EE 445/645: Physical Models in Remote Sensing (Spring 2026)

Chapter 01-Part 01: Basic Definitions – Notes (See C1-P1 PPTs)

Prof. Ranga B. Myneni, Boston University

ranga.myneni@gmail.com

Table of Contents

---

1. Electromagnetic Radiation: The Dual Nature

1.1 Wave Description

Definition: Electromagnetic radiation (EMR) consists of waves of the electromagnetic field that propagate through space carrying both momentum and electromagnetic radiant energy.

Key Characteristics of EM Waves:

1. **Self-Propagating**: EM waves consist of oscillating electric (E) and magnetic (B) fields that are perpendicular to each other and to the direction of propagation
2. **No Medium Required**: Unlike sound waves or water waves, EM waves can travel through vacuum
3. **Speed**: All electromagnetic waves travel at the speed of light (c) in vacuum:
    - c = 299,792,458 m/s (exact by definition)
    - Often approximated as $c \approx 3.0 \times 10^8$ m/s
4. **Wave Parameters**:
    - **Wavelength (λ)**: Distance between successive crests (units: meters, µm, nm)
    - **Frequency (ν)**: Number of oscillations per second (units: Hertz, Hz = s^{-1})
    - **Relationship**: $\lambda = c/\nu$ or $c = \lambda\nu$

The Electromagnetic Spectrum: EMR encompasses a continuous range of wavelengths and frequencies, traditionally categorized as:

- **Radio waves**: λ > 1 mm (ν < 300 GHz)
- **Microwaves**: 1 mm - 1 m (300 MHz - 300 GHz)

- **Infrared (IR)**: 700 nm - 1 mm (300 GHz - 430 THz)
- **Visible light**: 380 - 700 nm (430 - 790 THz)
- **Ultraviolet (UV)**: 10 - 380 nm (790 THz - 30 PHz)
- **X-rays**: 0.01 - 10 nm (30 PHz - 30 EHz)
- **Gamma rays**: $\lambda < 0.01$ nm ($\nu > 30$ EHz)

Wave Properties Important for Remote Sensing:

- **Diffraction**: Bending around obstacles (significant when obstacle size ~ λ)
- **Interference**: Constructive/destructive combination of waves
- **Polarization**: Orientation of the electric field oscillation
- **Reflection and Refraction**: Change in direction at interfaces
- **Dispersion**: Wavelength-dependent propagation speed in media

Historical Context:

- James Clerk Maxwell (1860s): Unified electricity and magnetism, predicted EM waves
- Heinrich Hertz (1887): Experimentally confirmed EM wave existence
- 19th century consensus: Light behaves as a wave phenomenon

1.2 Particle Description

Historical Development:

Max Planck (1900):

- Studied the spectrum of radiation emitted by heated objects (blackbody radiation)
- Classical wave theory predicted infinite energy emission at short wavelengths ("ultraviolet catastrophe")
- Planck's revolutionary solution: Energy is emitted/absorbed only in discrete packets called **quanta**
- Quantum hypothesis: $E = h\nu$, where h is Planck's constant ($h = 6.626 \times 10^{-34}$ J·s)

Albert Einstein (1905):

- Explained the photoelectric effect (electrons ejected from metals by light)
- Wave theory couldn't explain why:
    - Only light above a certain frequency could eject electrons (regardless of intensity)
    - Higher frequency light ejected electrons with more kinetic energy
    - Electron ejection was instantaneous
- Einstein proposed: Light consists of real particles (quanta) with energy $E = h\nu$
- Each quantum carries a discrete packet of energy
- Later named **photons** (by Gilbert Lewis, 1926)

Photon Properties:

1. **Energy**: $E = h\nu = hc/\lambda$

    - Higher frequency (shorter wavelength) = higher energy
    - Example: Blue photon has more energy than red photon
2. **Momentum**: $p = E/c = h/\lambda$
    - Despite zero rest mass, photons carry momentum
    - Explains radiation pressure
3. **Zero Rest Mass**: Photons always travel at speed c (in vacuum)
4. **Integer Spin**: Photons are bosons (spin = 1)
5. **Electrically Neutral**: No electric charge

Significance for Remote Sensing:

- Photon energy determines interaction with matter
- Detection involves counting individual photons (especially at low light levels)
- Quantum efficiency of detectors measures photon-to-electron conversion
- Shot noise arises from discrete nature of photon arrivals

1.3 Wave-Particle Duality

Central Concept: Quantum entities (photons, electrons, etc.) exhibit both wave and particle properties, but not simultaneously. Which aspect manifests depends on the experimental setup.

