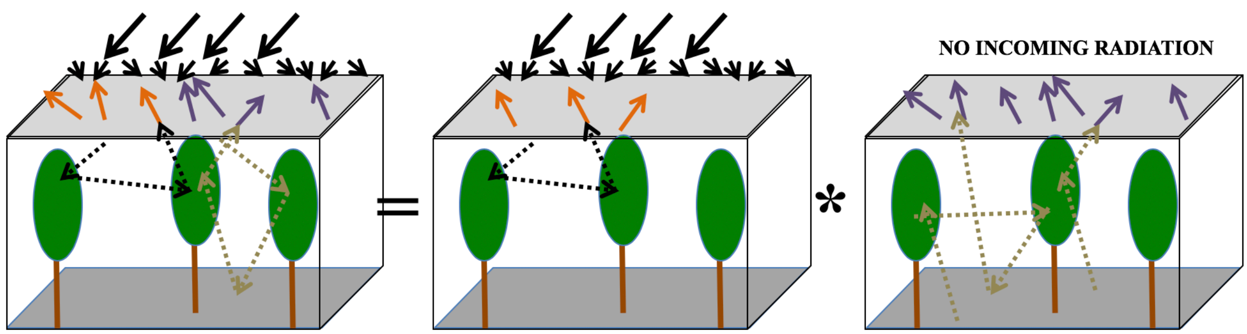
Chapter 3

Radiative Transfer in Vegetation Canopies

Knyazikhin et al.



$$\Omega \bullet \nabla I_\lambda + \sigma(r,\Omega) I_\lambda(r,\Omega) = \int_{4\pi} d\Omega' \sigma_{s,\lambda}(r,\Omega' \to \Omega) I_\lambda(r,\Omega')$$

$$I_\lambda(r_t,\Omega) = B_\lambda(r_t,\Omega),\ r_t \in \partial V_t,\ n(r_B) \bullet \Omega < 0$$

$$I(r_b,\Omega) = \frac{1}{\pi} \int_{\Omega' \bullet n(r_B) > 0} \rho_b \,|\, n(r_B') \bullet \Omega' \,|\, I(r_B',\Omega')\, d\Omega'$$

Chapter 3

Radiative Transfer in Vegetation Canopies

1. Radiative Transfer Equation for Vegetation Canopies

Solar radiation scattered from a vegetation canopy and measured by satellite sensors results from interaction of photons traversing through the foliage medium, bounded at the bottom by a radiatively participating surface. Therefore to estimate the canopy radiation regime, three important features must be carefully formulated. They are (1) the architecture of individual plant and the entire canopy; (2) optical properties of vegetation elements (leaves, stems) and soil; the former depends on physiological conditions (water status, pigment concentration); and (3) atmospheric conditions which determine the incident radiation field [Ross, 1981].

We idealize a vegetation canopy as a medium filled with small planar elements of negligible thickness. We ignore all organs other than green leaves. In addition, we neglect the finite size of vegetation canopy elements. Thus, the vegetation canopy is treated as a gas with non-dimensional planar scattering centers, i.e., a turbid medium. In other words, one cuts leaves residing in an elementary volume at a given spatial point r into "dimensionless pieces" and uniformly distributes them within the elementary volume. Three variables, the leaf area density distribution function $u_L(\underline{r})$, the leaf normal distribution, $g_L(\underline{r},\underline{\Omega}_L)$, and the leaf scattering phase function, $\gamma_L(\underline{r},\underline{\Omega}' \rightarrow \underline{\Omega},\underline{\Omega}_L)$ (Chapter 3) are used in the theory of radiative transfer in vegetation canopies to convey "information" about the total leaf area, leaf orientations and leaf optical properties in the elementary volume at r before "converting the leaves into the gas."

It should be emphasized that the turbid medium assumption is a mathematical idealization of canopy structure, which ignores finite size of leaves. In reality, finite size scatters can cast shadows. This causes a very sharp peak in reflected radiation about the retro-solar direction. This phenomenon is referred to as the "hot spot" effect. It is clear that point scatters cannot cast shadows and thus the turbid medium concept in its original formulation [Ross, 1981] fails to predict or duplicate experimental observation of exiting radiation about the retro-illumination direction. Zhang et al. [2002] showed that if the solution to the radiative transfer equation is treated as a Schwartz distribution, then an additional term must be added to the solution of the radiative transfer equation. This term describes the hot spot effect. This result justifies the use of the transport equation as the basis to model canopy radiation regime. Here we will follow classical radiative transfer theory in vegetation canopies proposed by Ross [1981]. For the mathematical theory of Schwartz distributions applicable to the transport equation, the reader is referred to Germogenova [1986], Choulli and Stefanov [1996] and Antyufeev [1996].

In addition to canopy structure and its optics a domain V in which the radiative transfer process is studied should be specified. In remote sensing application, a parallelepiped of horizontal dimensions X_S, Y_S, and height Z_S is usually taken as the domain V. The top δV_t, bottom δV_b, and lateral δV_l surfaces of the parallelepiped form the canopy boundary $\delta V = \delta V_t + \delta V_b + \delta V_l$. The height of a tallest plant in V can be taken as Z_S. The dimension of the upper boundary δV_t coincides with a footprint of the imagery. The function characterizing the radiative field in V is the specific intensity introduced in Chapter 2. Under condition of the absence of polarization, frequency shifting interaction, and emission processes within the canopy, the monochromatic specific intensity $I_\lambda(\underline{r},\underline{\Omega})$ is given by the stationery radiative transfer equation (Chapter 2, Eq. (24)) with $q_\lambda(\underline{r},\underline{\Omega}) = 0$. Substituting vegetation-specific coefficients (Chapter 3, Eqs (13) and (15)) into the transport equation (Chapter 2, Eq. (24)), one obtains the radiative transfer equation for a vegetation canopy occupying the domain V, namely,

$$\underline{\Omega} \bullet \nabla I_\lambda(\underline{r},\underline{\Omega}) + G(\underline{r},\underline{\Omega}) u_L(\underline{r}) I_\lambda(\underline{r},\underline{\Omega}) = \frac{u_L(\underline{r})}{\pi} \int_{4\pi} \Gamma_\lambda(\underline{r},\underline{\Omega}' \to \underline{\Omega}) I_\lambda(\underline{r},\underline{\Omega}') d\underline{\Omega}'. \tag{1}$$

The boundary condition for the radiative transfer problem is given by

$$I_\lambda(\underline{r}_b,\underline{\Omega}) = B_\lambda(\underline{r}_b,\underline{\Omega}), \quad \underline{r}_b \in \delta V, \quad \underline{n}(\underline{r}_b) \bullet \underline{\Omega} < 0. \tag{2}$$

Here $B_\lambda(\underline{r}_b,\underline{\Omega})$ is the intensity of radiation entering the domain V through a point $\underline{r}_B$ on the boundary δV in the direction $\underline{\Omega}$. Directions along which photons can enter the vegetation canopy through the point $\underline{r}_b$ satisfy the inequality $\underline{n}(\underline{r}_b) \bullet \underline{\Omega} < 0$ where $\underline{n}(\underline{r}_b)$ is an outward normal vector at $\underline{r}_b$.

The solution of the boundary value problem, Eqs (1)-(2), i.e., the monochromatic specific intensity $I_\lambda(\underline{r},\underline{\Omega})$, depends on wavelength, λ, location r, and direction $\underline{\Omega}$. Here, the position vector $\underline{r}$ denotes the triplet (x,y,z) with ($0<x<X_S$), ($0<y<Y_S$) and ($0<z<Z_S$) and is expressed in Cartesian coordinates with its origin, O=(0,0,0), at the top of the vegetation canopy and the *Z* axis directed down into the vegetation canopy. The unit vector $\underline{\Omega} \sim (\theta,\varphi)$ has an azimuthal angle φ measured in the (XY) plane from the positive X axis in a counterclockwise fashion and a

polar angle θ with respect to the polar axis that is opposite to the Z axis. In this Chapter we shall omit the sign λ in notations.

