Name:_____ Discussion Section :_____

MA 226 Section B – Final Exam - b

Fall 2014

Question Number	Possible Points	Student Score
1	12	
2	12	
3	10	
4	10	
5	12	
6	12	
7	6	
8	6	
9	12	
10	8	
Total Points	100	

You must show your work to receive full credit

Discussion Sections:

B2: Wednesday 9-10

B3: Wednesday 2-3

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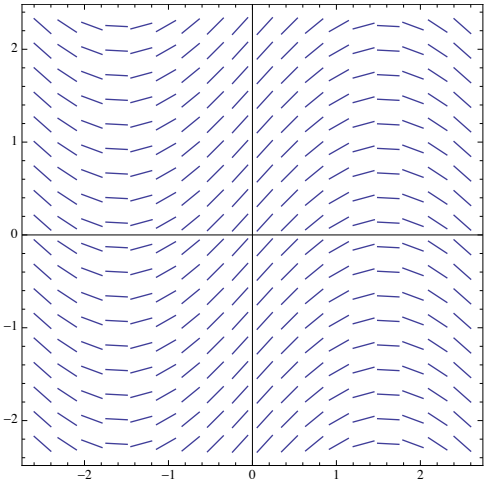
1. (3 pts each) Match the slope field with the corresponding equation

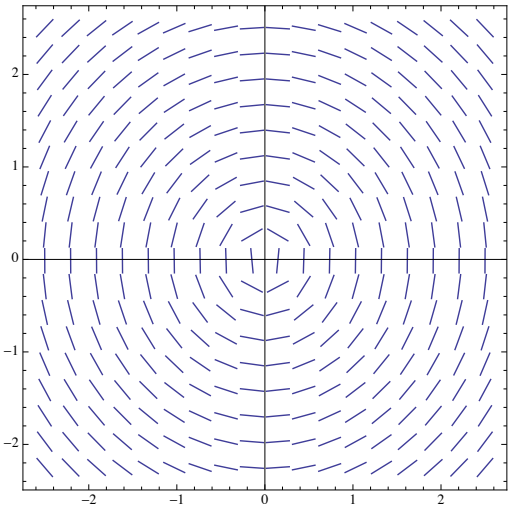
(i) $\frac{dy}{dt} = 2y(y-1)$ (ii) $\frac{dy}{dt} = -2y(y-1)$ (iii) $\frac{dy}{dt} = \cos(t)$ (iv) $\frac{dy}{dt} = \sin(t)$

(v) $\frac{dy}{dt} = \frac{y}{t}$ (vi) $\frac{dy}{dt} = -\frac{y}{t}$ (vii) $\frac{dy}{dt} = \frac{t}{y}$ (viii) $\frac{dy}{dt} = -\frac{t}{y}$

Slope Field A equation: _____ Reason:



Slope Field B equation: _____ Reason



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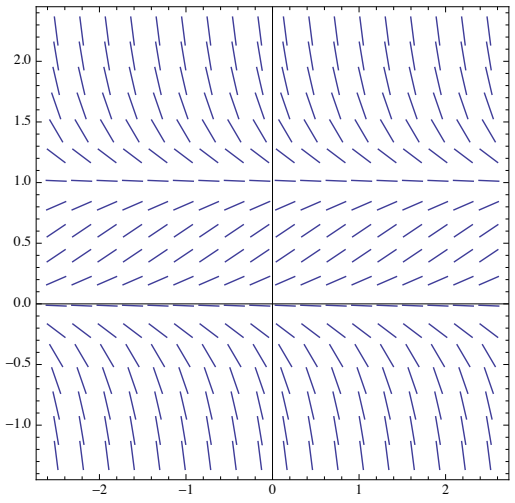
1.) (continued) Match the slope field with the equation

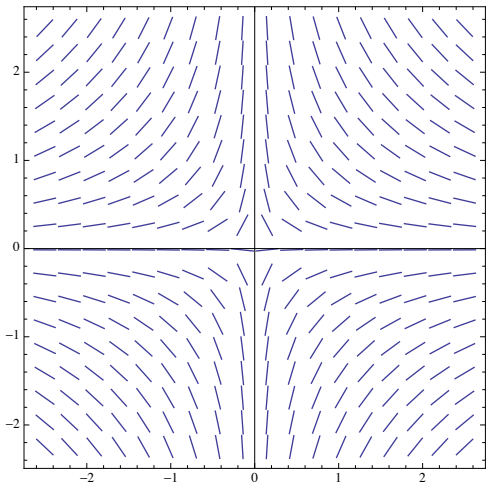
(i) $\frac{dy}{dt} = 2y(y-1)$ (ii) $\frac{dy}{dt} = -2y(y-1)$ (iii) $\frac{dy}{dt} = \cos(t)$ (iv) $\frac{dy}{dt} = \sin(t)$

(v) $\frac{dy}{dt} = \frac{y}{t}$ (vi) $\frac{dy}{dt} = -\frac{y}{t}$ (vii) $\frac{dy}{dt} = \frac{t}{y}$ (viii) $\frac{dy}{dt} = -\frac{t}{y}$

