

Why is Language Vague?¹

Barton L. Lipman

Department of Economics
Boston University

email: blipman@bu.edu

Current Draft

February 2025

¹I thank Eddie Dekel, Marie–Odile Yanelle, and Joel Sobel for useful comments. I wrote the first version of this paper in summer 1999, returning to it periodically over the years as I considered what to do with it. The key arguments and ideas here are unchanged from the 1999 version. Aside from a little tinkering, this paper is identical to the 2009 version but with the addition of a brief postscript to clarify a few points and comment on some more recent developments. I thank Joel Sobel for suggesting this approach to making the paper suitable for the present.

Abstract

I don't know.

“Is he — is he a tall man?”

“Who shall answer that question?” cried Emma. “My father would say, ‘Yes’; Mr. Knightly, ‘No’; and Miss Bates and I, that he is just the happy medium.”

— From *Emma* by Jane Austen.

1 Introduction

When one thinks about language as spoken by real people on a day-to-day basis, it is hard to ignore the fact that much of what is said is vague. Consider, for example, the word “tall.” There is no precise, known height which defines the line between a person who is tall and a person who is not. Why do we use a language in which such terms are so prevalent? Why don’t we simply adopt as a definition that “tall” will mean above, say, 6 foot 2? We could even adopt a context-specific definition, saying for example that “tall” for a newborn means above 15 inches, while “tall” for a professional basketball player means above 6 foot 10.

In this paper, I will argue that we cannot explain the prevalence of vague terms in natural language without a model of bounded rationality which is significantly different from anything in the existing literature. In a nutshell, the argument is that any model along existing lines will imply that a precise language like the one described above would Pareto dominate the vague language we see in every society in history. Of course, it seems rather far-fetched to conclude that we have simply tolerated a world-wide, several-thousand-year efficiency loss.¹ Further, even a moment’s reflection will suggest that it is easier to speak when one is allowed to use vague language than it would be if such language were banned. Hence this dominance surely tells us that there is something wrong with the model, not the world.

First, let me be more precise, though still informal, about what I mean by vagueness. Following Sainsbury [1996],² I will say that a word is *precise* if it describes a well-defined set of objects. By contrast, a word is *vague* if it is not precise. Hence the alternative definition given above for “tall” (with or without context specificity) would make this term precise, whereas it is vague in its current usage. While the conceptual distinction

¹Though I acknowledge that living in a country which has never managed to adopt the metric system might undercut my argument.

²A number of the papers cited here are contained in Keefe and Smith [1996], an excellent introduction to the philosophy literature on this topic. I refer to the versions of these papers in Keefe and Smith rather than the original versions.

here should be clear, as we will see, giving it formal content in a model is not a simple matter. Some readers may prefer to interpret the main claim of this paper as being that we cannot adequately give this idea formal meaning in something like a standard game-theoretic model.

I emphasize that vagueness (as I use the term) is *not* the same thing as less than full information. To say that a person's height is above six feet is precise in the sense that it defines a set of people unambiguously. It is less informative than saying that a person's height is 6 foot 2, but it is *not* vague as I use the term.

The classic illustration of vagueness is what is referred to in the philosophy literature as the *sorites paradox*. For one version of this paradox, note that two facts seem clear about the way people use the word "tall." First, anyone whose height is 10 feet is tall. Second, if one person is tall and a second person's height is within 1/1000 of an inch of the first, then the second person is tall as well. But then working backward from 10 feet, we eventually reach the absurd conclusion that a person whose height is 1 inch is tall. Of course, the source of the difficulty is the vagueness of the word "tall." Because there is no fixed set of heights which "tall" corresponds to, no line which separates "tall" from not, one has the feeling that a small change should not matter. Of course, many small changes add up to a big change, so this is not consistent.

Many other words are vague in this sense. Some words which can easily yield a sorites paradox are "bald," "red" (imagine a sequence of objects moving continuously from red to orange to yellow), "thin," "child" (imagine a sequence of people, each one second older than the previous), "many," and "probably." Some philosophers have constructed less obvious sorites paradoxes for "tadpole," "chair," and other seemingly clear-cut terms.

Many of these terms would be difficult to redefine in a precise way. On the other hand, many could be given such redefinitions. As noted above, "tall" could be defined to be over a specific height, "bald" could be defined by a specific fraction of the skull covered by hair, etc.³ In such cases, why is the precise redefinition not adopted?