2. Vegetated Surfaces Reflectance

Solution of the boundary value problem (Eqs. (1)-(2)) describes the radiative regime in a vegetation canopy and, as a consequence, reflectance properties of the vegetated surface. When describing surface reflectance, standard nomenclature [Nicodemus et al., 1977] dictates that the angular characteristics of the illumination are mentioned first, followed by the angular characteristics of the reflected radiance. In the definitions given below, the prefix hemispherical-directional implies an illumination which is hemispherical in directional extent and a reflected radiance in a single direction. Directional-hemispherical implies that the illumination is single directional and the reflected radiance is integrated over the hemisphere [Martonchik et al., 2000]. The following reflectance quantities are used in remote sensing to describe surface reflective properties.

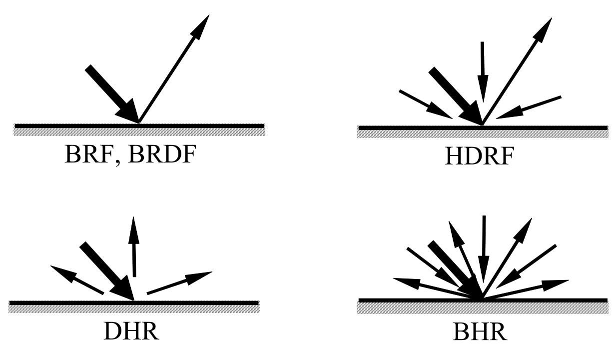


Figure 1. Reflectance nomenclature summary. The broad arrow represents an irradiance from a collima-ted beam. All other arrows represent incident and reflected radiance fields.

The hemispherical-directional reflectance factor (HDRF, dimensionless) for nonisotropic incident radiation is the ratio of the mean radiance leaving the top of the vegetation canopy to radiance reflected from an ideal Lambertian target into the same beam geometry and illuminated under identical atmospheric conditions [Martonchik et al., 2000]; this can be expressed in terms of the solution of Eq. (1)-(2) as

$$R = \frac{\int_0^{X_S}\int_0^{Y_S} I(x,y,0,\underline{\Omega})\,dx\,dy}{\frac{1}{\pi}\int_{2\pi-}|\mu'|\,d\underline{\Omega}'\int_0^{X_S}\int_0^{Y_S} I(x,y,0,\underline{\Omega}')\,dx\,dy} = \frac{< I(\underline{\Omega}) >_0}{\frac{1}{\pi}\int_{2\pi-} < I(\underline{\Omega}') >_0 d\underline{\Omega}'}, \quad \mu > 0. \qquad (3)$$

Here μ and μ' are the cosine of the polar angles of the upward (reflected) $\underline{\Omega}$ and downward (incident) $\underline{\Omega}'$ directions, respectively; the angle brackets $< >_0$ denotes the mean over the upper surface δV_t of the parallelepiped V. The HDRF depends on atmosphere conditions (i.e., the angular and spectral distribution of the incoming radiation), the surface properties (e.g., vegetation canopy below the boundary δV_t), the area of δV_t, and the direction $\underline{\Omega}$. For the condition of no atmosphere, i.e., the incident solar radiation at the upper canopy boundary δV_t is a parallel beam of light, the HDRF is termed a bidirectional reflectance factor (BRF, dimensionless). The BRF does not depend on atmosphere conditions and characterizes surface reflective properties. Its value varies with the directions, $\underline{\Omega}$ and $\underline{\Omega}'$, of reflected and incident radiation. The bidirectional reflectance distribution function (BRDF) is another reflectance quantity that describes the scattering of a parallel beam of incident radiation from one direction into another direction but, unlike the BRF, its values are expressed relative to the incident flux, i.e., the BRDF is the mean radiance leaving the upper boundary to the incident flux. The BRDF has units of sr^{-1} and is a factor of π smaller than BRF, i.e., $\mathrm{BRDF} = \pi^{-1}\mathrm{BRF}$.

The bihemispherical reflectance (BHR, dimensionless) for nonisotropic incident radiation is the ratio of the mean irradiance exitance to the incident irradiance [Martonchik et al., 1998], i.e.,

$$A = \frac{\int_{2\pi+} < I(\underline{\Omega}) >_0 |\mu| \, d\underline{\Omega}}{\int_{2\pi-} < I(\underline{\Omega}') >_0 |\mu'| \, d\underline{\Omega}'}. \tag{4}$$

For the condition of no atmosphere, the BHR becomes directional hemispherical reflectance (DHR, dimensionless). For Lambertian surfaces, HDRF=BRF=BHR=DHR.

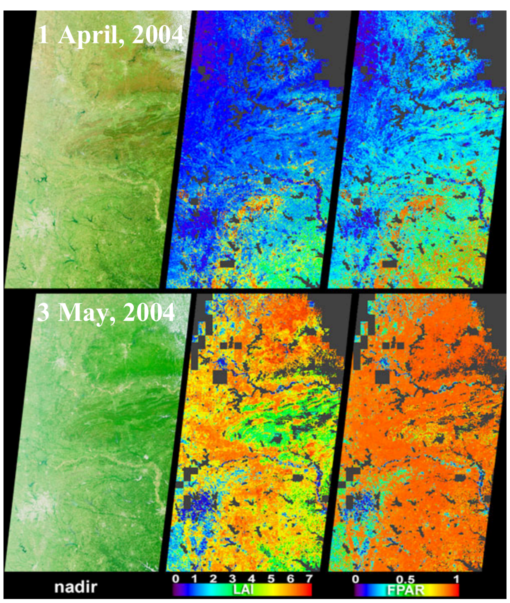


Figure 2. The multiangle imaging spectroradiometer (MISR) onboard the Earth Observing System (EOS) Terra platform provides global imagery at nine discrete viewangles and four visible/near-ibfrared spectral bands. MISR standard products include HDRF, BHR, BRF, DHR, leaf area index (LAI) and fraction of photosynthetically active radiation absorbed by vegetation (FPAR). Vigorous vegetation growth in the Southern United States (Texas, Oklahoma) after heavy rains fell during April and early May, 2004, is quantified in these images and LAI&FPAR product from MISR. The left-hand images are natural-color views from MISR's nadir camera acquired on April 1 (top set) and May 3 (bottom set). The middle and right-hand panels show MISR LAI and FPAR standard products. Data are at 1.1 km spatial resolution, i.e., the dimension of the upper boundary δV_t is 1.1 by 1.1 km.

All the reflectance quantities introduced above can be derived from data acquired by satellite-borne sensors (Fig. 2) which, in turn, are input to various techniques for retrieval of biophysical parameters from space. In remote sensing, the dimension of the upper boundary δV_t often coincides with a footprint of the imagery. Taking the size of δV_t to zero results in a BRF value defined at a spatial point $\underline{r}_t$. Given the bidirectional reflectance factor, $R_0(\underline{\Omega}',\underline{\Omega})$), and intensity, $I(\underline{r}_t,\underline{\Omega}')$, of radiation incident on a horizontal surface at $\underline{r}_t$, the intensity of reflected radiation, $I(\underline{r}_t,\underline{\Omega})$, can be calculated as

$$I(r_t,\underline{\Omega}) = \frac{1}{\pi}\int_{2\pi^-} R_0(\underline{\Omega}',\underline{\Omega})\,|\,\mu'\,|\,I(\underline{r}_t,\underline{\Omega}')\,d\underline{\Omega}' \,. \tag{5}$$

This equation is used to describe the lower boundary condition for the radiative transfer in the atmosphere.

3. Boundary Conditions

The boundary conditions for a three-dimensional canopy are also three-dimensional. Indeed, the radiation entering the canopy through the top, δV_t, through the bottom, δV_b, and through the lateral, δV_l, surfaces are different. Therefore we consider a very general form of boundary conditions (see Sect. 2.4), namely,

$$I(\underline{r}_b,\underline{\Omega}) = \frac{1}{\pi}\int_{\delta V} d\underline{r}'_b \int_{\underline{\Omega}'\bullet\underline{n}(\underline{r}_b)>0} \rho_b(\underline{r}'_b,\underline{\Omega}';\underline{r}_b,\underline{\Omega})\,|\,\underline{n}(\underline{r}'_b)\bullet\underline{\Omega}'\,|\,I(\underline{r}'_b,\underline{\Omega}')\,d\underline{\Omega}'$$
$$+\,q(\underline{r}_b,\underline{\Omega}), \quad +\,n(\underline{r}_b)\bullet\underline{\Omega} < 0. \tag{6}$$

Here $\underline{r}_B$ and $\underline{r}'_B$ are points on the canopy boundary δV; $\underline{n}(\underline{r}_B)$ is the outward normal at the point $\underline{r}_B$; $\rho_B(\underline{r}'_B,\underline{\Omega}';\underline{r}_B,\underline{\Omega})$ is the boundary scattering function; that is, the probability density that a photon having escaped from the canopy through the point $\underline{r}'_B \in \delta V$ and in the direction $\underline{\Omega}'$ will come back to it through the point $\underline{r}_B \in \delta V$ and in the direction $\underline{\Omega}$; and $q_B(\underline{r}_B,\underline{\Omega})$ is a photon source at the canopy boundary δV. Both ρ_B and q_B are wavelength dependent.

