

Name: Prof DeutschDate: Sept 17, 2014

MA 226 Quiz 3 - A

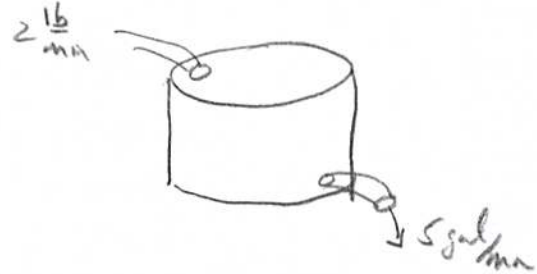
Please show your work.

1. (5 pts) A 100-gallon vat is full of pure water. Suppose we begin dumping salt into the vat at a rate of 2 pounds per minute. Also, we open the spigot so that 5 gallons per minute leaves the vat, and we add pure water to keep the vat full. Assume that the salt water solution is always well mixed.

a.) Write the initial value problem that models the amount of salt, $S(t)$, in the vat at time t .

$$\frac{dS}{dt} = 2 \frac{\text{lb}}{\text{min}} - 5 \frac{\text{gal}}{\text{min}} \cdot \frac{S}{100} \frac{\text{lb}}{\text{gal}}$$

$$\begin{cases} \frac{dS}{dt} = 2 - \frac{S}{20} = \frac{40-S}{20} \\ S(0) = 0 \end{cases}$$



b.) Solve the Initial Value Problem for $S(t)$

$S = 40$ is equilibrium solution

separation of variables

$$\frac{1}{40-S} dS = \frac{1}{20} dt \quad S \neq 40$$

$$\int \frac{1}{40-S} dS = \int \frac{1}{20} dt$$

$$-\ln|40-S| = \frac{1}{20}t + C$$

$$\ln|40-S| = -\frac{1}{20}t + C$$

$$|40-S| = C \cdot e^{-\frac{1}{20}t} \quad C > 0$$

$$40-S = C e^{-\frac{1}{20}t} \quad C \in \mathbb{R}$$

$$S(t) = 40 + C e^{-\frac{1}{20}t}$$

$$S(0) = 0 = 40 + C$$

$$\Rightarrow C = -40$$

$$S(t) = 40 - 40 e^{-\frac{1}{20}t}$$

$$S(t) = 40(1 - e^{-\frac{1}{20}t})$$

d.) What is $S(2)$?

3.8065

e.) What is $\lim_{t \rightarrow \infty} S(t) = 40$

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2. (5 pts) Given the initial value problem:

$$\frac{dy}{dt} = t - y^2 \text{ with } y(0) = 1$$

Use Euler's Method with a step size of .25 to approximate the value of $y(t)$ when $t = .75$. Create a table and show your work. Use 6 decimal places of accuracy.

k	t_k	y_k	$f(t_k, y_k) = t_k - y_k^2$	$y_{k+1} = y_k + f(t_k, y_k) \cdot .25$
0	0	1	-1	.75
1	.25	.75	-.3125	.671875
2	.5	.671875	.048584	.684021
3	.75	.684021		