Name:_____ Discussion Section : _____

MA 226 Section B – Exam 1

Question Number	Possible Points	Student Score
1	12	
2a	8	
2b	8	
2c	8	
2d	8	
3	16	
4	16	
5	16	
6	8	
Total Points	100	

You must show your work to receive full credit

Discussion Sections:

B2: Tuesday 4:30-5:30

B3: Tues : 3:30-4:30

B4: Weds: 9-10

B5: Weds: 10-11

B6: Weds: 4:30-5:30

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1. Short Answer (12 pts)

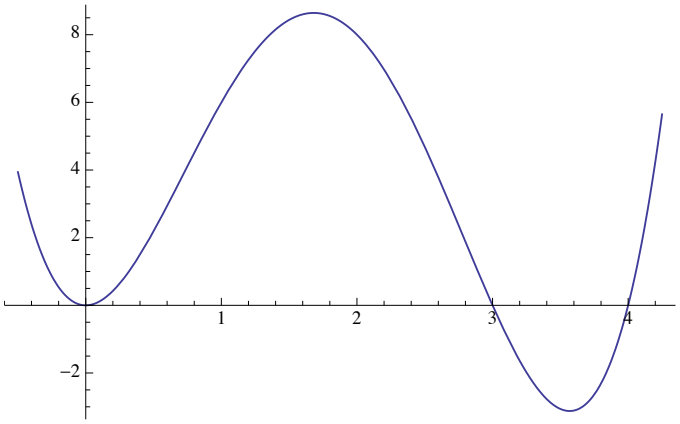
a) Given the differential equation : $\frac{dy}{dt} = 2y(y+3)(y-4)$. Let $y_1(t)$ be a solution that satisfies the initial condition $y_1(0) = 2$. Evaluate

$\lim_{t->\infty} y_1(t) =$ $\lim_{t->-\infty} y_1(t) =$

b) Find the equilibrium solutions of the differential equation

$$\frac{dy}{dt} = \frac{(t^2-4)(y^2-1)}{y^2-9}$$

c) Draw the phase line that corresponds to the differential equation $\frac{dy}{dt} = f(y)$ where $f(y)$ is given by the graph below



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2. Solving differential equations and initial value problems

a) (8 pts) Find the general solution of the equation: $\frac{dy}{dt} = 2y(2 - y)$

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b) (8 pts) Solve the initial value problem $\frac{dy}{dt} + 2y = \cos(4t)$ with $y(0) = 0$

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c) (8 pts) Solve the initial value problem $\frac{dy}{dt} - \frac{2}{t}y = t^2 e^{3t}$ with $y(1) = 0$

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d) (8 pts) Solve the initial value problem $\frac{dy}{dt} = 3y + 2e^{3t}$ with $y(0) = 4$

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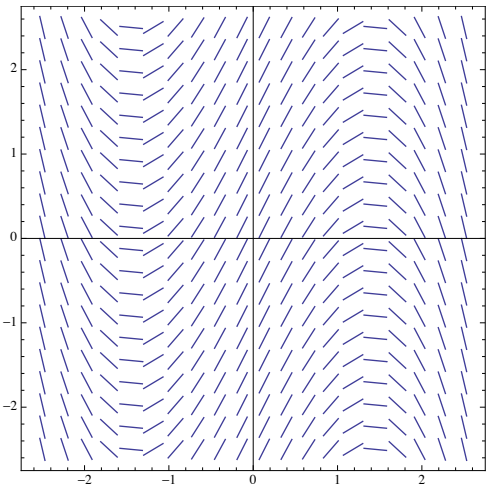
3. (16 pts) Matching slope fields

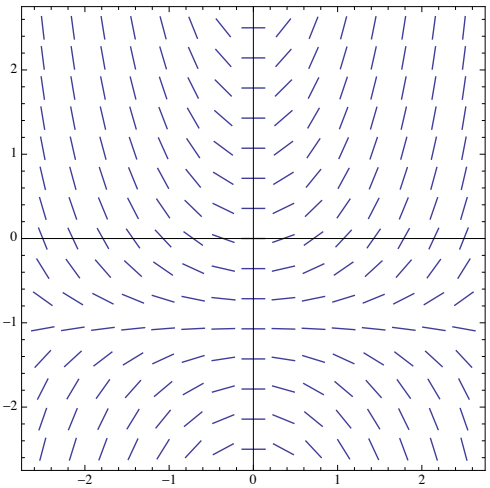
(i) $\frac{dy}{dt} = y^2 + y$ (ii) $\frac{dy}{dt} = y^2 - y$ (iii) $\frac{dy}{dt} = y^3 + y^2$ (iv) $\frac{dy}{dt} = 2 - t^2$