While it seems clear that it would be more difficult to converse in such a precise language, it is not obvious why. Some philosophers of language have tried to address this point. For example, Wright [1996, page 154] observes that "the utility and point of the classifications expressed by many vague predicates would be frustrated if they were supplied with sharp boundaries." Sainsbury [1996, page 251] more colorfully asks of vagueness: "is it so obvious that it is a Bad Thing, given the extent to which the throbbing centres of our lives appear to be describable only in vague terms?" With relatively few

³Of course, sometimes we do develop such definitions for legal purposes, most notably in the case of the word "child." On the other hand, it is clear that much common usage of this word outside the legal context is not based on such a precise definition.

exceptions, however, they have not explained what exactly it is about vague terms that make them preferred to their precise analogues.

In the next section, I give a simple but reasonably general model to analyze this question. Using what would seem to be the only available options for defining vagueness in this model, I show that the vague terms are Pareto dominated by precise ones in a certain sense. While it is very simple, the model illustrates that many seemingly obvious reasons why language is vague cannot be formalized using anything close to a standard model. I will argue that in some of these cases, the fault is with the model, not the argument for the optimality of vagueness. In Section 3, I argue that explaining the value of vagueness requires a new kind of model of bounded rationality. I suggest that such a model can potentially formalize some of the arguments in Section 2 which seem correct but are not consistent with the model.

2 The Suboptimality of “Vagueness”

In this section, I give a simple model and use this to show why many seemingly obvious advantages of vague terms do not fit into any standard model.

Consider the following two player game, a version of the standard Crawford–Sobel [1982] sender–receiver game.⁴ Player 1 observes h , a random draw from the set H with distribution function F . For example, he may observe the height of a person (which the letter h is intended to suggest). He then chooses a message m from a set M . 2 observes this message but not the value of h and then chooses an action a from a set A . I assume that messages themselves are costless (think of a message as 1 making a verbal statement to 2). Also, it is clear that if the objective functions of the agents differ, then 1 may wish to hide information from 2 and hence may wish to be vague in some sense. This does not seem to be the primary reason language is vague, so I rule this out by assuming that the two agents have the same utility function u over (h, a) pairs.⁵

A pure strategy for player 1 is a function $s_1 : H \rightarrow M$. A mixed strategy for 1 is a probability distribution σ_1 on the set of pure strategies.⁶ A pure strategy for player 2 is a function $s_2 : M \rightarrow A$, while a mixed strategy is a probability distribution σ_2 over the set of 2's pure strategies.

⁴One can also view this game as a generalization of Dow [1991].

⁵De Jaeger [2003] and Blume and Board [2014] show that noisy communication channels, interpreted as vagueness, can be useful in promoting information transmission when the agents have different preferences. While these results show a very intriguing role for vagueness, they do not explain the prevalence of vagueness in situations of common interests.

⁶It is more common to work with behavior strategies. Of course, it is equivalent to work with mixed strategies and this is more convenient for my purposes.

Clearly, if there are as many messages as there are possible values of h , the best equilibrium for 1 and 2 is for 1 to tell 2 the exact value of h . That is, 1's strategy should be invertible. Trivially, vagueness is suboptimal in such a model.

While this assumption is often viewed as natural, I think it is not particularly realistic as a model of language. Consider the case where h is the height of some individual. The set of possible heights presumably is uncountable, but the set of words we can use to state a height (in, say, less than a century) is finite. Even if one does not accept this view, I will argue below that models of bounded rationality in the literature can only be used to explain vagueness if vagueness is an implication of M being too small. Hence demonstrating that vagueness is suboptimal even when M is small is both less trivial and more important to the argument I wish to make. I make no assumptions about the size of M , but consider the interesting case to be where M is finite.

What would it mean to have a vague language in equilibrium? Note that any pure strategy for player 1 would give a precise language, not a vague one. A pure strategy specifies a message as a function of h . Given any message, then, 2 knows which set of h 's lead to this message. In this sense, the message has a precise meaning: it corresponds to a particular set of h 's.

The obvious way to try to obtain vagueness, then, would be for player 1 to use a (nondegenerate) mixed strategy. With such a randomization, player 2's interpretation of a given message would be that 1 is more likely to use this message to describe some h 's than others. In this sense, 1's message is vague. (I discuss an alternative formulation below.)

