

Sec 2.1 Modeling Via Systems

Predator-Prey System Revisited

$$\frac{dR}{dt} = 2R - 1.2RF$$

$$\frac{dF}{dt} = -F + 0.9RF$$

Equilibrium Solutions

Ask $\frac{dR}{dt} = 0$ and $\frac{dF}{dt} = 0$

$$\left. \begin{array}{l} R(2 - 1.2F) = 0 \\ F(-1 + 0.9F) = 0 \end{array} \right\} \begin{array}{l} \text{Two equilibrium points} \\ (R, F) = (0, 0) \\ (R, F) = \left(\frac{1}{.9}, \frac{2}{1.2}\right) \\ \approx (1.11, 1.67) \end{array}$$

Note: A system of two differential Equations requires two initial conditions, one for each function.

For example, consider the initial condition

$$R(0) = 1 \quad F(0) = .5$$

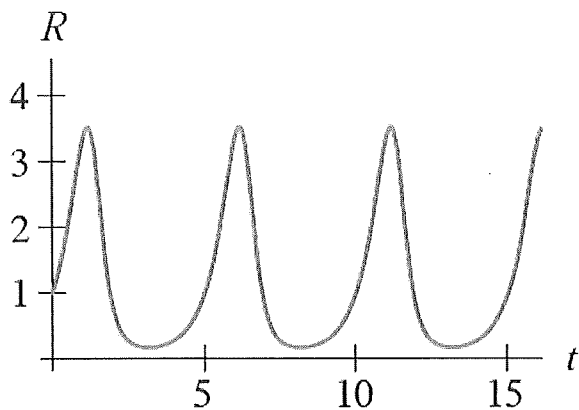


Figure 2.1
The $R(t)$ -graph if $R_0 = 1$ and $F_0 = 0.5$.