The radiative transfer problem can now be formulated as follows: find the intensity $I(\underline{r},\underline{\Omega})$ which satisfies the transport equation (Eq. (1)) within the domain V and the condition given by Eq. (6) on the canopy boundary δV. The maximum boundary reflectance, $\rho_0(\delta V)$, canopy optical path, $\tau_0(V)$, and single scattering albedo ϖ_0, (cf. Chapter 2, Section 11) are basic characteristics of boundary reflective properties, canopy structure and leaf optical properties. It follows from the uniqueness theorem that the conditions $\varpi_0 \le 1$, $\rho_0(\delta V) < 1$ and $\tau_0(\delta V) < \infty$ guarantee the existence and uniqueness of the solution to the boundary value problem given by Eq. (3) and (6). Specification of the boundary conditions for the upper, δV_t, lower, δV_b, and lateral, δV_l, surfaces of the parallelepiped are discussed below.

Canopy upper boundary. The upper canopy boundary δV_t is adjacent to the atmosphere. Therefore radiation penetrating into the canopy through the upper boundary δV_t is determined by atmospheric conditions, i.e., the upper canopy boundary is exposed to both direct solar irradiance and diffuse radiation from all points of the sky. The former is caused by photons in the solar parallel beam which arrive at the upper canopy boundary without experiencing a collision. The latter results from photon-atmosphere interactions. Thus, the boundary condition at the upper boundary δV_t can be written as

$$I(\underline{r}_t,\underline{\Omega}) = c_T(\underline{r}_t)\delta(\underline{\Omega}-\underline{\Omega}_0) + d(\underline{r}_t,\underline{\Omega}), \quad \underline{r}_t \in \delta V_t, \quad \mu < 0. \tag{7}$$

Here $c_T(\underline{r}_t,\underline{\Omega})$ and $d(\underline{r}_t,\underline{\Omega})$ are intensities of the solar beam and diffuse radiation at point $\underline{r}_t$ on the boundary δV_t and $\delta(\underline{\Omega}-\underline{\Omega}_0)$ is the Dirac delta function. Both $c_T(\underline{r}_t,\underline{\Omega})$ and $d(\underline{r}_t,\underline{\Omega})$ are wavelength dependent. The direction of the solar beam is given by the unit vector $\underline{\Omega}_0 \sim (\theta_0,\varphi_0)$. Since $\underline{n}(\underline{r}_t)\bullet\underline{\Omega}$ coincides with the cosine of the polar angle θ of the direction $\underline{\Omega}$, the condition $\underline{n}(\underline{r}_t)\bullet\underline{\Omega}<0$ for incoming directions can be written as $\mu<0$. In other words, the upper boundary condition is formulated for downward directions. In terms of notations used in Eq. (6), $\rho_B(\underline{r}'_b,\underline{\Omega}';\underline{r}_b,\underline{\Omega})=0$, $q(\underline{r}_b,\underline{\Omega}) = c_T(\underline{r}_b)\delta(\underline{\Omega}-\underline{\Omega}_0)+d(\underline{r}_b,\underline{\Omega})$, for $\underline{r}_b$ on the surface δV_t.

The canopy-radiation regime is sensitive to the partition between the mono-directional and diffuse components of the incoming radiation. The ratio f_{dir} of the mono-directional to the total radiation flux incident on the canopy is used to parameterize the partition; that is,

$$f_{dir}(\underline{r}_t) = \frac{F^{\downarrow}_{dir}(\underline{r}_t)}{F^{\downarrow}_{dir}(\underline{r}_t)+F^{\downarrow}_{diff}(\underline{r}_t)}, \quad \underline{r}_t \in \delta V_t. \tag{8}$$

Here $F^{\downarrow}_{dir}(\underline{r}_t)$ and $F^{\downarrow}_{dif}(\underline{r}_t)$ are monochromatic downward flux densities (irradiances) of mono-directional and diffuse components of the incident radiation, i.e.,

$$F^{\downarrow}_{dir}(\underline{r}_t) = c_T\,|\,\mu_0\,|, \quad F^{\downarrow}_{diff}(\underline{r}_t) = \int_{2\pi^-} d(\underline{r}_t,\underline{\Omega})\left|\underline{\Omega}\bullet\underline{n}(\underline{r}_t)\right|d\underline{\Omega}, \tag{9}$$

where $\mu_0=\cos\theta_0$. The ratio f_{dir} varies between 0 and 1 and depends on the direction of the mono-directional incident beam, wavelength and atmosphere conditions. This parameter along with the total downward flux $F^{\downarrow}(\underline{r}_t) = F^{\downarrow}_{dir}(\underline{r}_t)+F^{\downarrow}_{dir}(\underline{r}_t)$ can be derived from satellite data [Diner et al., 1999a]. It is conventional, therefore, to parameterize the upper boundary condition in terms of these variables, i.e.,

$$I(\underline{r}_t,\underline{\Omega}) = \left[\frac{f_{dir}}{|\mu_0|}\delta(\underline{\Omega}-\underline{\Omega}_0)+(1-f_{dir})\,d_0(\underline{r}_t,\underline{\Omega})\right]F^{\downarrow}(\underline{r}_t), \quad \underline{n}(\underline{r}_t)\bullet\underline{\Omega}<0, \tag{10}$$

where $d_0(\underline{r}_t,\underline{\Omega}) = d(\underline{r}_t,\underline{\Omega})/F^{\downarrow}_{dif}$ is the anisotropy factor of the diffuse radiation.

The HDRF and BHR defined in Eqs (3) and (4), respectively, are expressed in terms of solution of the transport equation with the upper boundary condition (Eq. (7)). If $f_{dir} = 0$, the HDRF and BHR become the BRF and DHR, respectively.

In many cases, the anisotropy of diffuse radiation can be assumed wavelength independent. A model of the anisotropy corresponding to clear–sky conditions proposed by Pokrowski [1929]

$$d_0(\underline{r}_t,\underline{\Omega}) = \left[1-\exp\left(\frac{-0.32}{|\mu|}\right)\right]\frac{1+\underline{\Omega}\bullet\underline{\Omega}_0}{1-\underline{\Omega}\bullet\underline{\Omega}_0}, \quad \mu<0,$$

is an example of the angular distribution of incoming diffuse radiation. In the case of the standard overcast sky ($f_{dir} = 0$), the intensity $d(\underline{r}_t,\underline{\Omega})$ of the incoming diffuse radiation in the photosynthetically active region of solar spectrum, 400–700 nm, can be approximated by

$$d(\underline{r}_t,\underline{\Omega}) = i(\pi)\frac{1+b|\mu|}{1+b}, \quad \mu<0,$$

where 1+b is the ratio between sky brightness in the zenith, $i(\pi)$, and at the horizon, $i(\pi/2)$ and it varies between 2.1 and 2.4 [Monteith and Unsworth, 1990]. Substituting the above equation into $d_0(\underline{r}_t,\underline{\Omega}) = d(\underline{r}_t,\underline{\Omega})/F^{\downarrow}_{dif}$ and taking into account Eq. (9) one can express $i(\pi)$ and d_0 as

$$i(\pi) = F^{\downarrow}(\underline{r}_t)\frac{1+b}{\pi(1+\frac{2}{3}b)}, \quad d_0(\underline{r}_t,\underline{\Omega}) = F^{\downarrow}(\underline{r}_t)\frac{1+b|\mu|}{\pi(1+\frac{2}{3}b)}. \tag{11}$$