Historical Evolution:

19th Century Understanding:

- Light: Behaves as waves (interference, diffraction proven)
- Matter: Consists of particles (atoms, molecules)
- Clear distinction between waves and particles

Early 20th Century Discoveries:

1. **Light Shows Particle Nature**:
    - Photoelectric effect (1905): Light behaves as photons
    - Compton scattering (1923): X-rays scatter like particles off electrons
2. **Matter Shows Wave Nature**:
    - Louis de Broglie (1924): Proposed matter waves, $\lambda = h/p$
    - Davisson-Germer (1927): Electron diffraction confirmed
    - Wave-like behavior of electrons, atoms, even molecules demonstrated

Resolution of the Paradox:

The confusion arose from trying to force quantum objects into classical categories. The resolution:

1. **Complementarity Principle** (Niels Bohr):
    - Wave and particle descriptions are complementary
    - Both needed for complete description
    - Which aspect appears depends on measurement type

2. **Experimental Context Determines Manifestation**:
    - **Wave experiments** (diffraction, interference): Wavelike behavior observed
    - **Particle experiments** (detection, counting): Particle behavior observed
    - Cannot simultaneously measure both aspects precisely

Examples Illustrating Duality:

Double-Slit Experiment:

- Single photons fired one at a time
- Each arrives as a particle (discrete detection)
- Pattern after many photons shows wave interference
- Paradox: Each photon "interferes with itself"

Photon Detection:

- Detection event: Particle-like (discrete, localized)
- Probability of detection location: Governed by wave equation

Remote Sensing Implications:

1. **Spectral Properties**: Wave description
    - Wavelength/frequency determines color
    - Dispersion, polarization, interference
2. **Detection**: Particle description
    - Discrete photon arrivals at detector
    - Photon counting, quantum efficiency
    - Statistical fluctuations (shot noise)
3. **Energy Transfer**: Particle description
    - Photoelectric effect in detectors
    - Discrete energy packets absorbed/emitted
4. **Propagation**: Wave description
    - Diffraction limits resolution
    - Antenna patterns
    - Scattering phenomena

Philosophical Implications:

- Classical concepts inadequate at quantum scale
- Reality not observer-independent at quantum level
- Measurement fundamentally affects system
- Uncertainty principle: Cannot simultaneously know position and momentum precisely

Practical Approach: In remote sensing, we use whichever description (wave or particle) is most convenient for the phenomenon being studied:

- Optical systems design: Wave optics
- Detector physics: Photon (particle) model
- Radiative transfer: Often hybrid approach

2. The Electromagnetic Spectrum in Remote Sensing

The electromagnetic spectrum spans an enormous range of wavelengths and frequencies. Different portions of the spectrum have unique properties and applications in remote sensing.

2.1 Spectrum Organization

Complete Range: From radio waves (kilometers) to gamma rays (picometers) - spanning ~20 orders of magnitude

Energy Relationships:

- Energy per photon: $E = h\nu = hc/\lambda$
- Higher frequency (shorter wavelength) = higher energy per photon
- Example energies:
  - Radio (1 m): $E \approx 2 \times 10^{-25}$ J ≈ 1.2 μeV
  - Visible (500 nm): $E \approx 4 \times 10^{-19}$ J ≈ 2.5 eV
  - X-ray (1 nm): $E \approx 2 \times 10^{-16}$ J ≈ 1.2 keV

2.2 Remote Sensing Bands

Radio and Microwave (1 mm - 1 m):

- **Advantages**: Penetrates clouds, vegetation, soil
- **Applications**:
  - Radar imaging (SAR)
  - Soil moisture measurement
  - Ice thickness
  - All-weather observation
- **Examples**: L-band (15-30 cm), C-band (4-8 cm), X-band (2.5-4 cm)

Thermal Infrared (3-15 μm):

- **Principle**: Detects thermal emission from surfaces
- **Applications**:
  - Land/sea surface temperature
  - Fire detection
  - Volcanic monitoring
  - Urban heat islands
- **Key bands**:
  - MWIR (3-5 μm): Less atmospheric absorption
  - LWIR (8-14 μm): Atmospheric window

Near Infrared (0.7-1.4 μm):

- **Key feature**: High vegetation reflectance
- **Applications**:
  - Vegetation health (NDVI)

    - Biomass estimation
    - Water content
    - Crop monitoring
- **Physical basis**: Leaf cellular structure scatters NIR strongly

Visible (0.38-0.7 µm):

- **Sub-bands**:
    - Blue (0.45-0.49 µm): Water penetration, atmospheric scattering
    - Green (0.50-0.58 µm): Vegetation reflectance peak
    - Red (0.63-0.69 µm): Chlorophyll absorption
- **Applications**: True color imagery, ocean color, land cover classification

Ultraviolet (0.01-0.38 µm):

- **Challenges**: Strong atmospheric absorption (ozone)
- **Applications**:
    - Ozone monitoring
    - Solar radiation
    - Fluorescence detection
- **Limited use**: Mostly from space platforms

2.3 Atmospheric Windows

Not all wavelengths can be used for remote sensing due to atmospheric absorption:

Major Windows:

1. **Visible/NIR** (0.3-1.3 µm): Excellent transmission
2. **SWIR** (1.5-1.8 µm, 2.0-2.5 µm): Good transmission between water absorption bands
3. **Thermal** (3-5 µm, 8-14 µm): Used for thermal remote sensing
4. **Microwave** (1 mm - 1 m): Excellent transmission, all-weather

Major Absorbers:

- **Water vapor (H_2O)**: Strong absorption at 1.4, 1.9, 2.7, 6.3 µm
- **Carbon dioxide (CO_2)**: 2.7, 4.3, 15 µm
- **Ozone (O_3)**: UV region (Hartley-Huggins bands)
- **Oxygen (O_2)**: UV, visible (weak), 0.76 µm (O_2-A band)

---

3. Boltzmann's Law (Stefan-Boltzmann Law)

3.1 Statement of the Law

Boltzmann's Law: The total radiant power emitted per unit area by a blackbody is proportional to the fourth power of its absolute temperature.