However, the following theorem demonstrates that this version of "vagueness" is suboptimal under very weak conditions. First, some notation: For any pair of mixed strategies, (σ_1, σ_2) , let $V(\sigma_1, \sigma_2)$ be the expected utility of player 1 (which equals the expected utility of 2) if these strategies are played. I will write $V(\sigma_1, \sigma_2) = V(s_1, \sigma_2)$ when σ_1 is a degenerate mixed strategy with probability 1 on the pure strategy s_1 and similarly for strategies of player 2. Let

$$V^* = \sup_{(\sigma_1, \sigma_2)} V(\sigma_1, \sigma_2).$$

Theorem 1 *If there is a pair of strategies (σ_1, σ_2) such that the supremum is attained, then every pair of pure strategies (s_1, s_2) in the support of (σ_1, σ_2) is a pure Nash equilibrium in which V^* is the expected payoff.*

Put differently, vagueness *cannot* have an advantage over specificity and, except in unusual cases, will be strictly worse.

The proof is almost trivial. Let (σ_1^*, σ_2^*) be a pair of strategies satisfying $V(\sigma_1^*, \sigma_2^*) = V^*$. Obviously, (σ_1^*, σ_2^*) is a Nash equilibrium — any deviation by either player (weakly) reduces V and hence the payoff to both players. Since σ_1^* is an optimal strategy for player 1, it must yield the same payoff as any⁷ pure strategy in its support. Hence for any such pure strategy s_1 , we have $V^* = V(s_1, \sigma_2^*)$. Again, this must be a Nash equilibrium since any deviation by either player leads to a weakly lower payoff for both. Again, since σ_2^* is optimal for 2, then for any pure strategy s_2 in its support, we have $V^* = V(s_1, s_2)$. By the same argument as before, (s_1, s_2) is a pure Nash equilibrium. ■

Remark 1 This result is not much more than Corollary 2.2 of Monderer and Shapley [1996].

Remark 2 One could give a similar result for the case where the supremum is not achievable. It is not hard to see that for any pair of strategies (σ_1, σ_2) achieving a payoff within ε of V^* , there is (s_1, s_2) in the support of (σ_1, σ_2) which is a pure ε -Nash equilibrium with a payoff within ε of V^* . A proof: Fix (σ_1, σ_2) with payoff within ε of V^* . Obviously, this must be a ε -Nash equilibrium. Player 1's payoff is, of course, a convex combination of the payoffs he gets from his various pure strategies. Hence there must be a pure strategy s_1 such that $V(s_1, \sigma_2) \geq V(\sigma_1, \sigma_2)$. Clearly, this yields both players payoffs within ε of V^* and so is a ε -Nash equilibrium. A similar argument for player 2 completes the proof.

Remark 3 These games often have no (nontrivial) mixed equilibria, even if we drop the assumption that the players have the same preferences over (h, a) pairs. For example, the original Crawford–Sobel [1982] model assumes that player 2's payoff function is strictly concave in his own action and that the set of actions is convex. Given this, 2's best reply is always pure. Given this, Crawford and Sobel's assumptions on 1's payoffs ensure that the set of types who are indifferent between inducing two different actions is of measure zero, implying that 1 (essentially) does not randomize.

One might object to my identification of vagueness with mixed strategies. While randomization by player 1 does give one way to formalize the idea that 2 doesn't know precisely what 1 means, other approaches might be possible. I explain a different approach here and return to a related alternative later. To see the idea, suppose $H = \{L, M, R\}$ and $M = \{m_1, m_2\}$. Suppose that L and M are “the same” in the sense that they induce the same preferences over actions. That is, letting u denote the utility function for 1 and 2 over (h, a) pairs, we have $u(L, a) > u(L, a')$ if and only if $u(M, a) > u(M, a')$. Suppose we have an equilibrium in which 1's strategy is $m(L) = m(R) = m_1$ and $m(M) = m_2$.

⁷Or almost any if the distribution is not discrete.

What does player 2 learn when he receives message m_1 ? In one sense, his information is precise: the signal received by 1 was either L or R . On the other hand, one could interpret this differently. One could say that what is really relevant is whether the signal is R or in $\{L, M\}$. In this sense, m_1 conveys noisy information. It lowers the probability of $\{L, M\}$ but does not refute it.

This kind of vagueness cannot be optimal either. To state this more precisely, for each h , define a preference relation over A by $a \succ_h a'$ if and only if $u(h, a) > u(h, a')$. Say that h and h' are equivalent, denoted $h \sim^* h'$ if they induce the same preferences over actions — that is, if $\succ_h = \succ_{h'}$. If h and h' are equivalent, then, neither player 1 nor player 2 care about which of these two signals is received.⁸ In light of this, I will say that strategy σ_1 induces a precise language if it is pure⁹ and $\sigma_1(h) = \sigma_1(h')$ whenever $h \sim^* h'$. Otherwise, I will say that σ_1 induces a vague language.