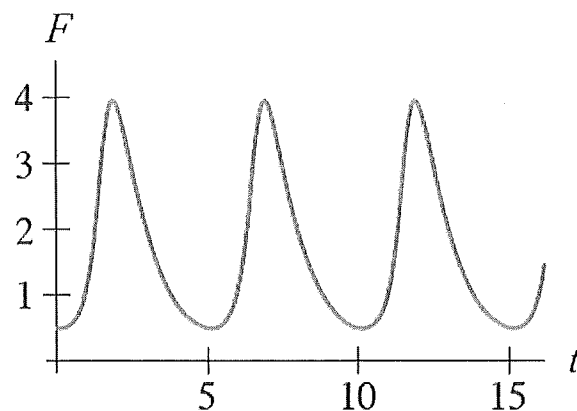
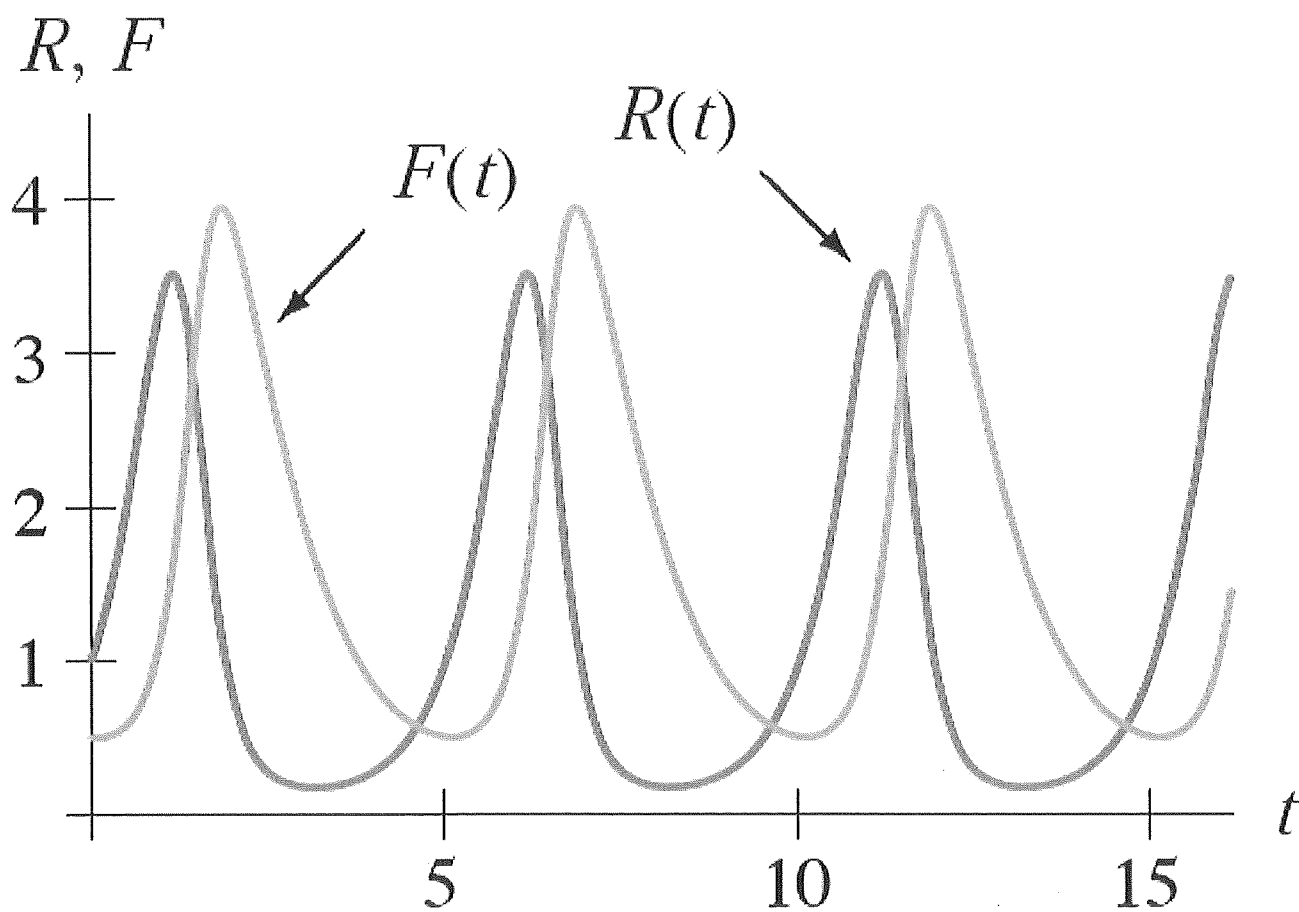