Mathematical Expression:

$M = \sigma T^4$

Where:

- **M**: Total radiant exitance (power per unit area) [W/m²]
- **σ**: Stefan-Boltzmann constant = 5.67×10^{-8} W/(m²·K⁴)
- **T**: Absolute temperature [Kelvin]

3.2 Physical Interpretation

What It Means:

1. **Temperature Dependence**: A small temperature increase causes dramatic increase in total emission
    - Double temperature → 16× more power ($2^4 = 16$)
    - Example: Object at 600 K emits 16× more than object at 300 K
2. **Total Emission**: Integrates over all wavelengths and all directions (hemisphere)
3. **Blackbody Assumption**: Maximum possible emission for given temperature
    - Real objects emit less: $M_{real} = \varepsilon \sigma T^4$ where ε is emissivity ($0 < \varepsilon \leq 1$)

3.3 Derivation from Planck's Law

Planck's Law (spectral radiance):

$B_\lambda(T) = (2hc^2/\lambda^5) \times 1/(e^{(hc/\lambda kT)} - 1)$

Integration Process: To obtain Boltzmann's law, we integrate Planck's function over:

1. All wavelengths: 0 to ∞
2. All directions over a hemisphere

Steps:

$M = \int_0^\infty \int_{hemisphere} B_\lambda(T) \cos(\theta)\, d\Omega\, d\lambda$

The cosine factor accounts for Lambert's law (projection effect).

Result (after mathematical manipulation):

$M = \sigma T^4$

Where σ emerges from the integration as:

$\sigma = (2\pi^5 k^4)/(15h^3c^2)$

This fundamental relationship shows that Boltzmann's law is not independent but rather a consequence of Planck's more fundamental quantum description.

3.4 Remote Sensing Applications

Thermal Remote Sensing:

1. **Temperature Retrieval**:
   - Measure radiance → Infer temperature
   - Requires knowledge of emissivity
   - $T = (M/\varepsilon\sigma)^{(1/4)}$
2. **Energy Balance**:
   - Earth's energy budget
   - Surface-atmosphere energy exchange
   - Climate modeling
3. **Target Detection**:
   - Hot targets (fires, volcanoes) emit much more radiation
   - Temperature contrast enables detection
4. **Time-of-Day Effects**:
   - Surface temperature changes throughout day
   - Thermal inertia affects heating/cooling rates
   - Diurnal temperature range contains information about materials

Examples:

- **Earth** ($T \approx 288$ K): $M \approx 390$ W/m²
- **Sun** ($T \approx 5800$ K): $M \approx 6.3 \times 10^7$ W/m²
- **Fire** ($T \approx 1000$ K): $M \approx 5.7 \times 10^4$ W/m² (146× Earth's emission)

---

4. Wien's Displacement Law

4.1 Statement of the Law

Wien's Law: The wavelength at which a blackbody emits the maximum spectral radiance is inversely proportional to its absolute temperature.

Mathematical Expression:

$\lambda_{max} = b/T$

Where:

- **λ_{max}**: Wavelength of peak emission [meters, µm, or nm]
- **b**: Wien's displacement constant = 2.898×10^{-3} m·K (or 2898 µm·K)
- **T**: Absolute temperature [Kelvin]

4.2 Physical Meaning

Key Insights:

1. **Inverse Relationship**: Higher temperature → Shorter peak wavelength
   - Hot objects emit primarily short wavelengths (blue-shifted)
   - Cool objects emit primarily long wavelengths (red-shifted)
2. **Color of Hot Objects**:
   - 800 K: Peak at 3.6 μm (infrared, appears red)
   - 5800 K: Peak at 0.5 μm (green, sun's color)
   - 10,000 K: Peak at 0.29 μm (UV, appears blue-white)
3. **Blackbody Curve Shifts**: Entire emission spectrum shifts as temperature changes

4.3 Derivation from Planck's Law

Planck's Law (wavelength form):

$$B_\lambda(T) = (2hc^2/\lambda^5) \times 1/(e^{(hc/\lambda kT)} - 1)$$

Finding the Maximum: To find λ_{max}, take derivative and set to zero:

$$dB_\lambda/d\lambda = 0$$

This yields a transcendental equation:

$$(1 - e^{(-hc/\lambda kT)}) = (hc)/(5\lambda kT)$$

Solution: Numerical solution gives:

$$\lambda_{max} = (hc)/(4.965kT) = b/T$$

Where $b = 2.898 \times 10^{-3}$ m·K

Important Note: Different definition in frequency domain!

- Wavelength domain: $\lambda_{max} = b/T$
- Frequency domain: $\nu_{max} \neq c/\lambda_{max}$ (due to Jacobian transformation)
- Frequency domain: $\nu_{max} = (5.879 \times 10^{10}$ Hz/K$) \times T$

This is an interesting consequence of the wavelength-frequency transformation and illustrates why the choice of variable (λ vs ν) matters in spectral descriptions.