Canopy lower boundary. At the bottom of the canopy, a fraction of the radiation can be reflected back into the canopy by the ground. In the remote sensing problems, reflective properties of the canopy lower boundary are often approximated as $\rho_B(\underline{r}'_b,\underline{\Omega}';\underline{r}_b,\underline{\Omega}) = \rho_b(\underline{r}'_b,\underline{\Omega}'\to\underline{\Omega})\delta(\underline{r}'_b-\underline{r}_b)$, $\underline{r}_b$, $\underline{r}'_b \in \delta V_b$ where $\rho_b(\underline{r}'_b,\underline{\Omega}'\to\underline{\Omega})$ is the bidirectional reflectance factor of the canopy ground. The canopy bottom does not emit the radiation at solar wavelengths and thus $q_b(\underline{r}_b,\underline{\Omega}) = 0$. Substituting these equations into Eq. (6) results in

$$I(\underline{r}_b,\underline{\Omega}) = \frac{1}{\pi}\int_{2\pi^-}\rho_b(\underline{r}_b,\underline{\Omega}'\to\underline{\Omega})\left|\underline{n}(\underline{r}_b)\bullet\underline{\Omega}'\right| I(\underline{r}_b,\underline{\Omega}')\,d\underline{\Omega}', \quad \mu>0. \tag{12}$$

Here $-\underline{n}(\underline{r}_b)\bullet\underline{\Omega}$ coincides with the cosine of the polar angle θ of the direction $\underline{\Omega}$. The condition $\underline{n}(\underline{r}_t)\bullet\underline{\Omega}<0$ for incoming directions therefore can be written as $\mu>0$, i.e., the lower boundary condition is formulated for upward directions.

Canopy lateral boundary. The radiation penetrating through the lateral sides of the canopy depends on the neighboring environment. Its influence on the radiation field within the canopy is especially pronounced near the lateral canopy boundary. Therefore inaccuracies in the lateral boundary conditions may cause distortions in the simulated radiation field within the domain V. These features should be taken into account when 3D radiation distribution in a vegetation canopy of a small area is investigated. The problem of photon transport in such canopies arises, for example, in the context of optimal planting and cutting of industrial wood, land surface climatology, and plant physiology.

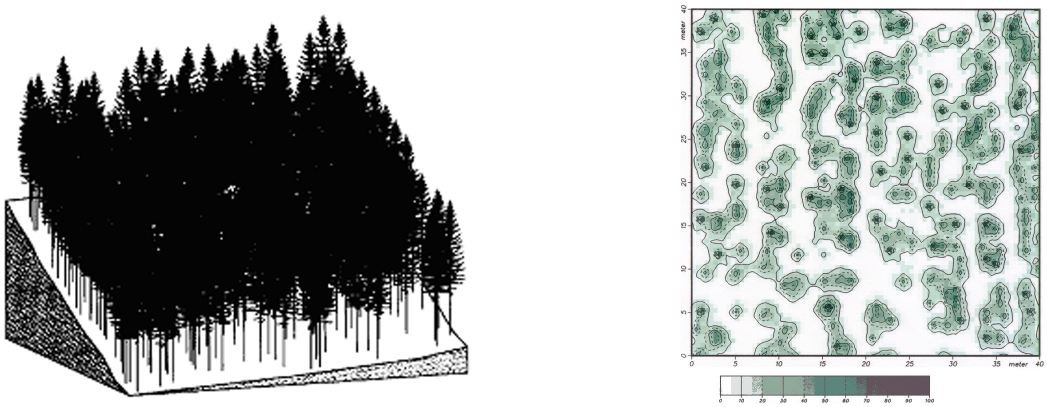


Figure 3. Computer simulated Norway spruce stand about 50 km near Goettingen, Germany, in the Harz mountains. The stand is about 45 years old and situated on the south slope. A 40×40 m^2 section of the stand with 297 trees was sampled for reconstruction. The stem diameters varied from 6 to 28 m and the tallest trees were about 12.5 m in height. The trees were divided into five groups with respect to stem diameter. A model of a Norway spruce based on fractal theory was used to build a representative of each group [Knyazikhin et al., 1996]. Given the distribution of tree stems in the stand, the diameter of each tree, the entire sample site was generated (left panel). The right panel shows the spatial distribution of leaf area index L(x,y) at spatial resolution of 50 cm^2, i.e., distribution of the mean leaf area index L(x,y) taken over each of 50 by 50 cm ground cells.

In order to demonstrate the range of the influence of the neighboring environment we simulate two extreme situations for a small 40 by 40m sample stand bounded from below by a black surface (i.e., ρ_b in Eq. (12) is set to zero). Canopy structure of this stand is shown in Fig. 3. In the first case, we "cut" the forest surrounding the sample plot. The incoming solar radiation can reach the sides of the sample stand without experiencing a collision in this case. The boundary condition (Eq. (6)) with $\rho_b = 0$ and $q(\underline{r}_l, \underline{\Omega}) = c_T(\underline{r}_l)\delta(\underline{\Omega} - \underline{\Omega}_0) + d(\underline{r}_l, \underline{\Omega})$, $\underline{n}(\underline{r}_l) \bullet \underline{\Omega} < 0$, $\underline{r}_L \in \delta V_l$ can be used to describe photons penetrating into the canopy through the lateral surface. In the second case, we "plant" a forest of an extremely high density around the sample stand so that no solar radiation can penetrate into the stand through the lateral boundary δV_l. The lateral boundary condition (Eq. (6)) takes the form $I(\underline{r}_L, \underline{\Omega}) = 0$, $\underline{n}(\underline{r}_l) \bullet \underline{\Omega} < 0$, $\underline{r}_L \in \delta V_l$. The radiative regimes in a real stand usually vary between these extreme situations. For each situation, the boundary value problem (Eqs. (1) and (6)) was solved and a vertical profile of mean downward radiation flux density was evaluated. Figure 4 demonstrates downward fluxes normalized by the incident flux at noon on both a cloudy and clear sunny day. A downward radiation flux density evaluated by averaging the extinction coefficient $u_L G$ and area scattering phase function Γ over the 40 by 40m area first and then solving the radiative transfer equation is also plotted in this figure. One can see that the radiative regime in the sample stand is more sensitive to the lateral boundary conditions during cloudy days ($f_{dir}=0$). In both cases, a 3D medium transmits more radiation than those predicted by the 1D transport equation.

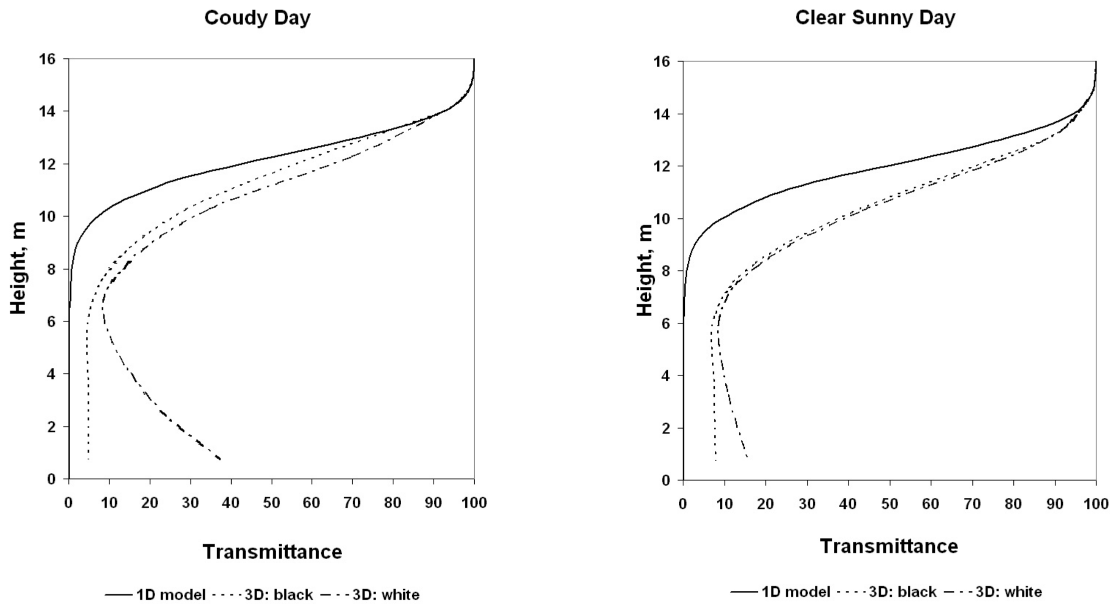


Figure 4. Vertical profile of the downward radiation flux normalized by the incident flux derived from the one-dimensional (1D model) and three-dimensional (3D: black and 3D: white) models on a cloudy day ($f_{dir}=0$) and on a clear sunny day ($f_{dir}=1$). Curves 3D: black correspond to a forest stand surrounded by the optically black lateral boundary, and curves 3D: white to an isolated forest stand of the same size and structure (from Knyazikhin et al., [1997]).