Slope Field C equation: _____ Reason



Slope Field D equation: _____ Reason



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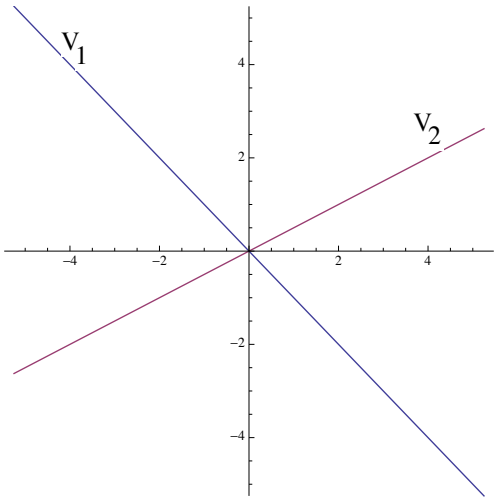
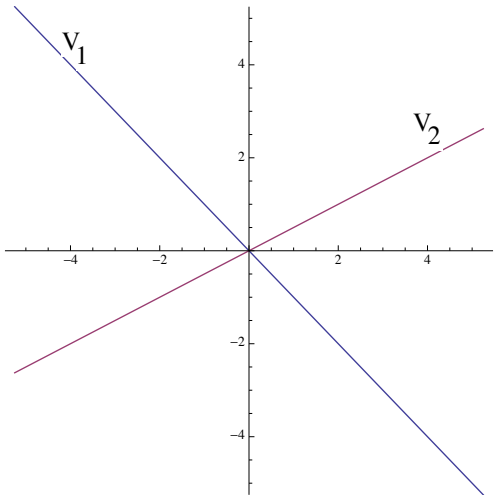
2. (3 pts each) Sketch phase portraits

a.) Given eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 3$ with corresponding eigenvectors $\vec{V_1} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\vec{V_2} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$



b.) Given eigenvalues $\lambda_1 = -3$ and $\lambda_2 = 0$ with corresponding eigenvectors $\vec{V_1} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\vec{V_2} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$



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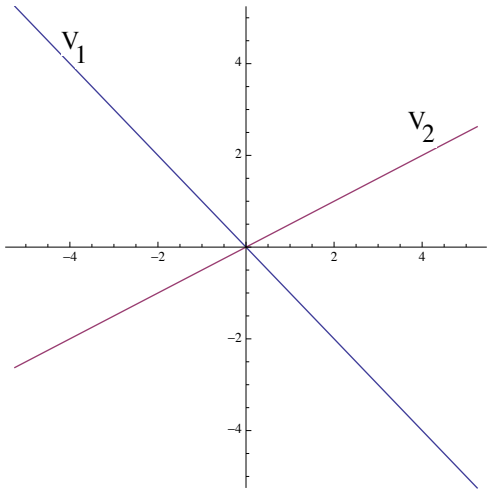
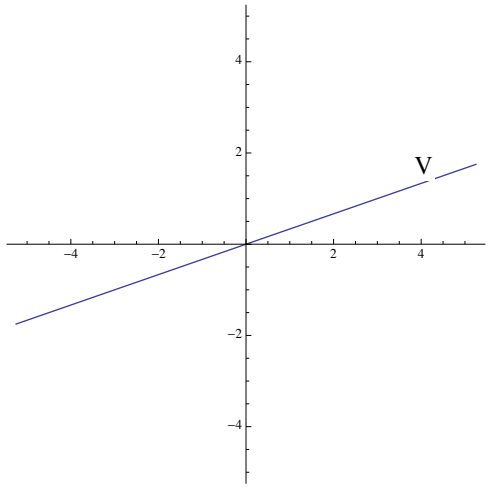
c.) Given eigenvalues $\lambda_1 = 3$ and $\lambda_2 = -1$ with corresponding eigenvectors $\vec{V}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\vec{V}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$



d.) Given the linear system $\frac{d\vec{Y}}{dt} = \begin{pmatrix} 3 & -18 \\ 2 & -9 \end{pmatrix}\vec{Y}$ with repeated eigenvalue $\lambda = -3$ and corresponding eigenvector $\vec{V} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. Draw the phase portrait for this system and show how you determined which orientation to use.



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3. Laplace Transforms

a.) (2 pts) Wrtie the Laplace transforms of : $u_3(t)(t-3)^2$

b.) (2 pts) Write the Laplace transform of : $e^{2t}\cos(5t)$

c.) (3 pts) Write the Laplace transform of: $u_4(t)e^{-2(t-4)}\cos(\pi(t-4))$

d.) (3 pts) Find the inverse Laplace transform of : $\dfrac{5e^{-3s}}{(s-2)^2+4}$

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4. (10 pts) Solve the initial value problem

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 9y = \delta_2(t) - 5\delta_3(t) \quad \text{and} \quad y(0) = 2\ ,\ \ y'(0) = 4$$

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5. (4 pts each) Short answer

a) Find all the equilibrium solutions of the system of differential equations

$$\frac{dx}{dt} = 4x - 2x^2 - 6xy$$
$$\frac{dy}{dt} = 12y - 4xy - 3y^2$$

b) Given the differential equation: $\frac{dy}{dt} = -3y(y+2)(y-2)$ draw the corresponding phase line.

