Name: Prof Deutsch Date: 9/17/14

MA 226 Quiz 3 – B

Please show your work.

1. (5 pts) A cup of hot chocolate is initially $175^{\circ}F$ and is left in a room with an ambient temperature of $75^{\circ}F$. Suppose that at $t=0$ it is cooling at a rate of $15^{\circ}F$ per minute.

a.) Assume the Newton's law of cooling applies: The rate of cooling is proportional to the difference between the current temperature and the ambient temperature. Write an initial value problem that models the temperature of the hot chocolate.

$$\begin{cases} \frac{dT}{dt} = K(T - T_A) \\ T(0) = 175 \end{cases}$$

$$\left.\frac{dT}{dt}\right|_{t=0} = -15 = K(175-75)$$

$$\frac{-15}{100} = K$$

$$\boxed{K = -.15}$$

b.) Solve the initial value problem for the temperature T(t).

T = 75 is the equilibrium solution

separation of variables

$$\frac{1}{T-75}\,dT = -.15\,dt$$

$$\int \frac{1}{T-75}\,dt = \int -.15\,dt$$

$$\ln|T-75| = -.15t + C$$

$$T - 75 = Ce^{-.15t}$$

$$T(t) = 75 + Ce^{-.15t}$$

$$T(0) = 175 = 75 + C$$

$$C = 100$$

$$\boxed{T(t) = 75 + 100e^{-.15t}}$$

c.) How long does it take the hot chocolate to cool to $115^{\circ}F$?

$$T(t) = 115$$

$$75 + 100e^{-.15t} = 115$$

$$e^{-.15t} = \frac{40}{100}$$

$$-.15t = \ln(.4)$$

$$t = \left(\frac{\ln .4}{-.15}\right) = \boxed{6.108 \text{ min}}$$

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2. (5 pts) Given the initial value problem:

$$\frac{dy}{dt} = t - y^2 \text{ with } y(0) = 1$$

Use Euler's Method with a step size of .25 to approximate the value of y(t) when t = .75. Create a table and show you work. Use 6 decimal places of accuracy.

$\Delta t = .25$

K	t_K	Y_K	$f(t_K, Y_K) = t_K - Y_K^2$	$Y_{K+1} = Y_K + f(t_K, Y_K) \cdot \Delta t$
0	0	1	-1	$1 - 1(.25) = .75$
1	.25	.75	$-.3125$	$.75 - .3125(.25) = .671875$
2	.5	.671875	.048584	$.671875 + .048584(.25) =$ .684021
3	.75	.684021		