Theorem 2 *Suppose there is a pair of strategies (σ_1, σ_2) such that $V(\sigma_1, \sigma_2) = V^*$. Then there must be an equilibrium (σ'_1, σ'_2) which also achieves payoff V^* with the property that σ'_1 induces a precise language.*

Proof. From the previous theorem, we know that given the condition stated, there is a pure strategy equilibrium achieving V^* . So fix such an equilibrium, say (s_1, s_2) . If s_1 does not induce a precise language, then, by definition, we must have some h and h' with $s_1(h) \neq s_1(h')$ even though $h \sim^* h'$. Note that for any such h and h' , it must be true that $u(h, s_2(s_1(h))) = u(h, s_2(s_1(h')))$. That is, player 1 must be indifferent between sending the message $s_1(h)$ or $s_1(h')$ when the signal is either h or h' — otherwise, he'd use the better of the two in response to either signal.

In light of this, construct a new strategy \hat{s}_1 as follows. Fix a subset of H , say H^* , with the property that for every $h \in H$, there is exactly one $h^* \in H^*$ with $h \sim^* h^*$. That is, H^* takes one representative from each equivalence class under \sim^* . Let $\hat{s}_1(h^*) = s_1(h^*)$ for all $h^* \in H^*$. For any other h , define $\hat{s}_1(h)$ to equal $\hat{s}_1(h^*)$ for that $h^* \in H^*$ with $h \sim^* h^*$. Clearly, \hat{s}_1 induces a precise language. (If s_1 already induced a precise language, $\hat{s}_1 = s_1$.)

Note also that $V(\hat{s}_1, s_2) = V(s_1, s_2) = V^*$ since \hat{s}_1 and s_1 differ only in which of several equally good messages to send. Hence, by definition of V^* , there cannot be any

⁸One might object to this statement, since I have defined h and h' to be equivalent if they induce the same preferences over A , not over lotteries over A . In other words, risk attitudes could differ between h and h' even if $h \sim^* h'$. As we will see, such a difference will be irrelevant.

⁹It is worth noting that one could equally well consider random precise languages. For example, if σ puts all probability on one set of messages when $h \in H_1$ and all probability on a disjoint set of messages when $h \in H_2$, this language precisely identifies whether h is in H_1 or H_2 . It is straightforward to extend Theorem 2 to allow a language that is precise in this sense.

s'_2 with $V(\hat{s}_1, s'_2) > V(\hat{s}_1, s_2)$. That is, (\hat{s}_1, s_2) is a Nash equilibrium. ■

An Example.

1 must describe Mr. X to 2 who must pick him up at the airport. 1 knows X's height; 2 doesn't. Height is continuously distributed on the interval $[0,1]$ with density function f and distribution function F , independently across people. There will be n people at the airport in addition to Mr. X. 2 must pick one of these $n + 1$ people and ask if he is X. If he guesses right, he and 1 get a payoff of 1. Otherwise, they both get 0. 2 cannot observe the exact heights of the people at the airport, but does observe relative heights. That is, he sees the height ranking of each person.

What is the optimal language when $\#M = 2$? Surprisingly, the answer is independent of n and depends on F only through its median. It is for 1 to say m_1 when X's height is below the population median and m_2 otherwise. 2's strategy is to try the shortest person in the first case and the tallest in the second. In fact, this is the unique equilibrium in which 2 uses a pure strategy.

To see this, let 2's strategy be to choose the $(k + 1)^{\text{th}}$ tallest person in response to one word and the $(k + d + 1)^{\text{th}}$ tallest in response to the other, where $d > 0$. It is not difficult to show that given k and d , there is a unique c with the property that 1's payoff to sending the first word exceeds his payoff to sending the second iff $h \in [0, c]$. Hence given any pure strategy by 2, 1's best reply will always partition $[0, 1]$ into two intervals. Therefore, we know that the language will have 1 send one message if $h \in [0, c]$ and the other if $h \in (c, 1]$ for some c .

