MA 226 Section B – Exam 1a

Question Number	Possible Points	Student Score
1	15	
2a	10	
2b	10	
2c	10	
2d	10	
3	12	
4	15	
5	10	
6	8	
Total Points	100	

You must show your work to receive full credit

Discussion Sections:

B2: Wednesday 9-10

B3: Wednesday 2-3

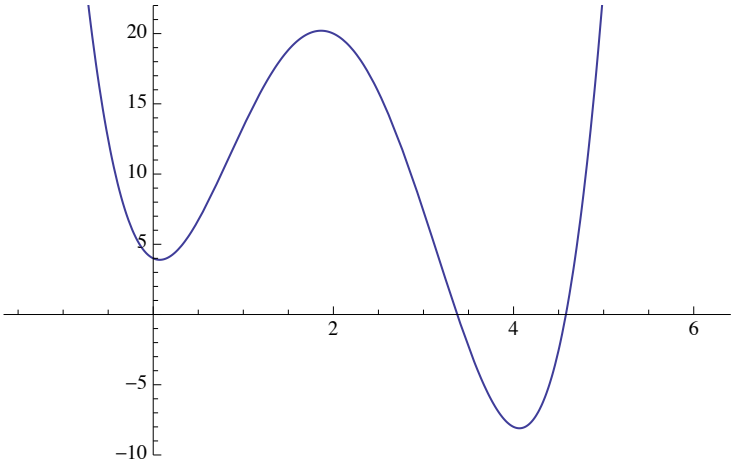
1. Short Answer (15 pts)

a) Given the differential equation : $\frac{dy}{dt} = 3(y-1)(y-5)(y-7)$. Let $y_1(t)$ be a solution that satisfies the initial condition $y_1(0) = 3$. Evaluate

$\lim_{t->\infty} y_1(t) =$ $\lim_{t->-\infty} y_1(t) =$

b) Find the equilibrium solutions of the differential equation $\frac{dy}{dt} = y^3 + 2y^2 - 8y$

c) Given the one parameter family of differential equations $\frac{dy}{dt} = f(y) + \alpha$ where $f(y)$ is given by the graph below, identify the bifurcation value(s). You **DO NOT** need to draw a bifurcation diagram just identify the bifurcation values. Note that the critical points on the graph of $f(y)$ occur at (0,4) , (2,20) and (4,-8)



2. Solving differential equations and initial value problems

a) (10 pts) Find the general solution of the equation: $\frac{dy}{dt} = \frac{5y(4-y)}{8}$

b) (10 pts) Solve the initial value problem $\frac{dy}{dt} + 3y = 12\sin(3t) + 9t$ with $y(0) = 0$

c) (10 pts) Solve the initial value problem $\frac{dy}{dt} = -2t \cdot y + 5e^{-t^2}$ with $y(0) = 6$

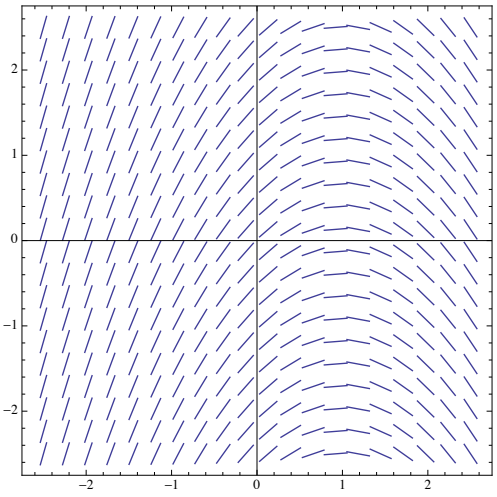
d) (10 pts) Solve the initial value problem with $\frac{dy}{dt} = \frac{y}{3} + 5e^{\frac{t}{3}}$ and $y(0) = 2$

3. (12 pts) Matching slope fields

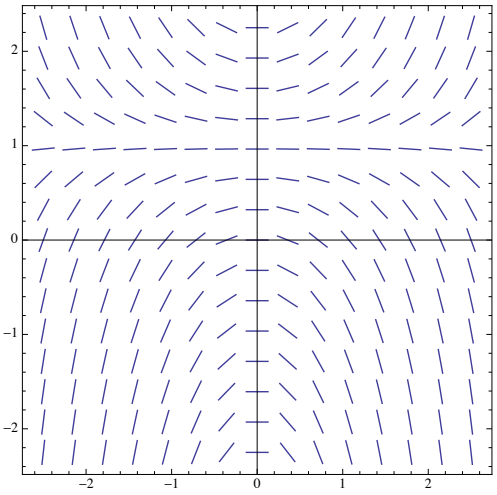
(i) $\frac{dy}{dt} = t - 1$ (ii) $\frac{dy}{dt} = y^2 - 1$ (iii) $\frac{dy}{dt} = ty - t$ (iv) $\frac{dy}{dt} = 1 - y^2$

(v) $\frac{dy}{dt} = 1 - t$ (vi) $\frac{dy}{dt} = 1 - y$ (vii) $\frac{dy}{dt} = y - t^2$ (viii) $\frac{dy}{dt} = y + t^2$

Slope Field A equation: _____ Reason:



