Name: Prof D Date: _____

MA 226 Quiz 5 – B

Please show your work.

1. (5 pts) Solve the initial value problem $\frac{dy}{dt} - 3y = 3t$, $y(0) = -1$.

step 1 : $y_h(t) = ke^{3t}$

step 2 : $y_p(t) = At + B$

$y_p'(t) = A$

plug into eqn. $A - 3(At + B) = 3t$

$-3At + (A - 3B) = 3t$

$-3A = 3 \Rightarrow \boxed{A = -1}$

$A - 3B = 0 \Rightarrow B = -\frac{1}{3}$

$\boxed{y_p(t) = -t - \frac{1}{3}}$

check: $y_p' - 3y_p = -1 - 3(-t - \frac{1}{3}) = -1 + 3t + 1 = 3t$ ✓

step 3 general solution $y(t) = y_h(t) + y_p(t)$

$\boxed{y(t) = ke^{3t} - t - \frac{1}{3}}$

step 4 Initial condition $y(0) = -1 = k - \frac{1}{3} \Rightarrow k = -\frac{2}{3}$

$\boxed{y(t) = -\frac{2}{3}e^{3t} - t - \frac{1}{3}}$

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2. (5 pts) Solve the initial value problem: $\frac{dy}{dt} = -\frac{y}{t} + 2$, $y(1) = 3$

Rewrite $\frac{dy}{dt} + \frac{y}{t} = 2$

$\mu(t) = e^{\int \frac{1}{t} dt} = e^{\ln t} = t$

$t \cdot \left(\frac{dy}{dt} + \frac{y}{t}\right) = t \cdot 2$

$\frac{d}{dt}(t \cdot y) = 2t$

$\int \frac{d}{dt}(t \cdot y)\, dt = \int 2t\, dt$

$t \cdot y = t^2 + C$

$\boxed{y(t) = \frac{t^2 + C}{t}}$

$y(1) = 3 \Rightarrow 3 = \frac{1 + C}{1} \Rightarrow \boxed{C = 2}$

$\boxed{y(t) = \frac{t^2 + 2}{t}}$

Check: $y(t) = t + \frac{C}{t}$

plug into eqn

$1 - \frac{C}{t^2} + \frac{1}{t}\left(t + \frac{C}{t}\right) \stackrel{?}{=} 2$

$1 - \frac{C}{t^2} + 1 + \frac{C}{t^2} \stackrel{?}{=} 2$

$2 = 2$ ✓