4.4 Remote Sensing Applications

Understanding Source Emissions:

1. **Solar Radiation** ($T_{sun} \approx 5800$ K):
   - $\lambda_{max} = 2898/5800 \approx 0.5$ μm (green, visible spectrum)
   - Explains why solar radiation peaks in visible range
   - Remote sensing in visible/NIR uses reflected sunlight

2. **Earth Emission** ($T_{earth} \approx 288$ K):
    - $\lambda_{max} = 2898/288 \approx 10$ μm (thermal infrared)
    - Earth's emission peaks in TIR window
    - Thermal remote sensing operates here
3. **Spectral Separation**:
    - Solar and terrestrial emission barely overlap
    - Wavelengths < 3 μm: Primarily reflected solar
    - Wavelengths > 4 μm: Primarily thermal emission
    - 3-4 μm: Transition region (both contribute)

Practical Examples:

- **Fire Detection** ($T \approx 800\text{-}1200$ K):
    - $\lambda_{max} \approx 2.4\text{-}3.6$ μm (MWIR)
    - Strong signal in 3-5 μm window
    - Background ($T \approx 300$ K) peaks at 10 μm
    - High contrast enables detection
- **Cloud-Top Temperature**:
    - Colder clouds → Longer wavelength peak
    - High clouds ($T \approx 230$ K): $\lambda_{max} \approx 12.6$ μm
    - Low clouds ($T \approx 280$ K): $\lambda_{max} \approx 10.3$ μm
- **Sensor Design**:
    - Detector spectral response matched to expected emission
    - Thermal cameras operate at object's emission peak
    - Optimizes signal-to-noise ratio

---

5. Planck's Law

5.1 The Fundamental Law of Blackbody Radiation

Planck's Law: Describes the spectral distribution of electromagnetic radiation emitted by a blackbody in thermal equilibrium at a given temperature.

Historical Significance:

- Solved the "ultraviolet catastrophe" (classical theory predicted infinite short-wavelength emission)
- Introduced quantum hypothesis (1900)
- Foundation of quantum mechanics
- One of the most important equations in physics

5.2 Mathematical Formulations

Wavelength Form (spectral radiance per unit wavelength):

$$B_\lambda(\lambda, T) = (2hc^2/\lambda^5) \times 1/(e^{(hc/\lambda kT)} - 1)$$

Frequency Form (spectral radiance per unit frequency):

$B\_\nu(\nu, T) = (2h\nu^3/c^2) \times 1/(e^{(h\nu/kT)} - 1)$

Where:

- **B_λ(λ,T)**: Spectral radiance [W/(m²·sr·m)] or [W/(m²·sr·μm)]
- **B_ν(ν,T)**: Spectral radiance [W/(m²·sr·Hz)]
- **h**: Planck's constant = 6.626×10^{-34} J·s
- **c**: Speed of light = 2.998×10^{8} m/s
- **k**: Boltzmann constant = 1.381×10^{-23} J/K
- **T**: Absolute temperature [K]
- **λ**: Wavelength [m]
- **ν**: Frequency [Hz]

5.3 Physical Components

Structure of Planck's Law:

1. **Prefactor**: $2hc^2/\lambda^5$ or $2h\nu^3/c^2$
   - Contains fundamental constants
   - Determines overall scale
   - Different forms for λ vs ν
2. **Exponential Term**: e^(hc/λkT) or e^(hν/kT)
   - Contains energy ratio: E_photon/(k_B T)
   - Compares photon energy to thermal energy
   - Governs temperature dependence
3. **Distribution Function**: 1/(e^x - 1)
   - Bose-Einstein distribution (photons are bosons)
   - Average occupation number of quantum states
   - No upper limit on photon number per state

5.4 Limiting Cases

High Temperature or Long Wavelength (hc/λkT << 1):

- Exponential ≈ 1 + hc/λkT
- Planck's law reduces to **Rayleigh-Jeans Law**:
- $B\_\lambda \approx (2ckT)/\lambda^4$
- Classical limit (no quantum effects)
- Valid for radio wavelengths

Low Temperature or Short Wavelength (hc/λkT >> 1):

- Exponential dominates
- Planck's law reduces to **Wien's Approximation**:
- $B\_\lambda \approx (2hc^2/\lambda^5)\, e^{(-hc/\lambda kT)}$
- Quantum effects dominant
- Exponential cutoff at high energies

5.5 Key Properties

Temperature Dependence:

1. **Amplitude Increases**: Higher T → More emission at all wavelengths
2. **Peak Shifts Blue**: Higher T → Shorter λ_{max} (Wien's law)
3. **Integrated Power**: Total emission $\propto T^4$ (Boltzmann's law)
4. **Shape Changes**: Curves for different T do not intersect

Spectral Distribution:

- Continuous spectrum (all wavelengths present)
- Single peak (location given by Wien's law)
- Long wavelength tail (Rayleigh-Jeans)
- Short wavelength cutoff (exponential)

5.6 Relationship to Other Laws

Planck's Law is Fundamental:

1. **Integration over all wavelengths → Boltzmann's Law**:
2. $M = \int_0^\infty \pi B_\lambda \, d\lambda = \sigma T^4$
3. **Maximum location → Wien's Displacement Law**:
4. $dB_\lambda/d\lambda = 0 \rightarrow \lambda_{max} = b/T$
5. **Long wavelength limit → Rayleigh-Jeans Law**
6. **Short wavelength limit → Wien's Approximation**