(v) $\frac{dy}{dt} = ty + ty^2$ (vi) $\frac{dy}{dt} = t^2 + t^2 y$ (vii) $\frac{dy}{dt} = t + ty$ (viii) $\frac{dy}{dt} = t^2 - 2$

Slope Field A equation: _____ Reason:



Slope Field B equation: _____ Reason



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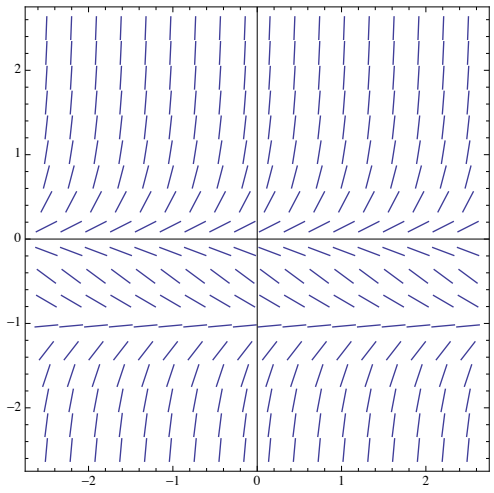
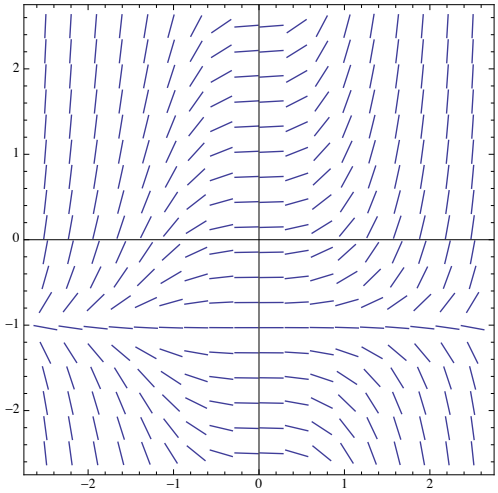
(i) $\frac{dy}{dt} = y^2 + y$ (ii) $\frac{dy}{dt} = y^2 - y$ (iii) $\frac{dy}{dt} = y^3 + y^2$ (iv) $\frac{dy}{dt} = 2 - t^2$

(v) $\frac{dy}{dt} = ty + ty^2$ (vi) $\frac{dy}{dt} = t^2 + t^2 y$ (vii) $\frac{dy}{dt} = t + ty$ (viii) $\frac{dy}{dt} = t^2 - 2$

Slope Field C equation: _____ Reason



Slope Field D equation: _____ Reason



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4. (16 pts) A college professor contributes $8,000 per year into her retirement fund by making many small deposits throughout the year. The fund grows at a rate of 7% per year compounded continuously. After 30 years, she retires and begins withdrawing from her fund at a rate of $5,000 per month. If she does not make any deposits after retirement, how long will the money last? [Hint: Solve the problem in two steps, before retirement and after]

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5. (16 pts) A cup of hot chocolate is initially $170^\circ F$ and is left in a room with an ambient temperature of $65^\circ F$. Suppose at time $t = 0$ it is cooling at a rate of 15° per minute.

a.) Assume the Newton's law of cooling applies: The rate of cooling is proportional to the difference between the current temperature and the ambient temperature. Write an initial value problem that models the temperature of the hot chocolate.

b.) Find a solution of the initial value problem.

c.) How long does it take for the hot chocolate to cool to 100°F? (show your work)

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6. (8 pts) Use Euler's Method with a step size of 0.5 to approximate the solution of the initial value problem $\frac{dy}{dt} = 2t - y$ and y(0)= 2 over the interval $0 \le t \le 2$.

Create a table that shows how you computed the values for t=.5, 1, 1.5, and 2