The following technique can be used to approximate the lateral boundary condition. One first calculates the radiative field in a vegetation canopy by solving a one-dimensional transport equation using average characteristics of canopy structure and optics over a given stand. Its solution, i.e. the vertical profile of the horizontally averaged radiation intensity, is then taken as the radiation penetrating through the lateral canopy boundary which, to some degree, accounts for photon interactions with both the stand and its neighboring environment. The size of an area impacted by such an approximation of the lateral boundary condition as a function of the adjoining vegetation and atmospheric conditions was studied by Kranigk [1996]. In particular, it has been shown that the "impacted area" consists of points being less than about 5m apart from the lateral boundary of a forest. Thus, the lateral boundary conditions can be expressed as

$$I(\underline{r}_l,\underline{\Omega}) = c_l(z)\delta(\underline{\Omega}-\underline{\Omega}_0)+i_l(z,\underline{\Omega}), \quad r_l=(x_l,y_l,z)\in\delta V_l, \quad \underline{n}(\underline{r}_l)\bullet\underline{\Omega}<0. \qquad (13)$$

Here $c_1(z)\delta(\underline{\Omega}-\underline{\Omega}_0)$ and i_1 are wavelength dependent mean intensities of the direct and diffuse radiation at depth z predicted with the 1D radiative transfer equation. In terms of notations used in Eq. (6), $\rho_B(\underline{r}'_b,\underline{\Omega}';\underline{r}_b,\underline{\Omega})=0$, $q(\underline{r}_b,\underline{\Omega})=c_T(z_1)\delta(\underline{\Omega}-\underline{\Omega}_0)+i_1(z_1,\underline{\Omega})$ if $\underline{r}_b$ belongs to δV_1. The lateral side effects, however, decrease with distance from this boundary toward the center of the domain. It has been shown that these lateral effects can be neglected when the radiation regime is analyzed in a rather extended canopy. A "zero" boundary condition for the lateral surface can then be used to simulate canopy radiation regime [Knyazikhin et al., 1997].

4. Decomposition of the Boundary Value Problem for Radiative Transfer Equation

The Green's function concept allows us to express the solution to the transport equation with arbitrary sources and boundary conditions as a superposition of the solutions of some basic subproblems. In this section we demonstrate this technique with an example for canopy-surface system. It will be shown that the three-dimensional radiative transfer problem with a reflecting boundary can be expressed as a superposition of the solutions of two radiative transfer sub-problems with purely absorbing boundaries ($\rho_0(\delta V)=0$). The first one is formulated for a vegetation canopy illuminated from above by the incident radiation and bounded from below by an absorbing surface. We term this sub-problem a "black soil problem." The second sub-problem, called "S problem," describes the radiative transfer in the same vegetation canopy which is illuminated from the bottom by anisotropic sources and bounded from above by a non-reflecting surface. Such a decomposition underlies the retrieval technique for operational producing global leaf area index from data provided by two instruments, the moderate resolution imaging spectroradiometer (MODIS) and multiangle imaging spectroradiometer (MISR), during the Earth Observing System (EOS) Terra mission ([Knyazikhin et al., 1998a,b], [Myneni et al., 2002]). The Green's function formalism described in Section 6 of Chapter 2 and operator notations introduced in Section 7 of Chapter 2 are required to follow this section.

Black Soil Problem. Consider the boundary value problem for the 3D radiative transfer equation in vegetation canopy $LI=SI+q$, $I^-=\mathcal{R}I^++q_B$ with the boundary scattering operator $\mathcal{R}$ and source q_B introduced in the previous section. We neglect lateral effects by assuming the zero boundary condition for the lateral surface δV_t. We represent a solution of the boundary value problem as the sum of two components, $I(\underline{r},\underline{\Omega})=I_{bs}(\underline{r},\underline{\Omega})+I_{rest}(\underline{r},\underline{\Omega})$. The first term describes intensity of radiation in the vegetation canopy bounded from below by a *non-reflecting surface* and satisfies

$$
\begin{aligned}
&LI_{bs}=SI_{bs},\\
&I^-_{bs}(\underline{r}_t,\underline{\Omega})=c_T\delta(\underline{\Omega}-\underline{\Omega}_0)+d(z_t,\underline{\Omega}),\\
&I^-_{bs}(\underline{r}_1,\underline{\Omega})=0,
\end{aligned}
$$

$$I^{-}_{bs}(\underline{r}_b,\underline{\Omega}) = 0. \tag{14a}$$

This is our first basic problem – the black soil problem.

Canopy-Surface Interaction. The function I_{rest} satisfies the radiative transfer equation

$$LI_{rest} = SI_{rest}$$

and the boundary conditions expressed as

$$\begin{aligned} I^{-}_{rest}(\underline{r}_t,\underline{\Omega}) &= 0, \\ I^{-}_{rest}(\underline{r}_l,\underline{\Omega}) &= 0, \\ I^{-}_{rest}(\underline{r}_b,\underline{\Omega}) &= \mathcal{R}\, I^{+}. \end{aligned} \tag{14b}$$

This boundary value problem describes radiation field due to the interaction between the underlying surface and the vegetation canopy. Unlike the black soil problem, I_{rest} depends on the solution of the "complete transport problem," i.e., $I = I_{bs} + I_{rest}$. And, therefore, requires further transformations to decompose it sub-problems with $\rho_0(\delta V) = 0$.

The lower boundary conditions can be rewritten as

$$I^{-}_{rest}(\underline{r}_b,\underline{\Omega}) = \frac{\mathcal{R}\, I^{+}}{T} T. \tag{15}$$

Here T is downward radiation flux density at the canopy bottom, i.e.,

$$T(r_b) = \int_{\underline{n}(\underline{r}_b)\bullet\underline{\Omega}'>0} I(\underline{r}_b,\underline{\Omega}')\left|\underline{\Omega}'\bullet n(\underline{r}_b)\right| d\underline{\Omega}', \quad \underline{r}_b \in \delta V_b. \tag{16}$$

Note that the ratio $\mathcal{R} I^{+}/T$ is a factor of π smaller than ground HDRF. A cosine-weighted integral of the ratio is the ground BHR

$$\rho_{eff}(\underline{r}_b) = \frac{\pi^{-1} \int_{\underline{n}(\underline{r}_b)\bullet\underline{\Omega}'>0} d\underline{\Omega}' \left|\underline{n}(\underline{r}_b)\bullet\underline{\Omega}'\right| I(\underline{r}_b,\underline{\Omega}') \int_{\underline{n}(\underline{r}_b)\bullet\underline{\Omega}<0} \rho_b(\underline{r}_b,\underline{\Omega}'\to\underline{\Omega})\left|\underline{\Omega}\bullet\underline{n}(\underline{r}_b)\right| d\underline{\Omega}}{\int_{n(r_b)\bullet\Omega>0} \left|\underline{\Omega}\bullet\underline{n}(\underline{r}_b)\right| I(\underline{r}_b,\underline{\Omega}')d\underline{\Omega}'}. \tag{17}$$

Here ρ_b is the BRF of the canopy ground (see Eq. (12)). It is clear that the ground BHR depends on ground reflective properties, vegetation canopy and radiation incident on the canopy upper boundary. For horizontally inhomogeneous vegetation canopies, the downward radiation flux T can vary significantly. However, it does not necessarily involve large variation in the BHR. As it follows from Eq. (17), its range of variation is given by

$$\inf_{\underline{n}(\underline{r}_b)\bullet\underline{\Omega}'>0} \frac{1}{\pi} \int_{n(r_b)\bullet\Omega<0} \rho_b(\underline{r}_b \underline{\Omega}' \to \underline{\Omega}) \left|\underline{\Omega} \bullet \underline{n}(\underline{r}_b)\right| d\underline{\Omega}$$

$$\le \rho_{eff}(\underline{r}_b)$$

$$\le \sup_{n(r_b)\bullet\Omega'>0} \frac{1}{\pi} \int_{n(r_b)\bullet\Omega<0} \rho_b(r_b\Omega' \to \Omega) \left|\Omega \bullet n(r_b)\right| d\Omega \tag{18}$$

If

$$\int_{\underline{n}(\underline{r}_B)\bullet\underline{\Omega}<0} \rho_b(\underline{r}_b, \underline{\Omega}' \to \underline{\Omega}) \left|\underline{n}(\underline{r}_b) \bullet \underline{\Omega}\right| \, d\underline{\Omega}$$

is independent on $\underline{\Omega}'$, the ground BHR becomes independent from I^+ and the ground BHR coincides with the ground DHR. Here we neglect the dependence of the ground BHR on the intensity of downward radiation at the canopy bottom within range given by the inequality, Eq. (18).

