2y}{dt^2} = -y \\ y(0) = 1 \\ \left.\frac{dy}{dt}\right|_{t=0} = 0 \end{cases}$$

When viewed as a system we have

$$\text{I.V.P.} \begin{cases} \frac{dy}{dt} = v \\ \frac{dv}{dt} = -y \\ y(0) = 1 \\ v(0) = 0 \end{cases}$$

Solution: $y(t) = \cos(t)$
$v(t) = -\sin(t)$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