EE 445/645: Physical Models in Remote Sensing (Spring 2026)

Chapter 01-Part 02: Radiative Quantities – Notes (See C1-P2 PPTs)

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1. Introduction to Photon Clouds

In remote sensing and radiative transfer theory, we conceptualize electromagnetic radiation as a **photon cloud** – a collection of discrete energy packets (photons) traveling through space. This particle-like description of light is essential for understanding how radiation interacts with matter and how sensors detect electromagnetic energy.

1.1 The Dual Nature of Light

Electromagnetic radiation exhibits both wave-like and particle-like properties. While classical electromagnetic theory treats radiation as waves, quantum mechanics reveals that light consists of discrete packets of energy called photons. In remote sensing applications, we often find it convenient to work with the photon description because:

• Photons carry quantized energy directly related to their frequency

• Many sensors (particularly LiDAR systems) actually count individual photons

• The photon picture naturally leads to radiometric quantities used in practice

1.2 Why 'Photon Cloud'?

The term 'photon cloud' emphasizes that we are dealing with a **statistical ensemble** of many photons rather than tracking individual particles. At any given location in space and time, there exists a distribution of photons traveling in various directions with different frequencies. This distribution changes as photons:

• Are emitted from sources (sun, Earth surface, artificial illumination)

• Propagate through the atmosphere

• Are scattered by atmospheric particles (molecules, aerosols, clouds)

• Are absorbed by atmospheric constituents

• Interact with Earth's surface (reflection, absorption, transmission)

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2. Photon Distribution Function

2.1 Counting Photons Precisely

To describe the photon cloud mathematically, we need a systematic way to count photons. However, we cannot simply count 'all photons' at a point – this would give us infinity or zero. Instead, we must specify:

Position: Where in space are we counting? (position vector r)

Time: When are we counting? (time t)

Direction: Which direction are photons traveling? (direction vector Ω)

Frequency: What is their electromagnetic frequency? (frequency ν)

2.2 The Distribution Function f

The **photon distribution function f(r, ν, Ω, t)** describes the number density of photons in a seven-dimensional space:

• Three spatial dimensions (x, y, z position)

• Two angular dimensions (direction of propagation)

• One frequency dimension

• One temporal dimension

Formally, f tells us the number of photons dN in an infinitesimal volume element around position r, traveling in direction Ω within solid angle $d\Omega$, with frequency in interval [ν, $\nu+d\nu$], at time t:

$dN = f(r, \nu, \Omega, t) \, d^3r \, d\Omega \, d\nu$

This function is the foundation for all radiative transfer calculations. It evolves in space and time according to the radiative transfer equation, which accounts for emission, absorption, and scattering processes.

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3. Energy Transport by Photons

3.1 From Photon Counts to Energy

While counting photons is conceptually clear, most radiometric instruments measure **energy**, not photon counts. The connection between the two is provided by Planck's relation:

$E\nu = h\nu$

where:

$E\nu$ is the energy of a single photon (in Joules)

h is Planck's constant ($h \approx 6.626 \times 10^{-34}$ J·s)

ν is the photon frequency (in Hz)

This equation reveals that higher-frequency (shorter wavelength) photons carry more energy. For example, a blue photon ($\lambda \approx 450$ nm, $\nu \approx 6.7 \times 10^{14}$ Hz) carries about 1.5 times more energy than a red photon ($\lambda \approx 650$ nm, $\nu \approx 4.6 \times 10^{14}$ Hz).

3.2 Energy Flow Requires Time

The photon distribution function f describes the photon density at an **instant** in time. However, to realize **energy transport** – the transfer of energy from one location to another – we must allow time to elapse. This is because energy transport is fundamentally a **rate** (energy per unit time).

Consider photons traveling through space at speed c. During a small time interval dt, photons move a distance c·dt. The photons that cross a surface element dS during this time interval are those within a cylinder of base dS and height c·dt·cos(θ), where θ is the angle between the photon direction and the surface normal.