Let $y_1(t)$ be a solution that satisfies the initial condition $y_1(0) = -3$. Evaluate the following limits:

Phase Line

$\lim_{t\to\infty} y_1(t) =$

$\lim_{t\to-\infty} y_1(t) =$

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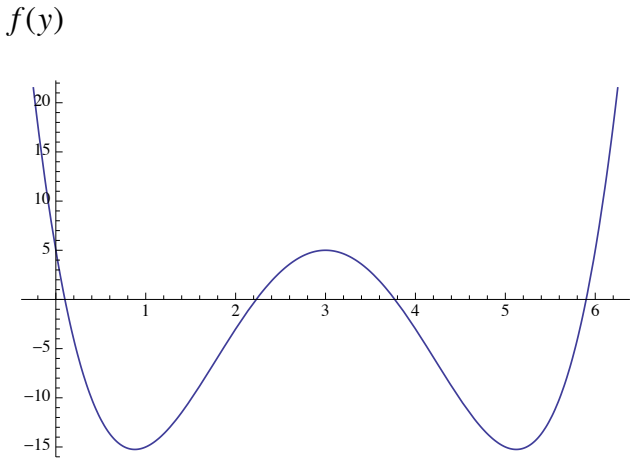
c) Given the one parameter family of differential equations $\frac{dy}{dt} = f(y) + \alpha$ where $f(y)$ is given by the graph below, identify the bifurcation value(s) and draw a bifurcation diagram. That is draw the phase lines at the bifurcation values as well as the phase lines for the values of α that are smaller and greater than the bifurcation values. The critical points on the graph of $f(y)$ occur at (1,-15), (3,5), and (5,-15)



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6. (12 pts) Solve the initial value problem: $\frac{d\vec{Y}}{dt} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}\vec{Y}$ with initial condition $\vec{Y}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

a.) Write the general solution

b.) Write the solution of the initial value problem

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6.) (continued)

c.) Make a sketch of the solution in the phase plane

d.) Make a sketch of $x(t)$ and $y(t)$ functions for the solution passing through the initial condition $\vec{Y}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Be sure to show the graph for both positive and negative values of t.

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7. (6 pts) Use Euler's Method with a step size of 0.2 to approximate the solution of the initial value problem $\frac{dy}{dt} = 2y - t$, $y(0) = 1$ over the time interval $0 \le t \le .6$.

Create a table that shows how you computed the approximate y for values of $t = .2,\ .4,\ \text{and}\ .6$.

Please use 4 decimal places of accuracy in our calculations.

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8. (6 pts) Find the general solution of : $\frac{dy}{dt} = 3y(y+2)$

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9. (12 pts) Consider a large vat containing sugar water that is to be made into soft drinks.

Let $y(t)$ represent the amount of sugar in the vat at time t.

- Initially the vat contains only pure water.

-The The vat contains 100 gals of liquid. Moreover, the amount flowing in is the same as the amount flowing out, so there are always 100 gallons in the vat.

- The vat is kept well mixed, so that the sugar concentration is uniform throughout the vat.

-Sugar water containing 3 tablespoons per gallon enters the vat through pipe A at a rate of 3 gallons per minute.

-Sugar water containing 4 tablespoons per gallon enters the vat through pipe B at rate of 2 gallon per minute.

-Sugar water leaves the vat through pipe C at a rate of 5 gallons per minute.

a.) (3 pts) : Write the initial value problem for $y(t)$ that describes the amount of sugar in the vat as a function of time.

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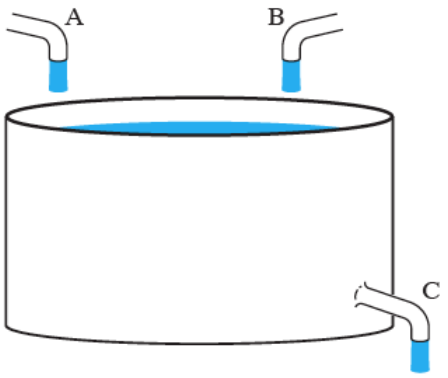

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b.) (5 pts) : Solve the initial value problem. You are free to use any technique to solve the problem.

c.) (2 pts): How much sugar will be in the tank after 10 minutes?

d.) (2 pts) : What is $\lim_{t\to\infty} y(t)$

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10. (8 pts) Given the one parameter family : $\frac{d\vec{Y}}{dt} = \begin{pmatrix} a & a^2 - a \\ 1 & a \end{pmatrix} \vec{Y}$ that depends on the parameter : a.

a.) (2 pts) Compute the trace and the determinant for the coefficient matrix and draw the corresponding curve in the trace determinant plane.

b.) (4 pts) List the different type of behaviors that are exhibited by the system as the parameter a is varied from $-\infty$ to $+\infty$.

c.)(2 pts) Identify the values of the parameter that correspond to bifurcation values.