Consider 2's best reply if he receives the message corresponding to $(c, 1]$. His payoff to choosing the $(k + 1)^{\text{th}}$ tallest person is proportional to

$$\Pr[h \in (c, 1] \text{ and } k \text{ people taller}] = \binom{n}{k} \int_c^1 [1 - F(h)]^k [F(h)]^{n-k} f(h) dh.$$

Define this expression to be $\varphi(k)$. For $k \geq 1$, we can integrate by parts to obtain

$$\begin{aligned} \varphi(k) = \binom{n}{k} & \frac{1}{n-k+1} [1 - F(h)]^k [F(h)]^{n-k+1} \Big|_c^1 \\ & + \binom{n}{k} \int_c^1 \frac{k}{n-k+1} [1 - F(h)]^{k-1} [F(h)]^{n-k+1} f(h) dh. \end{aligned}$$

But $F(1) = 1$ and

$$\binom{n}{k} \frac{k}{n-k+1} = \binom{n}{k-1},$$

so

$$\varphi(k) = \varphi(k-1) - \binom{n}{k} \frac{1}{n-k+1} [1 - F(c)]^k [F(c)]^{n-k+1}.$$

Because the term being subtracted on the right-hand side must be positive, we see that $\varphi(k)$ is decreasing in k . Hence the optimal choice for 2 is $k = 0$ — that is, to choose the tallest person. An analogous argument shows that when 2 conditions on $[0, c]$, the optimal choice is the shortest person.

Given these options, it is easy to see that 1 prefers 2 to pick the tallest person iff $[F(h)]^n > [1 - F(h)]^n$ or $F(h) > 1/2$. Thus we must have c equal to the median height. Because this is the only pure strategy equilibrium, Theorem 1 implies that it gives the optimal language.

This game may have mixed equilibria as well. For example, if the distribution is uniform and $n = 2$, there is an equilibrium where 1 sends m_1 if $h \in (.5 - (\sqrt{3}/6), .5 + (\sqrt{3}/6))$ and m_2 otherwise. In this equilibrium, 2 chooses the middle person when receiving m_1 and randomizes between the tallest and shortest when receiving m_2 . The expected payoff in this equilibrium is about .43, while it is .583 in the pure equilibrium. ■

This simple model illustrates my claim that many seemingly obvious advantages of vague terms cannot be made part of anything like a standard model. Perhaps the most obvious argument for the usefulness of vague terms is that one cannot observe the height of an individual precisely enough to be sure in all cases of how to classify someone in such a precise language. But there is an easy counterargument: Suppose I cannot observe height exactly. Then I should form subjective beliefs regarding which category an individual falls into. The theorem above then says that the optimal language will be precise about such probability distributions. More formally, reinterpret the h above not as height but as a signal of height. The above result then says that it is optimal to partition the set of signals in a precise way. This is equivalent to partitioning the set of induced beliefs and communicating them precisely.

An obvious reply to this is that real people do not form precise subjective beliefs. I believe this is true, but it cannot fit into anything like a standard model of bounded rationality. For example, it is not enough to replace probability distributions with nonadditive probabilities or sets of probability beliefs. If agents have nonadditive probabilities, then it is surely optimal to partition the set of such beliefs precisely and analogously for sets of probabilities.

Another seemingly obvious advantage to vague language is that it makes context-sensitivity easier. For example, if “tall” is not given a precise definition, I can use it to describe a newborn whose height is 2 feet or a professional basketball player whose height is 7 feet. Again, this objection cannot fit: we could make the precise definitions context-specific. That is, “tall” could be defined to be greater than or equal to 15 inches for a newborn and greater than or equal to 6 foot 10 for a professional basketball player. In terms of the model above, it would be simple to add a random variable which is observed by both players and interpreted as the context. The result above would imply that in

each context, it is optimal to have a precise language, though the language might vary with the context in general.

A natural objection to this point is that it is cognitively difficult to remember all the relevant cutoff points. Of course, the key question is not whether it is difficult to remember the cutoff points corresponding to words in a precise language but whether it is more difficult than remembering a vague meaning. Again, this requires a model of bounded rationality different from anything in the existing literature.

The existing models often consider the complexity of strategies or rules¹⁰ and so, in principle, could be used to formalize a notion of the complexity of identifying the appropriate cutoffs for a given context or the complexity of the language in other senses. However, this is unlikely to deliver the conclusion that vagueness is optimal. Such a model could well deliver the conclusion that fewer words are used than would be used in the absence of complexity costs since this would be a way of simplifying. However, this effectively gives only a model of how many messages there are. As noted earlier, the size of the message set has no bearing on the conclusion that vagueness is suboptimal. The real problem is that none of these models include options which are naturally interpreted as simplifying the language by introducing some form of vagueness.

