Sec 1.4 Euler's Method

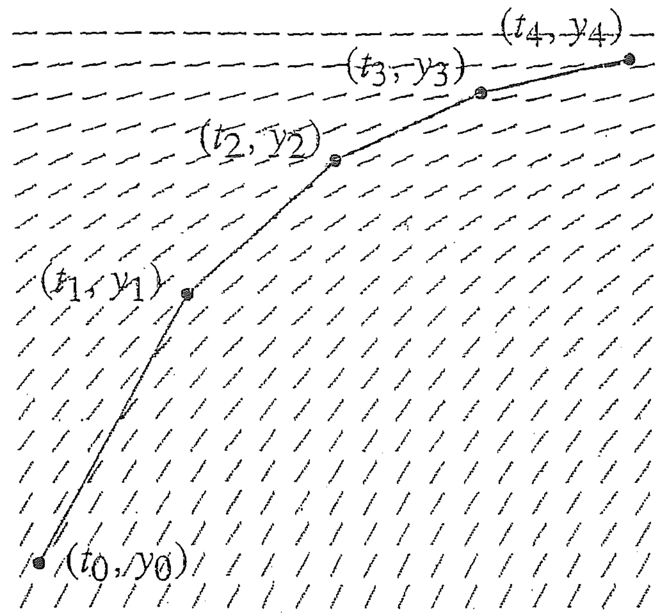


Figure 1.31
Stepping along the slope field.

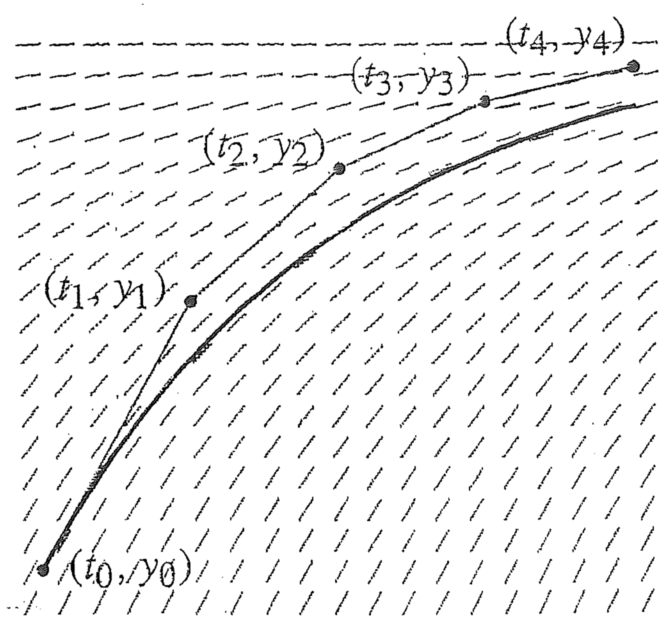
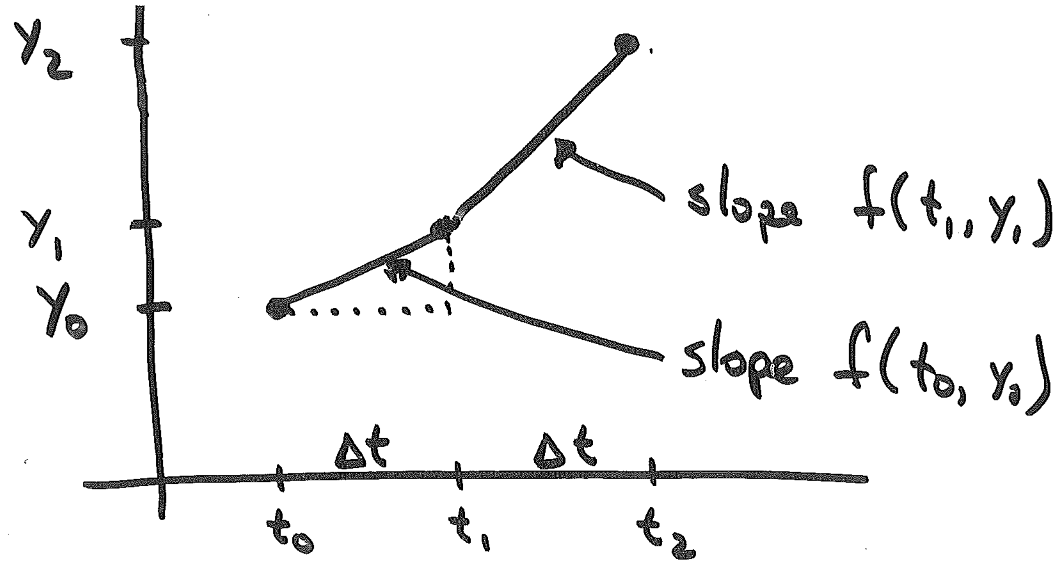