3.3 The Importance of Geometry

Energy transport in three-dimensional space requires careful geometric consideration. Key geometric concepts include:

Solid angle: Measures the 'cone' of directions (unit: steradian, sr)

Surface normal: The perpendicular direction to a surface

Projection factor: The factor cos(θ) that accounts for oblique incidence

Hemispherical integration: Summing contributions from all directions in a hemisphere

4. Time and Space in Electromagnetic Radiation

4.1 The Speed of Light as a Fundamental Constant

The speed of electromagnetic radiation in vacuum is one of the most precisely known physical constants:

c = 299,792,458 m/s (exactly)

This value is not merely measured – it is **defined**. Since 1983, the meter has been defined as the distance light travels in 1/299,792,458 of a second. This means:

Time defines space: A meter is the distance light travels in 1/299,792,458 seconds

Space defines time: A second is the time light takes to travel 299,792,458 meters

4.2 Three-Dimensional Space

Our universe appears to have three spatial dimensions. This seemingly obvious fact has profound implications for radiative transfer:

• Radiation spreads in three dimensions, causing the inverse-square law for point sources

• A full sphere subtends 4π steradians of solid angle

• A hemisphere (relevant for flux calculations) subtends 2π steradians

• Geometric dilution of radiation follows specific mathematical relationships

The three-dimensionality of space also necessitates the use of **spherical coordinates** (or equivalent angular parameterizations) to describe radiation directions. This leads to the concepts of polar angle θ and azimuthal angle φ that are fundamental to radiative transfer theory.

5. The Nature of EMR Propagation

5.1 Self-Propagating Waves

Electromagnetic radiation has a remarkable property: it is **self-propagating**. Unlike mechanical waves (sound, water waves) that require a medium, EMR can travel through perfect vacuum. This is possible due to a 'leapfrog' mechanism between electric and magnetic fields:

Step 1: A changing (time-varying) magnetic field creates an electric field in the surrounding space

Step 2: That newly created electric field is also changing, which generates a magnetic field

This mutual generation allows the electromagnetic wave to propagate indefinitely. Each field creates the other slightly ahead in space, enabling the wave to 'pull itself forward' through empty space.

5.2 Field Waves vs. Mechanical Waves

Most waves we encounter in everyday life are **mechanical waves**:

• Ocean waves propagate through water molecules pushing against each other

• Sound waves propagate through air molecules vibrating and colliding

• Seismic waves propagate through rock and soil particles

EMR is fundamentally different – it is a **field wave**. Fields (electromagnetic, gravitational) exist throughout space, even in vacuum. An electromagnetic wave is not 'stuff' moving through space; rather, it is a **disturbance in the electromagnetic field** that propagates through space at speed c.

5.3 Maxwell's Equations

The behavior of electromagnetic fields is governed by Maxwell's equations, which in vacuum can be written as:

• Gauss's law for electricity: $\nabla \cdot E = 0$

• Gauss's law for magnetism: $\nabla \cdot B = 0$

• Faraday's law: $\nabla \times E = -\partial B / \partial t$

• Ampère-Maxwell law: $\nabla \times B = (1/c^2)\partial E / \partial t$

These equations predict wave solutions that propagate at exactly the speed c, confirming that light is an electromagnetic phenomenon.

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6. Geometric Considerations in 3D Space

6.1 Solid Angle and Direction

In three-dimensional space, we need to specify directions. A direction can be thought of as a point on the unit sphere. The **solid angle** $d\Omega$ represents a small patch on this sphere, measured in steradians (sr).

Using spherical coordinates with polar angle θ (measured from the vertical) and azimuthal angle φ (measured from a reference direction), the solid angle element is:

$d\Omega = sin(\theta)\ d\theta\ d\varphi$

Alternatively, using the directional cosine $\mu = cos(\theta)$:

$d\Omega = d\mu \, d\varphi$

6.2 Surface Projection Factor

When radiation traveling in direction Ω crosses a surface with normal vector n, the **effective cross-sectional area** depends on the angle between Ω and n. This is described by the projection factor:

$|n \cdot \Omega| = |cos(\theta)| = |\mu|$

This factor explains why:

• Radiation at normal incidence ($\theta = 0°$) has maximum effect ($\cos(0°) = 1$)

• Radiation at grazing incidence ($\theta \rightarrow 90°$) has minimal effect ($\cos(90°) = 0$)

• Solar heating varies with latitude and season (sun angle effect)

6.3 Hemispherical Integration

Many radiometric quantities require integrating over a hemisphere of directions. For a horizontal surface, the upward hemisphere is described by:

• θ ranging from 0° (zenith) to 90° (horizon)

• φ ranging from 0° to 360° (full rotation)

• Equivalently: μ from 0 to 1, φ from 0 to 2π

The integral over the upward hemisphere of any quantity $F(\theta,\varphi)$ is:

$\iint\_hemisphere \, F \, d\Omega = \int_0^{2p} d\varphi \int_0^1 F(\mu,\varphi) \, d\mu$

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7. Specific Intensity (Radiance)

7.1 Definition and Physical Meaning

The **specific intensity** (also called **radiance**), denoted I(r, v, Ω, t), is the fundamental radiometric quantity in remote sensing. It represents the **energy flow per unit time, per unit area, per unit solid angle, per unit frequency interval**.