5.7 Remote Sensing Applications

Radiometric Calibration:

- Blackbody sources used to calibrate thermal sensors
- Known T → Predictable radiance via Planck's law
- Establishes relationship between detector response and scene radiance

Brightness Temperature:

- Measured radiance inverted using Planck's law
- Gives "brightness temperature" T_B
- For non-blackbodies: $T_B < T_{physical}$ due to emissivity < 1

Atmospheric Correction:

- Atmospheric emission modeled as blackbody at atmospheric temperature
- Path radiance includes thermal emission from atmosphere
- Must be removed to retrieve surface temperature

Mixed Reflectance/Emission:

- At 3-4 μm: Both reflected solar and thermal emission contribute

- Planck's law helps separate these components
- Critical for fire detection, high-temperature events

Forward Modeling:

- Predict expected sensor signal given surface/atmosphere properties
- Compare with observations for validation
- Retrieve unknown parameters (temperature, emissivity)

Examples:

- **MODIS bands**: Centered where Planck function is sensitive to temperature
- **Thermal cameras**: Response weighted by Planck's law at object temperature
- **Climate models**: Radiation budget calculations use Planck's law

---

6. Photon Properties and Characterization

6.1 Energy of a Photon

Fundamental Relation:

$E_\nu = h\nu = hc/\lambda$

Where:

- **E_ν**: Energy of a photon [Joules]
- **h**: Planck's constant = 6.626×10^{-34} J·s
- **ν**: Frequency [Hz = s^{-1}]
- **λ**: Wavelength [m]
- **c**: Speed of light = 2.998×10^8 m/s

Key Insights:

1. **Quantization**: Energy comes in discrete packets
   - Cannot have arbitrary energy
   - $E_{photon} = n \times h\nu$ where n is integer
2. **Frequency Dependence**: Higher frequency = Higher energy
   - Blue photons more energetic than red
   - X-rays more energetic than visible light
3. **Wavelength Inverse**: Shorter wavelength = Higher energy
   - UV photons can damage DNA
   - IR photons felt as heat

Units and Conversions:

- **SI unit**: Joule (J)
- **Often used**: Electron volt (eV)
  - 1 eV = 1.602×10^{-19} J

- Energy gained by electron in 1 V potential
- Convenient for atomic/molecular processes

Example Calculations:

1. **Red light** (λ = 650 nm):
2. $E = (6.626 \times 10^{-34} \times 2.998 \times 10^{8})/(650 \times 10^{-9})$
3. $E = 3.06 \times 10^{-19}$ J = 1.91 eV
4. **Blue light** (λ = 450 nm):
5. $E = 4.42 \times 10^{-19}$ J = 2.76 eV

   (44% more energy than red)

6. **Thermal IR** (λ = 10 μm):
7. $E = 1.99 \times 10^{-20}$ J = 0.124 eV

   (Much less than visible)

6.2 Characterization of a Photon

A photon's complete state is specified by **seven dimensions**:

Spatial Coordinates (3 dimensions):

- **Position**: r = (x, y, z)
- Location in 3D space
- Units: meters

Directional Coordinates (2 dimensions):

- **Direction**: $\Omega = (\Omega_x, \Omega_y, \Omega_z)$ with constraint $\Omega_x^2 + \Omega_y^2 + \Omega_z^2 = 1$
- Direction of propagation
- Can be specified by two angles (θ, φ) in spherical coordinates
- Units: dimensionless (direction cosines) or radians (angles)

Spectral Coordinate (1 dimension):

- **Frequency**: ν [Hz] or **Wavelength**: λ [m]
- Determines photon energy and color
- Spectral variable

Temporal Coordinate (1 dimension):

- **Time**: t [seconds]
- When the photon passed through the location
- Important for time-varying phenomena

Complete Description:

Photon state = $(x, y, z, \theta, \varphi, \nu, t)$

or equivalently:

Photon state = $(x, y, z, \Omega\_x, \Omega\_y, \Omega\_z, \lambda, t)$

Seven-Dimensional Phase Space:

- Radiative transfer operates in this 7D space
- Each photon occupies a point in this space
- Radiance describes photon density in phase space

Additional Properties (implicit):

- **Polarization**: Orientation of electric field oscillation
  - Linear, circular, or elliptical
  - 2 additional parameters (Stokes vector has 4 components)
- **Phase**: Position in oscillation cycle
  - Usually not tracked in incoherent radiation
- **Momentum**: $p = h/\lambda$ (follows from other properties)

6.3 Implications for Remote Sensing

Spatial Resolution:

- Detector location determines which (x,y,z) photons are collected
- Pixel size defines spatial sampling

Angular Resolution:

- Sensor field of view determines which Ω directions are sampled
- Determines spatial coverage from given altitude

Spectral Resolution:

- Filters/dispersive elements select specific ν or λ ranges
- Multispectral vs. hyperspectral sensors

Temporal Resolution:

- Revisit time determines temporal sampling
- Important for monitoring changing phenomena

Radiative Transfer Complexity:

- Must track photons through 7D space
- Scattering changes Ω
- Absorption removes photons
- Emission adds photons
- Position changes along ray path

7. Geometry: Solid Angle and Coordinate Systems

7.1 Solid Angle (dΩ)

Definition: The three-dimensional analog of planar angle, measuring the extent of a direction as viewed from a point.