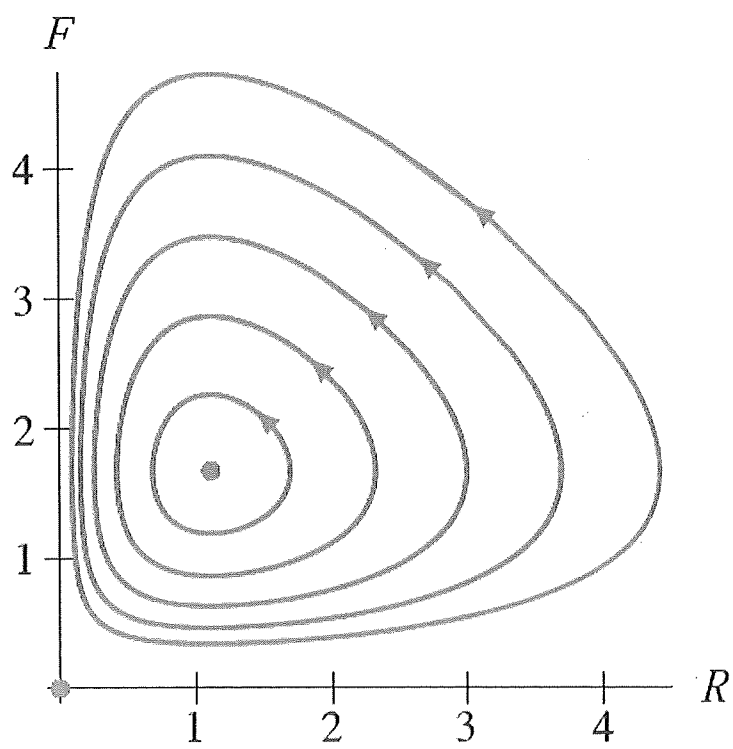
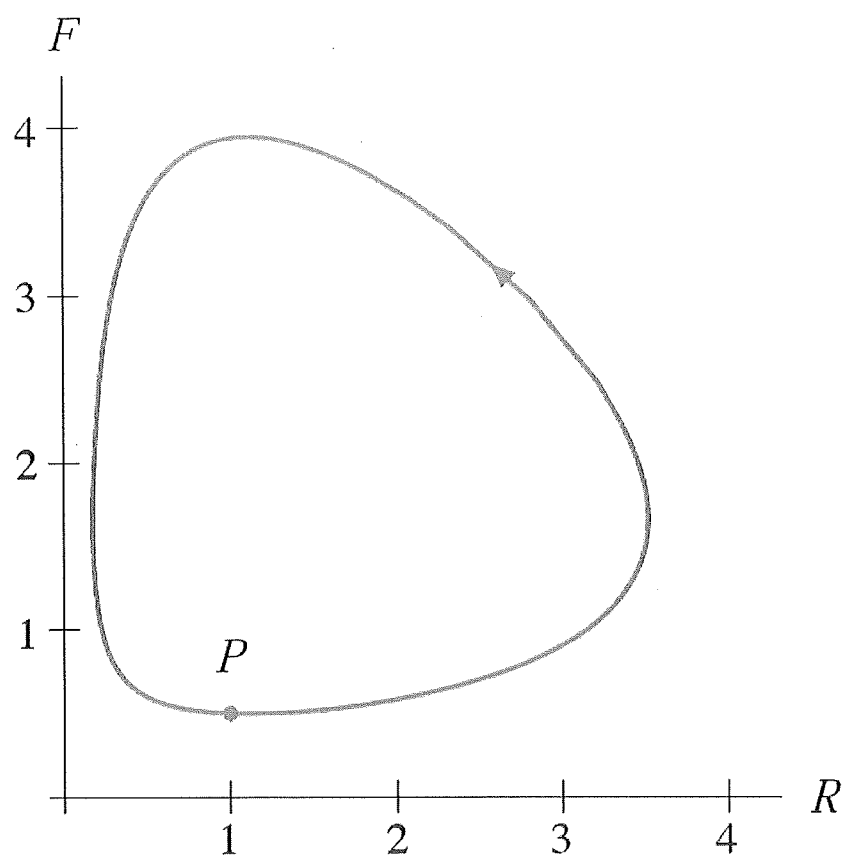