An effective ground anisotropy is another parameter used to characterize the canopy–ground interaction. This parameter, $d_b(\underline{r}_b, \underline{\Omega})$, is defined as

$$d_b(\underline{r}_b, \underline{\Omega}) = \frac{RI^+}{\rho_{eff} T} = \frac{1}{\rho_{eff}(\underline{r}_b)} \cdot \frac{\pi^{-1} \int_{2\pi+} \rho_b(\underline{r}_b, \underline{\Omega}' \to \underline{\Omega}) \left|\underline{n}(\underline{r}_b) \bullet \underline{\Omega}'\right| I(\underline{r}_b, \underline{\Omega}') \, d\underline{\Omega}'}{\int_{2\pi^-} \left|\underline{\Omega}' \bullet \underline{n}(\underline{r}_b)\right| I(\underline{r}_b, \underline{\Omega}') d\underline{\Omega}'} \tag{19}$$

Its cosine-weighted integral over downward directions is unity. In terms of these notations, the lower boundary conditions, Eq. (15), can be rewritten as

$$I^-_{rest}(\underline{r}_b, \underline{\Omega}) = \rho_{eff}(\underline{r}_b) d_b(\underline{r}_b, \underline{\Omega}) T(\underline{r}_b) \,. \tag{20}$$

We neglect variations in ρ_b and d_b due to variation in I^+. However, the downward radiation flux density $T(\underline{r}_b)$ is sensitive to both ground reflectance properties and radiation regime within the vegetation canopy. This variable must be carefully specified.

We use Eq. (28) of Chapter 2 to express I_{rest} in terms of the Green's function, namely,

$$I_{rest}(\underline{r}, \underline{\Omega}) = \int_{\delta V_b} d\underline{r}'_b \int_{\underline{n}(\underline{r}'_b)\bullet\underline{\Omega}'<0} d\underline{\Omega}' G_S(\underline{r}, \underline{\Omega}; \underline{r}'_b, \underline{\Omega}') \rho_{eff}(\underline{r}'_b) d_b(\underline{r}'_b, \underline{\Omega}') T(\underline{r}'_b) \,. \tag{21}$$

Substituting this equation into $I(\underline{r}, \underline{\Omega}) = I_{bs}(\underline{r}, \underline{\Omega}) + I_{rest}(\underline{r}, \underline{\Omega})$ results in

$$I(\underline{r},\underline{\Omega}) = I_{bs}(\underline{r},\underline{\Omega}) + \int_{\delta V_b} d\underline{r}'_b\, \rho_{eff}(\underline{r}'_b) T(\underline{r}'_b) J_S(\underline{r},\underline{\Omega};\underline{r}'_b), \tag{22a}$$

where

$$J_S(\underline{r},\underline{\Omega},\underline{r}'_b) = \int_{\underline{n}(\underline{r}'_b)\bullet\underline{\Omega}'<0} d\underline{\Omega}'\, G_S(\underline{r},\underline{\Omega};\underline{r}'_b,\underline{\Omega}')\, d_b(\underline{r}'_b,\underline{\Omega}'). \tag{22b}$$

is the intensity of radiation field at r generated by a point anisotropic source $d_b(\underline{r}'_b,\underline{\Omega}')\delta(\underline{r}_b - \underline{r}'_b)$ located at $\underline{r}'_b$. Substituting Eq. (22) into Eq. (16) one obtains an integral equation for T

$$T(\underline{r}_b) = T_{bs}(\underline{r}_b) + \int_{\delta V_b} G_d(\underline{r}_b,\underline{r}'_b)\rho_{eff}(\underline{r}'_b) T(\underline{r}'_b)\, d\underline{r}'_b. \tag{23}$$

Here T_{bs} is the downward radiation flux density at the lower boundary for the case of a black surface underneath the vegetation canopy, i.e.,

$$T_{bs}(\underline{r}_b) = \int_{\underline{n}(\underline{r}_b)\bullet\underline{\Omega}'>0} I_{bs}(\underline{r}_b,\underline{\Omega}') |\Omega' \bullet \underline{n}(\underline{r}_b)| d\underline{\Omega}'. \tag{24}$$

The kernel G_d is the downward radiation flux density at $\underline{r}_d$ due to the point anisotropic source $d_b(\underline{r}_b,\underline{\Omega}')\delta(\underline{r}_b - \underline{r}'_b)$,

$$G_d(\underline{r}_b,\underline{r}'_b) = \int_{\underline{n}(\underline{r}_b)\bullet\underline{\Omega}>0} d\underline{\Omega} |\underline{\Omega} \bullet \underline{n}(\underline{r}_b)| \int_{\underline{n}(\underline{r}'_b)\bullet\underline{\Omega}'<0} G_S(\underline{r}_b,\underline{\Omega};\underline{r}'_b,\underline{\Omega}')\, d_b(\underline{r}'_b,\underline{\Omega}') d\underline{\Omega}'. \tag{25}$$

Note that

$$I_S(\underline{r},\underline{\Omega}) = \int_{\delta V_b} d\underline{r}'_b \int_{\underline{n}(\underline{r}'_b)\bullet\underline{\Omega}'<0} G_S(\underline{r},\underline{\Omega};\underline{r}'_b,\underline{\Omega}')\, d_b(\underline{r}'_b,\underline{\Omega}') d\underline{\Omega}'. \tag{26}$$

is the intensity of radiation field in vegetation canopy generated by the isotropic homogeneous sources d_b located at the canopy bottom. It satisfies the equation $LI_S = SI_S$ and boundary condition $I_S^-(\underline{r}_t,\underline{\Omega}) = 0,\ I_S^-(\underline{r}_l,\underline{\Omega}) = 0,\ I^-(\underline{r}_b,\underline{\Omega}) = d_b(\underline{r}_b,\underline{\Omega})$. This is our second "basic problem" – the "S problem." It follows from this property that

$$\int_{\delta V_b} G_d(\underline{r}_b,\underline{r}'_b)\, d\underline{r}'_b = \int_{\underline{n}(\underline{r}_b)\bullet\underline{\Omega}>0} I_S(\underline{r}_b,\underline{\Omega}) |\underline{\Omega} \bullet \underline{n}(\underline{r}_b)| d\underline{\Omega}. \tag{27}$$

Thus integration of G_d over the lower boundary results in downward flux at $\underline{r}_b \in \delta$ which accounts for contribution from all anisotropic sources at the canopy bottom.

Equations (22) and (23) are basic equations which describe canopy-ground interaction. They are parameterized in terms of ground reflectance properties (the ground BHR and effective ground anisotropy) which are independent on the vegetation canopy; the radiation field in the vegetation canopy bounded at the bottom by a black surface (black soil problem) and radiation field in the vegetation canopy generated by anisotropic heterogeneous source d_b located at the surface underneath the canopy (S problem).