Analogy:

- **Planar angle**: Arc length / radius → radians
- **Solid angle**: Surface area / radius² → steradians

Mathematical Expression:

$d\Omega = dA/r^2$

Where:

- **dΩ**: Infinitesimal solid angle [steradians, sr]
- **dA**: Area element on sphere of radius r
- **r**: Radius of sphere

Units: Steradian (sr)

- **Dimensionless** (area/area = dimensionless)
- Full sphere: 4π sr
- Hemisphere: 2π sr
- Full circle (planar): 2π radians

In Spherical Coordinates:

$d\Omega = \sin(\theta)\, d\theta\, d\varphi$

Where:

- **θ**: Polar angle (zenith angle, 0 to π)
- **φ**: Azimuthal angle (0 to 2π)

Geometric Interpretation:

- Cone with half-angle α: $\Omega = 2\pi(1 - \cos \alpha)$
- Small angle approximation: $\Omega \approx \pi \alpha^2$ (for small α)

7.2 Direction Vectors and Directional Cosines

Unit Direction Vector:

$\Omega = (\Omega_x, \Omega_y, \Omega_z)$

with $|\Omega| = 1$

Directional Cosines: The components of Ω are the cosines of angles with coordinate axes:

- **Ω_x** = $\cos(\alpha)$ where α is angle with x-axis
- **Ω_y** = $\cos(\beta)$ where β is angle with y-axis
- **Ω_z** = $\cos(\gamma)$ where γ is angle with z-axis

Normalization Condition:

$$\Omega_x^2 + \Omega_y^2 + \Omega_z^2 = 1$$

This means only 2 of the 3 components are independent.

Relation to Spherical Coordinates:

$\Omega_x = \sin(\theta) \cos(\varphi)$
$\Omega_y = \sin(\theta) \sin(\varphi)$
$\Omega_z = \cos(\theta)$

Physical Meaning:

- Ω points in direction of photon propagation
- Ω_z often represents vertical component (nadir direction)
- $|\Omega_z| = \cos(\theta)$ is the cosine of zenith angle

7.3 Polar Coordinates

Two-Angle System:

1. **Zenith Angle (θ)**:
   - Angle from vertical (z-axis)
   - Range: 0 to π (or 0° to 180°)
   - $\theta = 0$: Looking straight up (or down if z is down)
   - $\theta = \pi/2$: Horizontal direction
   - $\theta = \pi$: Looking straight down (or up)
2. **Azimuthal Angle (φ)**:
   - Angle in horizontal plane from x-axis
   - Range: 0 to 2π (or 0° to 360°)
   - $\varphi = 0$: Pointing along +x direction
   - $\varphi = \pi/2$: Pointing along +y direction

Solid Angle Element:

$$d\Omega = \sin(\theta)\, d\theta\, d\varphi$$

Why sin(θ)?

- Area element on sphere: $dA = r^2 \sin(\theta)\, d\theta\, d\varphi$
- Solid angle: $d\Omega = dA/r^2 = \sin(\theta)\, d\theta\, d\varphi$

- $\sin(\theta)$ is the "Jacobian" of the transformation
- Larger at equator ($\theta = \pi/2$), smaller at poles ($\theta = 0, \pi$)

Integration Examples:

1. **Full sphere**:
2. $\iint_{sphere} d\Omega = \int_0^{2\pi} \int_0^{\pi} \sin(\theta)\, d\theta\, d\varphi = 4\pi$ sr
3. **Hemisphere** (upper, θ from 0 to $\pi/2$):
4. $\iint_{hemisphere} d\Omega = \int_0^{2\pi} \int_0^{\pi/2} \sin(\theta)\, d\theta\, d\varphi = 2\pi$ sr
5. **Cone** (θ from 0 to θ_{max}):
6. $\Omega_{cone} = \int_0^{2\pi} \int_0^{\theta_{max}} \sin(\theta)\, d\theta\, d\varphi = 2\pi(1 - \cos \theta_{max})$

7.4 Remote Sensing Applications

Sensor Field of View:

- Instantaneous field of view (IFOV): Solid angle of single detector
- Total field of view: Solid angle of entire sensor
- Determines spatial resolution and coverage

Radiometric Quantities:

- **Radiance**: Power per unit area per unit solid angle $[W/(m^2 \cdot sr)]$
- Must specify direction Ω
- Integration over Ω gives irradiance

Bidirectional Reflectance:

- BRDF: $f(\Omega_i \to \Omega_r)$
- Depends on incident direction Ω_i
- Depends on reflected direction Ω_r
- Both specified using (θ, φ) or directional cosines

Scattering Phase Function:

- $P(\Omega \to \Omega')$: Probability of scattering from Ω to Ω'
- Often depends only on scattering angle between Ω and Ω'
- Integrated over all Ω' gives total scattering probability

Atmospheric Path Radiance:

- Radiation scattered into sensor from atmosphere
- Integral over all directions:
- $L_{path} = \iint P(\Omega' \to \Omega_{sensor}) L(\Omega')\, d\Omega'$

