Name: Prof D Date: Sept 10, 2014

MA 226 Quiz 2 – A

Please show your work.

1. (5 pts) Consider the differential equation $\frac{dy}{dt} = y^3 - 3y^2 - 4y = y(y^2 - 3y - 4)$

$= y(y-4)(y+1)$

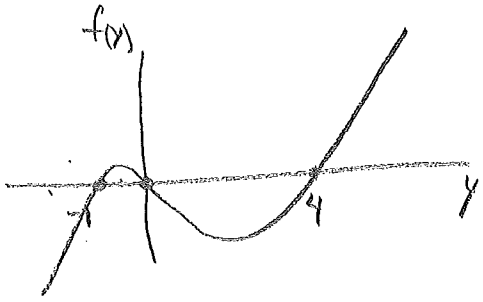
$= f(y)$

a.) For what values of y is y(t) in equilibrium?

$y = 0$

$y = -1$

$y = 4$



b.) For what values of y is y(t) increasing?

y(t) is increasing for

$-1 < y < 0$ and $y > 4$

c.) For what values of y is y(t) decreasing?

$0 < y < 4$ and $y < -1$

2. (5 pts) MacQuarie Island is a small island about half-way between Antarctica and New Zealand. Between 2000 and 2006 , the population of rabbits on the island rose from 4,000 to 380,000. Model the growth in rabbit population R(t) at time t using an exponential growth model $\frac{dR}{dt} = kR$ where t = 0 corresponds to the year 2000. What is an appropriate value of the growth rate parameter k and what does the model predict the rabbit population will be in the year 2010?

$R(t) = Ce^{kt}$

$R(0) = 4000 = Ce^{k \cdot 0}$

$C = 4000$

$R(6) = 4000\, e^{k \cdot 6} = 380{,}000$

$e^{k \cdot 6} = \frac{380{,}000}{4{,}000} = 95$

$k \cdot 6 = \ln(95)$

$k = \frac{1}{6}\ln(95) = .75898$

$\boxed{R(t) = 4000\, e^{.75898\, t}}$

$R(10) = 4000\, e^{(.75898)10}$

$R(10) = 7911671.56$

Population in 2010 is approx $\boxed{7{,}911{,}671}$

or

$\boxed{7.911671 \times 10^6}$