Decomposition Equations. Given $G_d(\underline{r}_b, \underline{r}'_b)$ one can resolve the integral equation (23) and substitute it into Eq. (21). As a results one obtains a solution to the three-dimensional radiative transfer problem with the reflecting lower boundary δV_t. The integral equation (23) allows for an analytical solution in the case of a horizontally homogeneous vegetation canopy bounded from below by a homogeneous Lambertian surface (i.e., the ground BRF is constant with respect to angular variable and points on the canopy bottom). The radiation fluxes T, T_{bs} become independent of horizontal coordinates x and y. The ground BHR is independent of on the intensity of downward radiation at the canopy bottom and coincides with the ground BRF, i.e., $\rho_{eff} = \rho_b$. The effective ground anisotropy is also independent on $I^+(\underline{r}_b, \underline{\Omega})$ and equal to $1/\pi$. The solution of Eq. (23) is then

$$T = \frac{T_{BS}}{1 - \rho_b R_S}, \tag{28}$$

where

$$R_S = \int_{\delta V_b} G_d(\underline{r}_b, \underline{r}'_b)\, d\underline{r}'_b$$

is the downward flux density at $\underline{r}_b \in \delta V_b$ generated by isotropic sources $1/\pi$ distributed over the lower boundary and is given by Eq. (27). This variable is independent of points on the canopy bottom and varies between 0 and 1. Substituting Eq. (26) into Eq. (22) and accounting for Eq. (28) one gets the following decomposition of the boundary value problem into solutions of the black soil and S problems

$$I(z, \underline{\Omega}) = I_{bs}(z, \underline{\Omega}) + \frac{\rho_b T_{BS}}{1 - \rho_b R_S} I_S(z, \underline{\Omega}). \tag{29}$$

In the case of horizontally inhomogeneous medium, however, Eq. (23) needs to be solved in order to decompose the solution of the boundary value problem. The following approximation to I_{rest} can be performed. Consider the ratio

$$R_S(\underline{r}_b) = \frac{\int\limits_{\delta V_b} G_d(\underline{r}_b, \underline{r}'_b)\rho_{eff}(\underline{r}'_b)T(\underline{r}'_b)\,d\underline{r}'_b}{\rho_{eff}(\underline{r}_b)T(\underline{r}_b)}, \tag{30}$$

which is the BHR calculated for the vegetation canopy illuminated by anisotropic sources $\rho_{eff}(\underline{r}'_b)T(\underline{r}'_b)d_b(\underline{r}'_b,\underline{\Omega}')$ from below. We term the ratio, Eq. (30) a "bottom-of-canopy reflectance". For horizontally inhomogeneous vegetation canopies, the downward radiation flux T can vary significantly. However, it does not necessarily involve large variation in R_S. A theoretical explanation of this result can be found in the linear operator analysis [Krein, 1967] and, specifically, in its applications to radiative transfer theory ([Knyazikhin, 1991]; [Kaufmann et al., 2000]; [Zhang et al., 2002]; [Lyapustin and Knyazikhin, 2002]). One of the theorems of the operator theory states that for a continuous positive linear operator B, minimum, m_n, and maximum, M_n, values of the function $\eta_n = \sqrt[n]{B^n u}$ converge to the maximum eigenvalue, $\rho(B)$, of the operator B from below and above for any arbitrarily chosen positive function u, i.e., $m_n \le \rho(B) \le M_n$ and $M_n - m_n$ tends to zero as n tends to infinity. For example, for the problem of atmospheric radiative transfer over common land–surface types, including vegetation, soil sand, and snow, the proximity of m_n and M_n to a high accuracy holds at $n \ge 2$ [Lyapustin and Knyazikhin, 2002]. Here the numerator in Eq. (30) can be treated as a positive integral operator B with a kernel $G_d(\underline{r}_b,\underline{r}'_b)$. For n=1, the ratio (30) varies within an interval [m_1, M_1] around the maximum eigenvalue which is independent of $\rho_{eff}T$. We will demonstrate the eigenvalue technique with an example for canopy spectral response to the incident radiation in next section. Here we assume that an acceptable accuracy takes place at n=1 and we replace $R_S(\underline{r}_b)$ with the maximum eigenvalue $\overline{R}_S$ of an integral operator with the kernel $G_d(\underline{r}_b,\underline{r}'_b)$.

Equation (23) can be rewritten as

$$T(\underline{r}_b) = T_{bs}(\underline{r}_b) + \overline{R}_S\,\rho_{eff}(\underline{r}_b)T(\underline{r}_b). \tag{31}$$

Solving this equation for T and substituting it into Eq. (21) one gets

$$I(\underline{r},\underline{\Omega}) = I_{bs}(\underline{r},\underline{\Omega}) + \int\limits_{\delta V_b} \frac{\rho_{eff}(\underline{r}'_b)T_{bs}(\underline{r}'_b)}{1-\overline{R}_S\rho_{eff}(\underline{r}'_b)} J_S(\underline{r},\underline{\Omega},\underline{r}'_b)\,d\underline{r}'_b, \tag{32}$$

where $J_S(\underline{r},\underline{\Omega},\underline{r}'_b)$ is given by Eq. (22b). Further simplification can be done by either replacing $J_S(\underline{r},\underline{\Omega},\underline{r}'_b)$ or the ratio in the integral term of Eq. (32) with a mean value over the canopy bottom. In the former case we have

$$I(\underline{r},\underline{\Omega}) = I_{bs}(\underline{r},\underline{\Omega}) + \frac{\overline{\rho}_{eff}\overline{T}_{bs}}{1-\overline{R}_S\overline{\rho}_{eff}} I_S(\underline{r},\underline{\Omega}). \tag{33}$$

where I_S is the solution of the S problem and given by Eq. (26); $\bar{\rho}_{eff}$ and $\bar{T}_{bs}$ are mean values of the ground BHR and downward flux density for the black soil problem over the lower boundary δV_b, respectively. Thus we have parameterized the solution of the transport problem in terms of the ground BHR and solutions of the "black-soil problem," I_{bs}, and "*S* problem," I_s. The solution of the "black-soil problem" depends on Sun-view geometry, canopy architecture, and spectral properties of the leaves. The "*S* problem" depends on spectral properties of the leaves and canopy structure only. Substituting Eq. (33) into definitions of the surface HDRF, BHR, BRF, BRDF, and DHR, one obtains the following decompositions of these reflectance quantities

$$R = R_{bs}(\underline{r},\underline{\Omega}) + \frac{\bar{\rho}_{eff} t_{bs}}{1 - \bar{R}_S \bar{\rho}_{eff}} T_S(\underline{\Omega}), \tag{34}$$

$$A = A_{bs} + \frac{\bar{\rho}_{eff} t_{bs}}{1 - \bar{R}_S \bar{\rho}_{eff}} t_S. \tag{35}$$

Here R_{bs} and A_{bs} are HDRF and BHR calculated for a vegetation canopy bounded from below by a black surface; t_{bs} is the canopy transmittance defined as the ratio of the mean downward flux $< T_{bs} >_b$ at the canopy lower boundary to mean incident irradiance $< F_{dir}^{\downarrow} + F_{dif}^{\downarrow} >_0$ where the angle brackets denotes the mean over the upper (subscript "t") or lower boundary (subscript "b") of the parallelepiperd V. If the ratio f_{dir} of the mono-directional to the total incident radiation flux is unity, the HDRF and BHR become BRF and DHR, respectively. The transmittance quantity $T_s(\underline{\Omega})$ is the ratio of the mean radiance, $< I_s >_0$, leaving the top of the vegetation canopy to mean radiance reflected from an ideal Lambertian surface into the same beam geometry and illuminated by anisotropic sources located at the canopy bottom, i.e.,

$$T_S(\underline{\Omega}) = \frac{< I_S(\underline{\Omega}) >_0}{\frac{1}{\pi} \int\limits_{\underline{n}(\underline{r}_b) \bullet \underline{\Omega} < 0} < d_b(\underline{\Omega}') >_b | \underline{n}(\underline{r}_b) \bullet \underline{\Omega}' | d\underline{\Omega}'}. \tag{36}$$