---

8. Mathematical Foundations

8.1 Greek Symbols (Reference)

The slides provide a reference table of Greek symbols commonly used in remote sensing:

Lowercase:

- α (alpha), β (beta), γ (gamma), δ (delta), ε (epsilon)
- ζ (zeta), η (eta), θ (theta), ι (iota), κ (kappa)
- λ (lambda), μ (mu), ν (nu), ξ (xi), ο (omicron)
- π (pi), ρ (rho), σ (sigma), τ (tau), υ (upsilon)
- φ (phi), χ (chi), ψ (psi), ω (omega)

Uppercase:

- Γ (Gamma), Δ (Delta), Θ (Theta), Λ (Lambda)
- Ξ (Xi), Π (Pi), Σ (Sigma), Φ (Phi)
- Ψ (Psi), Ω (Omega)

Common Uses in Remote Sensing:

- λ: Wavelength
- ν: Frequency
- θ: Zenith angle
- φ: Azimuthal angle
- Ω: Direction vector or solid angle
- σ: Scattering/extinction coefficient, Stefan-Boltzmann constant
- τ: Optical depth
- ρ: Reflectance
- ε: Emissivity
- α: Absorption coefficient

8.2 Angular Integrals

Simple Examples (from slides):

The slides mention exercises to evaluate simple angular integrals, which are fundamental to radiative transfer:

Example 1: Integrate over full sphere

$$\iint_{4\pi} d\Omega = \int_0^{2\pi} \int_0^{\pi} \sin(\theta)\, d\theta\, d\varphi$$
$$= 2\pi \times [-\cos(\theta)]_0^{\pi}$$
$$= 2\pi \times (-\cos \pi + \cos 0)$$
$$= 2\pi \times (1 + 1) = 4\pi \text{ sr}$$

Example 2: Integrate cos(θ) over hemisphere

$$\iint_{2\pi} \cos(\theta)\, d\Omega = \int_0^{2\pi} \int_0^{\pi/2} \cos(\theta) \sin(\theta)\, d\theta\, d\varphi$$
$$= 2\pi \times [\sin^2(\theta)/2]_0^{\pi/2}$$
$$= 2\pi \times (1/2 - 0) = \pi$$

This integral appears in:

- Lambertian reflectance
- Radiative transfer (projection factor)
- Flux calculations from radiance

Example 3: Average directional cosine

$$\langle \cos(\theta) \rangle = (1/2\pi) \iint_{2\pi} \cos(\theta)\, d\Omega = \pi/(2\pi) = 1/2$$

Physical Significance:

- Average projection factor over hemisphere = 1/2
- Connects radiance to irradiance
- Appears in diffuse reflectance calculations

More Complex Integrals:

- Scattering phase functions: $\int_{4\pi} P(\theta)\, d\Omega = 4\pi$
- Asymmetry parameter: $g = (1/4\pi) \int_{4\pi} P(\theta) \cos(\theta)\, d\Omega$
- Radiative transfer source terms involve complex angular integrals

8.3 Visual and Computational Tools

Visualizations Mentioned: The course includes computational visualizations to help understand these concepts, though the specific tools aren't detailed in the extracted text.

Importance:

- Abstract concepts (7D phase space, solid angles) difficult to visualize
- Interactive tools help build intuition
- Computational practice reinforces mathematical understanding

---

9. Assumptions for Mathematical Modeling

The slides list key assumptions used to create tractable mathematical models:

9.1 Sun as Point Source

Assumption: The sun is treated as a distant point source of electromagnetic radiation.

Justification:

- Sun-Earth distance: $\sim 1.5 \times 10^8$ km (1 AU)

- Sun's angular diameter: ~0.5°
- From Earth, sun subtends solid angle: ~6.8×10^{-5} sr
- Much smaller than typical sensor fields of view
- Simplifies geometric calculations

Implications:

- Solar illumination can be treated as collimated beam
- Single incident direction (θ_{sun}, φ_{sun})
- Simplifies bidirectional reflectance calculations
- Shadows have sharp edges (in absence of atmosphere)

Limitations:

- Not valid for instruments with extremely narrow FOV
- Solar disk's finite size causes:
    - Penumbra (partial shadow)
    - Limb darkening effects
    - Aureole in atmospheric scattering

9.2 Energy Carried by Photons

Assumption: All electromagnetic energy is carried by discrete photons.

Justification:

- Quantum nature of light
- Especially important at low light levels
- Explains photoelectric effect in detectors

Implications:

- Detection is fundamentally a counting process
- Shot noise arises from discrete photon arrivals
- Quantum efficiency measures photon-to-electron conversion
- Energy per photon: $E = h\nu$

When It Matters:

- Low-light imaging (night-time, deep ocean)
- Single-photon counting detectors
- Quantum remote sensing
- Noise analysis

9.3 Photon Properties

Point Particles:

- Zero spatial extent
- Simplifies tracking through media

- Position is well-defined

Massless:

- Always travel at speed c (in vacuum)
- Energy-momentum relation: E = pc
- No rest frame exists

Neutral (No Charge):

- Don't interact via electromagnetic force
- No deflection by E or B fields
- Interact only through absorption/scattering with matter

Straight-Line Propagation:

- Between collisions, photons travel in straight lines
- No curvature (in flat spacetime, no strong gravity)
- Simplifies ray tracing
- Path length = distance

Constant Speed:

- c = 299,792,458 m/s (exactly, by definition)
- In media: v = c/n where n is refractive index
- For remote sensing in atmosphere: $n \approx 1$ (excellent approximation)

9.4 No Photon-Photon Collisions

Assumption: Photons do not collide with each other.