Figure 2.2
The $F(t)$ -graph if $R_0 = 1$ and $F_0 = 0.5$.





A Modified Predator Prey Model

$$\frac{dR}{dt} = 2R \left(1 - \frac{R}{2}\right) - 1.2RF$$

$$\frac{dF}{dt} = -F + 0.9RF$$

Equilibrium Values

Ask $\frac{dR}{dt} = 0$ and $\frac{dF}{dt} = 0$

$$R(2 - R - 1.2F) = 0$$

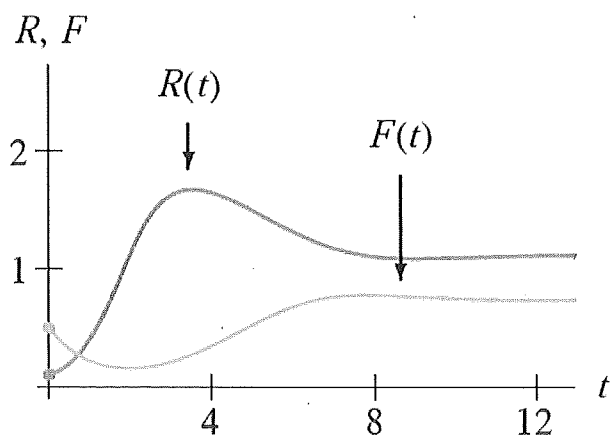
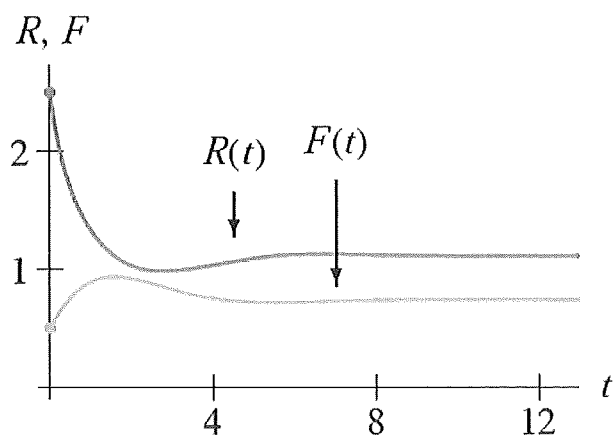
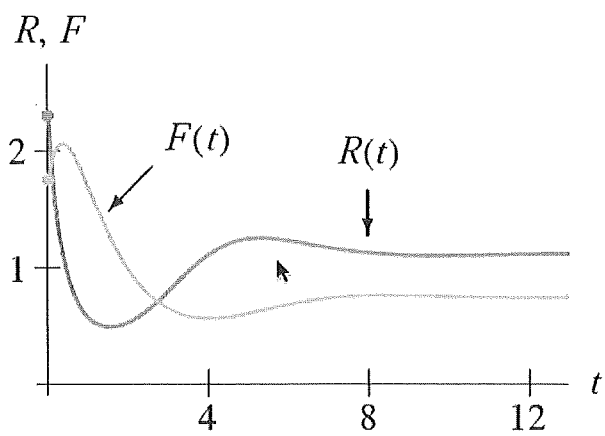
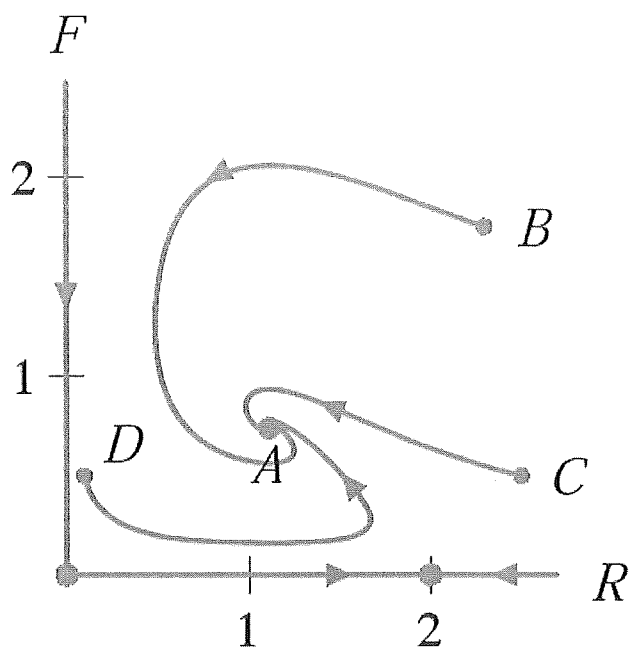
$$F(-1 + 0.9R) = 0$$

Three equilibrium points:

$$(R, F) = (0, 0)$$

$$(R, F) = (2, 0)$$

$$(R, F) \approx (1.11, 0.74)$$



Motion of a mass attached to a spring

Hooke's Law: The restoring force exerted by a spring is linearly proportional to the spring's displacement from its rest position and is directed toward the rest position.

Let $y(t)$ = displacement $\Rightarrow \frac{d^2 y}{dt^2}$ = acceleration

$$\text{Force} = m \frac{d^2 y}{dt^2} = -ky$$

m = mass

k = spring constant.