Finally, t_s is the transmittance of the vegetation canopy illuminated from below by anisotropic sources, i.e., the ratio of the mean irradiance exitance to the mean irradiance of the radiation incident on the canopy from below,

$$t_S(\underline{\Omega}) = \frac{\int\limits_{\underline{n}(\underline{r}_b) \bullet \underline{\Omega}' > 0} < I_S(\underline{\Omega}) >_0 | \underline{n}(\underline{r}_b) \bullet \underline{\Omega}' | d\underline{\Omega}'}{\frac{1}{\pi} \int\limits_{\underline{n}(\underline{r}_b) \bullet \underline{\Omega} < 0} < d_b(\underline{\Omega}') >_b | \underline{n}(\underline{r}_b) \bullet \underline{\Omega}' | d\underline{\Omega}'} = \frac{1}{\pi} \int\limits_{\underline{n}(\underline{r}_b) \bullet \underline{\Omega}' > 0} T_S(\underline{\Omega}') | \underline{n}(\underline{r}_b) \bullet \underline{\Omega}' | d\underline{\Omega}'. \tag{37}$$

Thus, the three-dimensional radiation field can be expressed in terms of ground reflectance properties which are independent on the medium; the radiation field in the medium bounded at the bottom by a black surface (black soil problem); and the radiation field in the medium generated by anisotropic heterogeneous sources located at the surface underneath the medium (S problem). Solutions to the black soil and S problems are surface independent parameters since no multiple interaction of radiation between the medium and underlying surface is possible and, therefore, have intrinsic canopy information. This decomposition of underlay the retrieval technique for operational producing global leaf area index from data provided by the MODIS and MISR instruments.

Problem Sets

- **Problem 1**. Show that HDRF=BRF=BHR=DHR for Lambertian surfaces.
- **Problem 2**. Show the validity of Eq. (5).
- **Problem 3**. Derive the BRF and DHR for a mirror.
- **Problem 4**. Let a vegetation canopy located in the parallelepiped *V* is isotropically illuminated from above and bounded from below and lateral sides by a black surface, i.e., $B(\underline{r}_B,\underline{\Omega}) = 1/\pi$ if $\underline{r}_B \in \delta V_t$ and $B(\underline{r}_B,\underline{\Omega}) = 0$, otherwise. Prove that the BHR is less than 1. Use Theorem I.1.
- **Problem 5**. Prove that the BHR is an increasing function with respect to single scattering albedo. Do not use the assumptions of the Problem 4. Use Theorem I.1.
- **Problem 6**. Derive Eq. (11). Explain why Eq. (11) is not used in remote sensing of vegetation.
- **Problem 7**. Show that $\rho_{B,V}$ for the boundary conditions (Eqs (7), (10) and (13)) is given by
$$\rho_{B,V} = \sup_{\underline{r}'\in\delta V_t;\underline{\Omega}'\bullet\underline{n}(\underline{r}')>0} \frac{1}{\pi} \int_{\underline{\Omega}'\bullet\underline{n}(\underline{r}')<0} \rho_b(\underline{r}',\underline{\Omega}' \to \underline{\Omega})\left|\underline{\Omega} \bullet \underline{n}(\underline{r}')\right| d\underline{\Omega}.$$
This is albedo of the surface underneath the canopy.
- **Problem 8**. Show that $\rho_{eff}(r_b) \le \rho_0(\delta V)$.
- **Problem 9**. Demonstrate the validity of Eq. (19).
- **Problem 10**. Find the effective ground reflectance for the case of Lambertian surface underneath the canopy.
- **Problem 11**. Show that
$$\int_{2\pi^-} d_b(\underline{r}_b,\underline{\Omega})\left|\underline{\Omega} \bullet \underline{n}(\underline{r}_b)\right| d\underline{\Omega} = 1.$$
- **Problem 12**. Find the effective ground anisotropy for the case of Lambertian surface underneath the canopy.
- **Problem 13**. Show that the function
$$I_d(\underline{r},\underline{\Omega}) = \int_{\delta V_b} d\underline{r}'_b \int_{\underline{n}(\underline{r}'_b)\bullet\underline{\Omega}'<0} G_S(\underline{r},\underline{\Omega};\underline{r}'_b,\underline{\Omega}')\, d_b(\underline{r}'_b,\underline{\Omega}')\, d\underline{\Omega}'$$
is the intensity of radiation field in vegetation canopy generated by the anisotropic heterogeneous source db located at the canopy bottom. It follows from this property that
$$\int_{\delta V_b} G_d(\underline{r}_b,\underline{r}'_b)\, d\underline{r}'_b = \int_{\underline{n}(\underline{r}_b)\bullet\underline{\Omega}>0} I_d(\underline{r}_b,\underline{\Omega})\left|\underline{\Omega} \bullet \underline{n}(\underline{r}_b)\right| d\underline{\Omega}$$

- **Problem 14**. Show that for a horizontally homogeneous vegetation canopy bounded from below by a homogeneous Lambertian surface the downward flux density,

$$R_S = \int_{\delta V_b} G_d(\underline{r}_b, \underline{r}'_b)\, d\underline{r}'_b$$

does not depend on r_b and varies between 0 and 1.
- **Problem 15**. Show that for a horizontally homogeneous vegetation canopy bounded from below by a homogeneous Lambertian surface Eq. (26) does not depend on horizontal variables x and y.

References

Bell, G. I., and S. Glasstone (1970). *Nuclear Reactor Theory*, Van Nostrand Reinholt, New York.

Case, K. M., and P.F. Zweifel (1967). *Linear Transport Theory*, Addison-Wesley Publishing Company, Reading, Massachusetts.

Diner, D.J., J.V.Martonchik, C. Borel, S.A.W. Gerstl, H.R. Gordon, Y. Knyazikhin, R. Myneni, B. Pinty, and M.M. Verstraete (1998) MISR: Level 2 surface retrieval algorithm theoretical basis, *JPL Internal Doc. D-11401, Rev. C*, Calif. Inst. of Technol., Jet Propul. Lab., Pasadena.

Germogenova, T.A. (1986). *The Local Properties of the Solution of the Transport Equation* Nauka, Moscow (in Russian).

Knyazikhin, Y., G. Miessen, O. Panfyorov, and G. Gravenhorst (1997). Small-Scale Study of Three-Dimensional Distribution of Photosynthetically Active Radiation in a Forest, *Agric. For. Meteorol*., **88**, 215-239.

Kranigk, J. (1996). *Ein Model für den Strahlungstransport in Fichtenbeständen*, Cuvillier, Göttingen, Germany.

Krein, S.G (ed.) (1972). *Functional Analysis*, Groningen, Wolters-Noordhoff.

Martonchik, J.V., C.J. Bruegge, and A.H. Strahler (2000). A review of reflectance nomenclature used in remote sensing, *Remote Sens. Rev.*, **19,** 9-20.

Monteith, J.L. and M.H. Unsworth (1990). *Principles of Environmental Physics*. E.Arnold, London.

Nicodemus, F.E., J.C. Richmond, I.J. Hsia, I.W. Ginsberg, and T. Limperis (1997). Geometrical consideration and nomenclature for reflectance, NBS Monograph, 160, National Bureau of Standards, US Department of Commerce, Washington, D.C

Pokrowski, G.I. (1929). Über die Helligkeitsverteilung am Himmel. *Physikal. Zeitschr*., **30**, 697-700.

Richards, P. I. (1959). *Manual of Mathematical Physics*. A Pergamon Press Book, New York.

Ross, J. (1981). *The Radiation Regime and Architecture of Plant Stands*. Dr. W. Junk Publishers, Delft, The Netherlands.

Shabanov, N.V., Y. Knyazikhin, F. Baret, and R.B. Myneni (2000). Stochastic modeling of radiation regime in discontinuous vegetation canopy, *Remote Sens. Environ*, **74**, 125-144.

Vladimirov, V.S. (1963). Mathematical problems in the one-velocity theory of particle transport, *Tech. Rep. AECL-1661*, At. Energy of Can. Ltd., Chalk River, Ontario.

Further Readings

Davis, A. and Y. Knyazikhin, *A Primer in Three-Dimensional Radiative Transfer*, in A. Davis and A. Marshak (eds.) (2006). "Three-dimensional Radiative Transfer in the Cloudy Atmosphere," Springer Verlag (in print).

Bell, G. I., and S. Glasstone (1970). *Nuclear Reactor Theory*. Van Nostrand Reinholt, New York.

Case, K. M., and P.F. Zweifel, (1967). *Linear Transport Theory*, Addison-Wesley Publishing Company, Reading, Mass.