Justification:

- Linear superposition in classical electromagnetism
- Photon-photon scattering requires quantum electrodynamics
- Cross-section is extremely small ($\sim 10^{-65}$ m^2)
- Requires very high intensities (not found in remote sensing)

Implications:

- Radiative transfer is linear in intensity
- Multiple beams pass through each other without interaction
- Scattering from particles is independent of other photons present
- Greatly simplifies radiative transfer equations

When It Breaks Down:

- Extremely high-intensity laser beams

- Astrophysical contexts (neutron star magnetospheres)
- Not relevant for Earth remote sensing

9.5 Summary of Modeling Framework

These assumptions create a tractable mathematical framework:

1. **Geometric Optics**: Ray tracing, straight-line propagation
2. **Radiative Transfer**: Linear equations for photon transport
3. **Detector Response**: Photon counting statistics
4. **Scattering**: Single-particle scattering, no collective effects

Validity: Excellent for all Earth remote sensing applications

Limitations:

- Break down for coherent phenomena (laser radar interferometry)
- Ignore wave effects (diffraction in some cases)
- Don't account for quantum entanglement (not relevant anyway)

---

10. Summary and Integration

10.1 Key Concepts Hierarchy

Fundamental Physics:

1. **Wave-Particle Duality**: EM radiation exhibits both natures
2. **Planck's Law**: Quantum description of blackbody radiation
3. **Derived Laws**: Boltzmann's and Wien's laws follow from Planck's

Photon Description:

1. **Energy**: $E = h\nu = hc/\lambda$
2. **Seven Dimensions**: $(x,y,z,\theta,\varphi,\nu,t)$ or (x,y,z,Ω,ν,t)
3. **Properties**: Massless, neutral, straight-line travel at c

Geometric Framework:

1. **Direction**: Specified by unit vector Ω or angles (θ,φ)
2. **Solid Angle**: $d\Omega = \sin(\theta)d\theta d\varphi$ measures directional extent
3. **Directional Cosines**: Components of Ω

Modeling Assumptions:

1. Point source sun
2. Photon energy carriers
3. No photon-photon interactions
4. Straight-line propagation

10.2 How These Concepts Connect

From Theory to Practice:

```
Planck's Law (fundamental quantum description)
   ↓
   ↓ Integration over wavelength
   ↓
Boltzmann's Law (total emission)    Wien's Law (peak wavelength)
   ↓                          ↓
   ↓                          ↓
   └─────────────── Applications ──────────────────┘
              ↓
      Thermal Remote Sensing
      Temperature Retrieval
      Radiometric Calibration
      Energy Balance Studies
```

Photon Framework:

```
Energy (E = hν) + Position (r) + Direction (Ω) + Time (t)
              ↓
         Complete Description
              ↓
         Radiative Transfer
              ↓
      Predict/Interpret Observations
```

Geometric Framework:

```
Solid Angle (dΩ) + Directional Cosines (Ω)
              ↓
         Angular Integration
              ↓
      Radiance → Irradiance
      BRDF Calculations
      Scattering Integration
```

10.3 Remote Sensing Applications Summary

Spectral Domain (λ or ν):

- Determine photon energy
- Select spectral bands
- Identify materials via spectral signatures

Spatial Domain (x, y, z):

- Locate targets
- Map surface properties

- Determine spatial resolution

Angular Domain (θ, φ or Ω):

- Bidirectional reflectance (BRDF)
- View geometry effects
- Atmospheric path radiance

Temporal Domain (t):

- Monitor changes
- Track moving targets
- Time-series analysis

Radiometric Domain (photon flux):

- Calibration using blackbody sources (Planck's law)
- Temperature retrieval (Boltzmann's and Wien's laws)
- Energy budget calculations

Conclusion

This chapter lays the essential groundwork for understanding physical models in remote sensing. The key takeaways are:

1. **Electromagnetic radiation** has both wave and particle nature (wave-particle duality)
2. **Planck's Law** is the fundamental quantum description of blackbody radiation, from which both **Boltzmann's Law** (total emission) and **Wien's Law** (peak wavelength) can be derived
3. **Photons** are characterized by seven dimensions: 3 spatial (x,y,z), 2 directional (θ,φ), 1 spectral (ν or λ), and 1 temporal (t)
4. **Geometric concepts** (solid angle, directional cosines, spherical coordinates) are essential for describing directional radiation
5. **Modeling assumptions** (point source sun, photon energy carriers, no photon-photon interactions, straight-line propagation) create a tractable mathematical framework

These fundamental concepts provide the vocabulary and theoretical framework for understanding more advanced topics in radiative transfer, atmospheric correction, and quantitative remote sensing that will be developed in subsequent chapters.

The mathematical rigor introduced here - from quantum mechanics (Planck) to geometric optics (solid angles) to statistical mechanics (blackbody radiation) - forms the foundation for quantitative, physics-based remote sensing rather than purely empirical approaches.