$$m \frac{d^2 y}{dt^2} = -ky$$

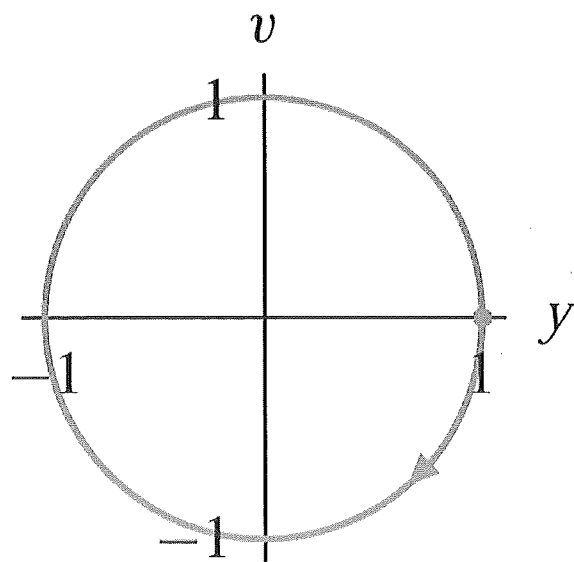
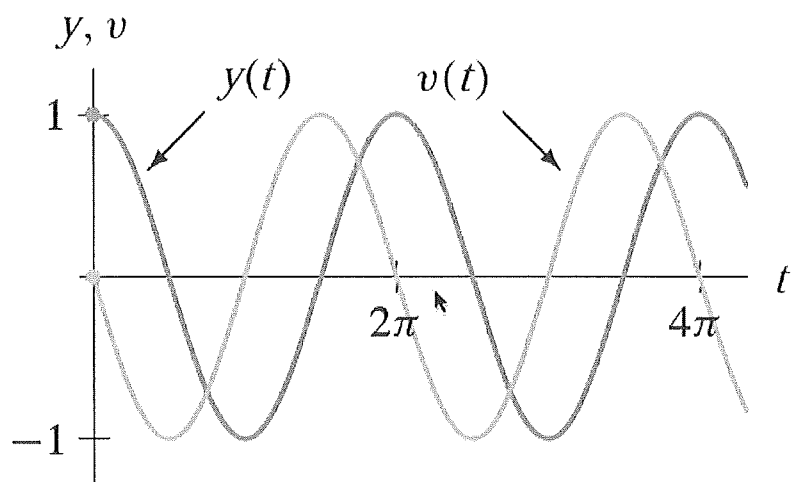
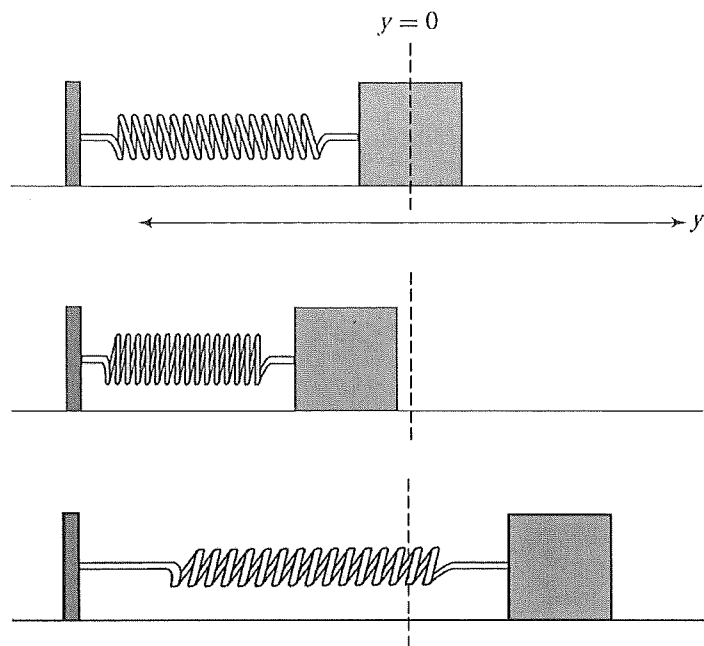
$$m \frac{d^2 y}{dt^2} + ky = 0$$

$$\frac{d^2 y}{dt^2} + \frac{k}{m} y = 0$$

Note: $\frac{dy}{dt} = v$ and $\frac{dv}{dt} = \frac{d^2 y}{dt^2} = -\frac{k}{m} y$

Equivalent
System

$$\begin{aligned} \frac{dy}{dt} &= v \\ \frac{dv}{dt} &= -\frac{k}{m} y \end{aligned}$$



An example of the motion of a mass attached to a spring when viewed as a system

Suppose $\frac{k}{m} = 1$ and $y(0) = 1$ and $v(0) = 0$

The 2nd order equation

$$\text{I.V.P.} \begin{cases} \frac{d^2 y}{dt^2} = -y \\ y(0) = 1 \\ \left. \frac{dy}{dt} \right|_{t=0} = 0 \end{cases}$$

When viewed as a system we have

$$\text{I.V.P.} \begin{cases} \frac{dy}{dt} = v \\ \frac{dv}{dt} = -y \\ y(0) = 1 \\ v(0) = 0 \end{cases}$$

Solution: $y(t) = \cos(t)$
 $v(t) = -\sin(t)$

