

Sec 1.4 Euler's Method

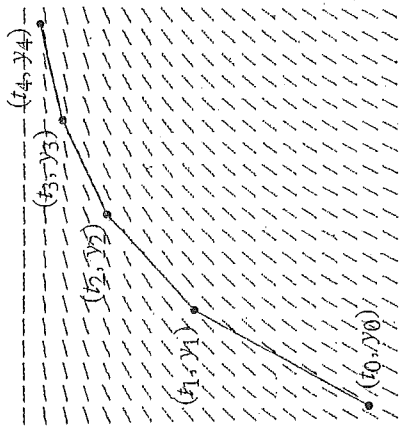


Figure 1.31
Stepping along the slope field.

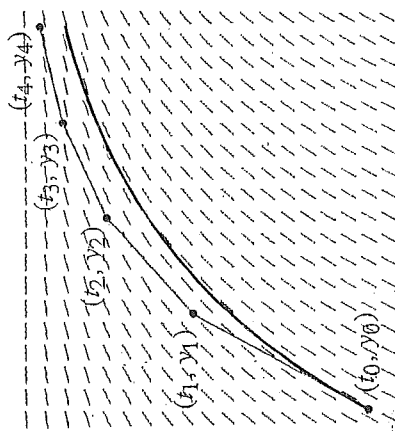
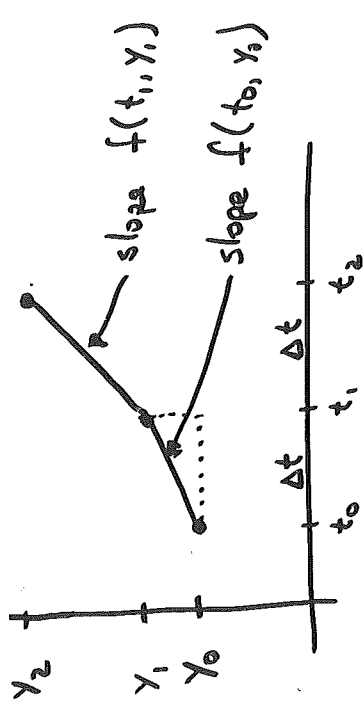


Figure 1.32
The graph of a solution and its approximation obtained using Euler's method.



Given $\frac{dy}{dt} = f(t, y)$ $y(t_0) = y_0$

Basic Slope Calculation

$$\frac{y_1 - y_0}{t_1 - t_0} = \text{slope} = f(t_0, y_0)$$

Euler's Method

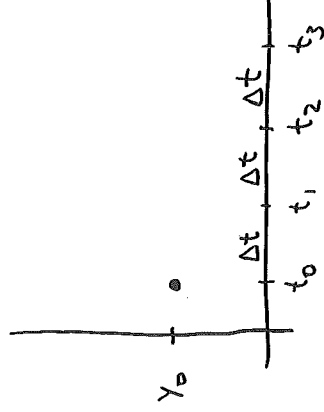
at (t_0, y_0) we know slope $\frac{dy}{dt} = f(t_0, y_0) = m$

Write the equation of the line passing through (t_0, y_0) (use point-slope form)

$$y - y_0 = m(t - t_0)$$

$$y = y_0 + m(t - t_0)$$

$$y = y_0 + f(t_0, y_0)(t - t_0)$$

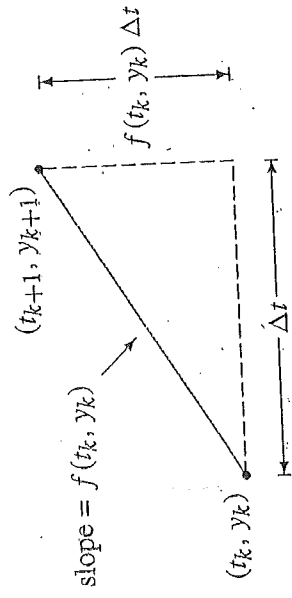


Evaluate the line at $t = t_1$

$$y(t_1) = y_0 + f(t_0, y_0)(t_1 - t_0)$$

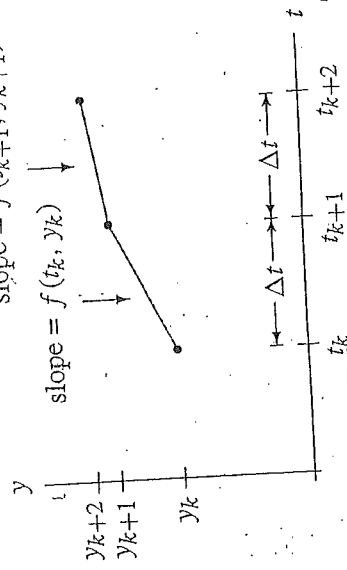
$$y_1 = y_0 + f(t_0, y_0) \Delta t$$

In general $y_{k+1} = y_k + f(t_k, y_k) \Delta t$



$$\text{slope} = f(t_{k+1}, y_{k+1})$$

$$\text{slope} = f(t_k, y_k)$$



Euler's Method in Table Form

k	t_k	y_k	$f(t_k, y_k)$	$y_{k+1} = y_k + f(t_k, y_k) \Delta t$
0	0	1		
1	.1			
2	.2			
3	.3			
4	.4			

Let use the example $\frac{dy}{dt} = -2ty^2$ $y(0) = 1$