Mathematically, the energy dE flowing through surface element dS, in direction Ω within solid angle dΩ, at frequency v within interval dv, during time interval dt, is:

$dE = I(r, v, \Omega, t) \, |cos(\theta)| \, dS \, d\Omega \, dv \, dt$

where θ is the angle between Ω and the surface normal.

7.2 Units and Dimensions

The SI units of specific intensity are:

$$[I] = W\,m^{-2}\,sr^{-1}\,Hz^{-1}$$

Breaking this down:

W (watts): Energy per unit time

m^{-2}: Per unit area

sr^{-1}: Per unit solid angle (per steradian)

Hz^{-1}: Per unit frequency interval

Sometimes specific intensity is expressed per unit wavelength instead of per unit frequency, with units $W\,m^{-2}\,sr^{-1}\,nm^{-1}$ or $W\,m^{-2}\,sr^{-1}\,\mu m^{-1}$.

7.3 Key Properties of Specific Intensity

Specific intensity has several important properties:

Directional: I depends on the direction Ω. Different directions can have vastly different intensities

Frequency-dependent: The spectrum $I(\nu)$ contains information about the source and intervening medium

Conserved along rays: In vacuum, I remains constant along a ray (no geometric dilution)

Measurable: Instruments with narrow field-of-view measure specific intensity directly

7.4 Specific Intensity on Tilted Surfaces

Consider a surface tilted at an angle θ relative to the direction of incoming radiation. The energy flux through the surface depends on the **projected area** perpendicular to the radiation direction. This gives rise to Lambert's cosine law:

Flux $\propto I\,cos(\theta)$

This explains why a surface receives maximum solar heating when the sun is directly overhead ($\theta = 0°$) and minimal heating at sunrise/sunset ($\theta \to 90°$).

8. Flux Density and Irradiance

8.1 From Intensity to Flux

While specific intensity describes radiation in a specific direction, we often want to know the **total radiation** crossing a surface from all directions. This leads to the concept of **flux density**.

8.2 Net Flux Density F(r, ν, t)

The **net flux density** (or net irradiance) at position r, frequency ν, and time t is the rate of energy flow across a unit area, per unit frequency:

$$F(r, ν, t) = \int_{4π} I(r, ν, Ω, t) (n·Ω) \, dΩ$$

The integral is over the full sphere ($4π$ steradians), and the dot product $n·Ω$ accounts for the projection factor. Positive contributions come from directions where radiation flows in the +n direction; negative contributions from the -n direction.

Units: W m^{-2} Hz^{-1}

8.3 Upward Flux Density F⁺

For a horizontal surface, it is often useful to separate upward and downward components. The **upward flux density** is:

$$F^+(r, ν, t) = \int_{n·Ω>0} I(r, ν, Ω, t) \, |n·Ω| \, dΩ$$

This can be written in spherical coordinates as:

$$F^+(r, ν, t) = \int_0^{2π} dφ \int_0^1 I(r, ν, μ, φ, t) \, μ \, dμ$$

where $μ = \cos(θ)$ is the directional cosine. The limits $μ \in [0,1]$ correspond to the upward hemisphere ($θ$ from 0° to 90°).

Physical interpretation: F⁺ represents the total energy (per unit time, area, and frequency) flowing upward through a horizontal surface, summing contributions from all upward directions weighted by their projection factor $μ$.

8.4 Downward Flux Density F⁻

Similarly, the **downward flux density** is:

$$F^-(r, ν, t) = \int_{n·Ω<0} I(r, ν, Ω, t) \, |n·Ω| \, dΩ$$

In spherical coordinates (with $μ$ now representing the magnitude of the cosine):

$$F^-(r, ν, t) = \int_0^{2π} dφ \int_{(-1)}^0 I(r, ν, -μ, φ, t) \, |μ| \, dμ$$

The net flux density is then:

$F(r, v, t) = F^+(r, v, t) + F^-(r, v, t)$

8.5 Applications in Remote Sensing

Flux density measurements are central to many remote sensing applications:

Solar irradiance: F^- at the top of atmosphere gives the solar constant (~1361 W/m²)

Surface radiation budget: Net flux $F = F^+ - F^-$ determines heating/cooling

Albedo: Ratio F^+/F^- (for solar radiation) characterizes surface reflectance

Plant photosynthesis: PAR (photosynthetically active radiation) is F^- integrated over 400-700 nm

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9. Radiant Power

9.1 Total Hemispherical Irradiance

The **total hemispherical irradiance** (sometimes called total irradiance or broadband irradiance) is obtained by integrating the monochromatic flux density over all frequencies:

$$E(r, t) = \int_0^\infty F(r, v, t) \, dv$$

Units: W m⁻² (watts per square meter)

This quantity represents the total radiative energy (across all wavelengths/frequencies) flowing through a unit area per unit time. It is the quantity measured by broadband radiometers such as pyranometers (for solar radiation) and pyrgeometers (for terrestrial longwave radiation).