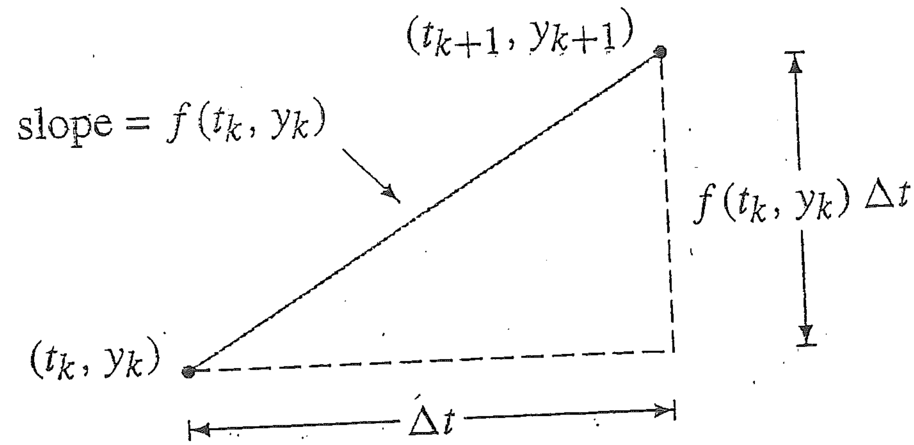
Figure 1.32
The graph of a solution and its approximation obtained using Euler's method.



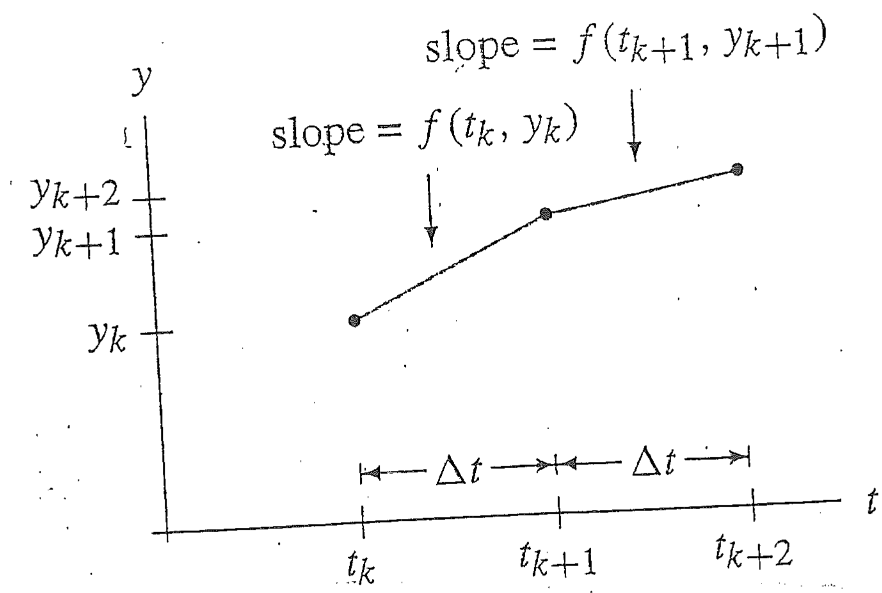
Given $\quad \dfrac{dy}{dt} = f(t, y) \qquad y(t_0) = y_0$

Basic Slope Calculation

$$\frac{y_1 - y_0}{t_1 - t_0} = \text{slope} = f(t_0, y_0)$$



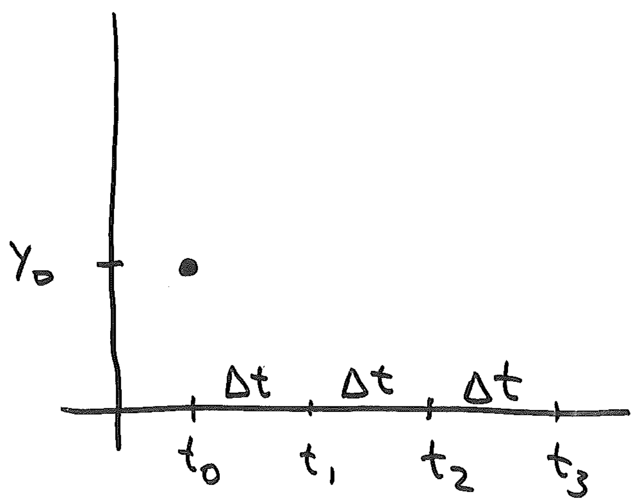
(t_{k+1}, y_{k+1})
slope = $f(t_k, y_k)$
$f(t_k, y_k)\,\Delta t$
(t_k, y_k)
Δt



slope = $f(t_{k+1}, y_{k+1})$
y
slope = $f(t_k, y_k)$
y_{k+2}
y_{k+1}
y_k
Δt
Δt
t
t_k
t_{k+1}
t_{k+2}

Euler's Method

at (t_0, y_0) we know slope $\frac{dy}{dt} = f(t_0, y_0) = m$



Write the equation of the line passing through (t_0, y_0) (use point-slope form)

$$y - y_0 = m(t - t_0)$$

$$y = y_0 + m(t - t_0)$$

$$y = y_0 + f(t_0, y_0)(t - t_0)$$

Evaluate the line at $t = t_1$

$$y(t_1) = y_1 = y_0 + f(t_0, y_0)(t_1 - t_0)$$

$$y_1 = y_0 + f(t_0, y_0)\Delta t$$

In general $\quad y_{k+1} = y_k + f(t_k, y_k)\Delta t$

Euler's Method in Table Form

k	t_k	y_k	$f(t_k, y_k)$	$y_{k+1} = y_k + f(t_k, y_k)\Delta t$
0	0	1		
1	.1			
2	.2			
3	.3			
4	.4			

Let use the example $\frac{dy}{dt} = -2ty^2 \quad y(0) = 1$