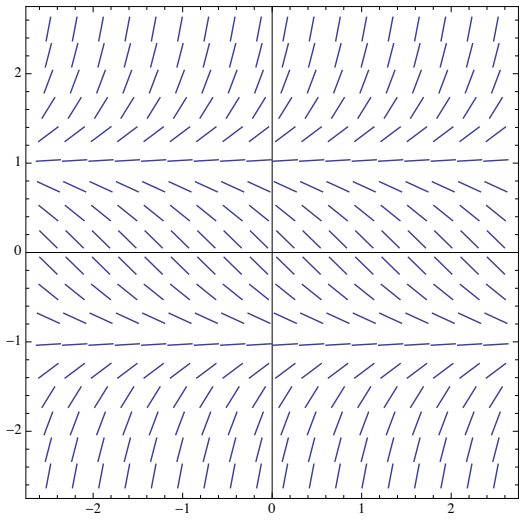
Slope Field B equation: _____ Reason



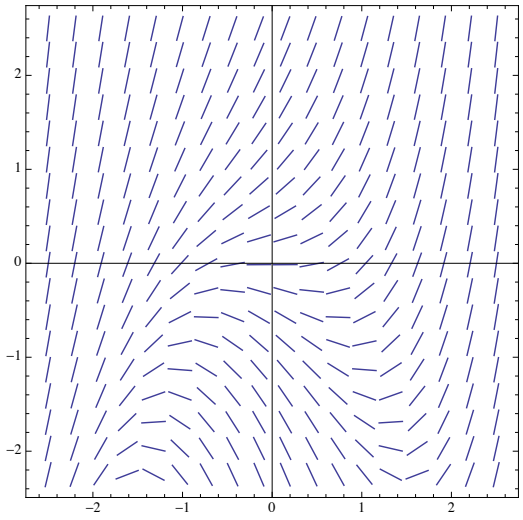
(i) $\frac{dy}{dt} = t - 1$ (ii) $\frac{dy}{dt} = y^2 - 1$ (iii) $\frac{dy}{dt} = ty - t$ (iv) $\frac{dy}{dt} = 1 - y^2$

(v) $\frac{dy}{dt} = 1 - t$ (vi) $\frac{dy}{dt} = 1 - y$ (vii) $\frac{dy}{dt} = y - t^2$ (viii) $\frac{dy}{dt} = y + t^2$

Slope Field C equation: _____ Reason



Slope Field D equation: _____ Reason



4. (15 pts) Consider a large vat containing sugar water that is to be made into soft drinks.

- Initially the vat contains only pure water.

-The The vat contains 100 gals of liquid. Moreover, the amount flowing in is the same as the amount flowing out, so there are always 100 gallons in the vat.

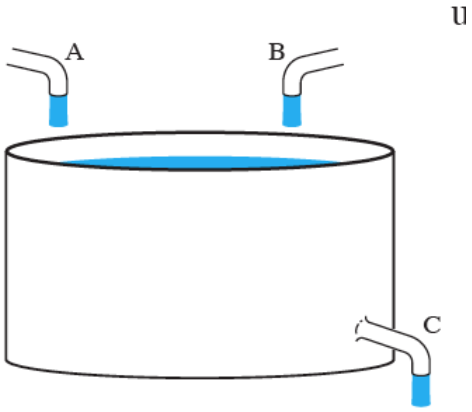
- The vat is kept well mixed, so that the sugar concentration is uniform throughout the vat.

-Sugar water containing 6 tablespoons per gallon enters the vat through pipe A at a rate of 3 gallons per minute.

-Sugar water containing 7 tablespoons per gallon enters the vat through pipe B at at rate of 2 gallon per minute.

-Sugar water leaves the vat through pipe C at a rate of 5 gallons per minute.

a.) (4 pts) : Write the initial value problem that describes the amount of sugar in the vat as a function of time.



b.) (7 pts) : Solve the initial value problem

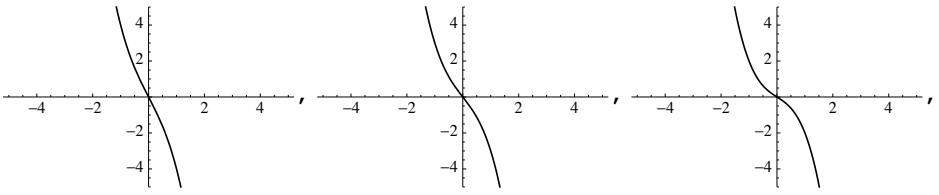
c.) (2 pts): How much sugar will be in the tank after 30 minutes?

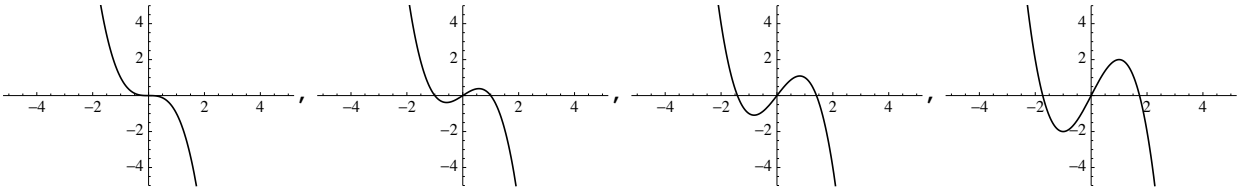
d.) (2 pts) : What is $\lim_{t\to\infty} S(t)$

5. (10 pts) Given the one parameter family of differential equations:

$\frac{dy}{dt} = Ay - y^3$ and a parameter study for values of the parameter A ranging from A = -3 to 3 create a bifurcation diagram and indicate the bifurcation value(s).

A = -3 A = -2 A=-1





A = 0 A = 1 A = 2 A = 3

6. (8 pts) Use Euler's Method with a step size of 0.25 to approximate the solution of the initial value problem $\frac{dy}{dt} = y^2 - 2t$ and $y(0) = 1$ over the interval $0 \le t \le .75$.

Create a table that shows how you computed the approximate y values for values of t= .25, .5, and .75

Please use 6 places of accuracy in your calculations.