9.2 Radiant Power (Total Flux)

The **radiant power** or **total flux** through a surface of area A is:

$$\Phi(t) = \iint_A E(r, t) \, dA$$

Units: W (watts)

If the irradiance is uniform over the surface, this simplifies to:

$$\Phi = E \times A$$

9.3 Energy Balance Applications

Radiant power is the appropriate quantity for energy balance calculations:

Solar power incident on Earth: $\Phi_{solar} \approx 1361$ W/m² $\times \pi R_{Earth}^2 \approx 1.74 \times 10^{17}$ W

Power absorbed by a leaf: $\Phi_{absorbed} = (1-albedo) \times E_{incident} \times A_{leaf}$

Sensor power collection: Determines signal-to-noise ratio and integration time

Greenhouse effect: Balance between solar input and terrestrial output

9.4 Relationship Between Radiometric Quantities

To summarize the hierarchy of radiometric quantities:

Specific intensity I(r,ν,Ω,t): Most detailed – directional and spectral

Flux density F(r,ν,t): Integrate I over all directions

Irradiance E(r,t): Integrate F over all frequencies

Radiant power Φ(t): Integrate E over surface area

Each successive integration reduces the dimensionality but loses information about direction, spectrum, or spatial variation.

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10. Summary and Applications

10.1 Key Concepts Review

This chapter has introduced the fundamental radiometric quantities used in remote sensing:

Photon clouds: Statistical ensembles of photons described by distribution functions

Energy transport: Requires time evolution and geometric considerations

EMR propagation: Self-sustaining electromagnetic waves in 3D space

Specific intensity: Fundamental directional and spectral quantity

Flux density: Integration over directions

Irradiance and power: Further integration over frequency and area

10.2 Remote Sensing Applications

Understanding these radiative quantities is essential for:

Atmospheric correction: Accounting for scattering and absorption in satellite imagery

Surface energy budget: Climate modeling and weather prediction

Vegetation studies: Photosynthesis, evapotranspiration, biomass estimation

Ocean color: Phytoplankton, water quality, ocean productivity

Sensor design: Optimizing field-of-view, spectral bands, and sensitivity

10.3 Looking Ahead

The radiative quantities introduced here form the foundation for the radiative transfer equation (RTE), which describes how radiation propagates through absorbing, emitting, and scattering media. Future topics will include:

• Source functions and emission processes

• Absorption and extinction coefficients

• Scattering phase functions

• Solutions to the RTE in various geometries

• Applications to specific remote sensing problems

10.4 Further Reading

For deeper understanding of radiative transfer theory and remote sensing applications, consult:

• Chandrasekhar, S. (1960). Radiative Transfer. Dover Publications.

Appendix: Mathematical Notation Summary

Symbols and Notation

c = speed of light in vacuum = 299,792,458 m/s

h = Planck's constant $\approx 6.626 \times 10^{-34}$ J·s

v = frequency (Hz)

λ = wavelength (m)

r = position vector

t = time

Ω = direction unit vector

θ = polar angle (measured from zenith)

φ = azimuthal angle

$\mu = \cos(\theta)$, directional cosine

$d\Omega$ = solid angle element (sr)

n = surface normal unit vector

$I(r,v,\Omega,t)$ = specific intensity (W m^{-2} sr^{-1} Hz^{-1})

$F(r,v,t)$ = net flux density (W m^{-2} Hz^{-1})

$F^+(r,v,t)$ = upward flux density (W m⁻² Hz⁻¹)

$F^-(r,v,t)$ = downward flux density (W m⁻² Hz⁻¹)

$E(r,t)$ = irradiance (W m⁻²)

$\Phi(t)$ = radiant power (W)

Key Equations

Photon energy: $Ev = hv$

Solid angle element: $d\Omega = sin(\theta)d\theta d\varphi = d\mu d\varphi$

Upward flux density: $F^+ = \int_0^{2P} d\varphi \int_0^1 d\mu\, I(\mu,\varphi)\mu$

Net flux density: $F = F^+ - F^-$

Total irradiance: $E = \int_0^\infty F(v)dv$

Radiant power: $\Phi = \iint_A E\, dA$