Another argument might be that the model above ignores the fact that people must learn to use a language. In particular, one explanation for the prevalence of vague terms which has been suggested in the philosophy literature is that vague terms might be easier to learn. Sainsbury [1996, page 262] argues that “we acquire the concept from the inside, working outwards from central cases . . . rather than from the outside, identifying boundaries and moving inwards.” As I understand the argument, the idea is rather natural. Suppose that I enter a population of people who have agreed on the cutoff height between “short” and “tall.” The only way I can learn about this cutoff, however, is to observe how they classify various people. Naturally, unless I get very lucky and observe the classification of two individuals very close to but on opposite sides of the boundary, I will never learn the cutoff being used. Instead, I will end up learning that people below, say, 5 foot 5, are short, while people above, say, 6 foot 6, are tall and I would have no clear idea about people in the middle. In this sense, what I have learned is a vague term. Hence, even if we begin with a precise term, as new agents come into the population and old ones die out, the term will become vague.

While this idea has some appeal for terms based on variables that are difficult to quantify (such as “nice” or perhaps Sainsbury’s example of “red”), it is hard to accept when applied to a notion like height. Surely in the fictitious world described above, I should be able to open a dictionary and get out a tape measure to see exactly how tall

¹⁰See, for example, Rubinstein [1998], Chapters 8 and 9.

is “tall.”¹¹

A related but different critique is that words are vague because we don’t know the strategies of other people. If 2 does not know 1’s pure strategy, then 2 does not know the intended meaning of the words 1 uses. This idea can also be incorporated into the model. Suppose we add a payoff irrelevant signal to what player 1 observes and assume this is not observed by 2. In such a model, there might be equilibria in which player 1’s usage of words to describe the h he has seen depends on this signal. Since 2 does not observe the signal, 2 does not know how 1 is using words. In other words, player 1 may use a pure strategy, but his pure strategy choice depends on the payoff irrelevant signal.

On the other hand, it is not hard to see that Theorem 2 implies that an equilibrium of this kind is at least weakly Pareto inefficient. That is, the best possible outcome can be achieved by an equilibrium where 1 does not condition his choice of words on the payoff irrelevant signal. Hence an equilibrium in which people aren’t sure of the pure strategies chosen by others is Pareto inefficient.¹²

Another version of this argument would be to suppose that player 1 observes *two* payoff relevant signals. This would make the set H above a product space. In such a world, even a precise language will not give a precise statement on each dimension. To see the point, return to the airport example and suppose 1 observes both height and weight of Mr. X. A partition of the set of height–weight pairs will not necessarily induce a clear partition on the set of heights. For example, it might be optimal to lump height and weight together into some general comparison of “large” versus “small.”

I think this objection also misses the mark for two reasons. First, the optimal language here is precise, even if it is not precise in either dimension. Real language is not precise in this sense. Second, “tall” seems to be a clearly one dimensional notion and this argument cannot explain vagueness in one dimension. Many other vague terms (“thin” and “bald” for example) seem similarly one dimensional.

Some readers may object to my argument on the grounds that the formulation of

¹¹Another problem is that it is hard to see why the language won’t ultimately collapse. After all, each generation learns a vague version of the previous generation’s usage.

¹²One could argue that the players might not have a common prior over these various languages 1 might use. More precisely, they may have the same prior over H but different priors over the payoff irrelevant signal. If so, they have different beliefs about the likelihood that 1 uses a particular message for a particular reason. On the other hand, 1 observes h and the payoff irrelevant signal before choosing his message, so his prior over the pair is irrelevant to determining whether particular strategies form an equilibrium or not. Hence for the purposes of characterizing equilibria, we may as well assume the players have the same prior. On the other hand, 1’s prior does affect his ex ante expected payoff in an equilibrium and hence differences in priors could induce different preferences over equilibria. However, as suggested above, surely differences in preferences (induced by differences in priors or more “fundamental” disagreements) are not the primary reason for the prevalence of vague languages.

vagueness as an issue in game theory is incorrect. The sorites problem, for example, does not seem to depend on communication between agents, only the inability of a single agent in isolation to make a categorization. I believe such a criticism is misplaced. There is no meaning to the word “tall” aside from what people interpret it to be. If a person says he cannot say for sure whether a particular person is “tall,” surely this means he is not sure how most people would categorize this person or how he would best describe this person to others, not that he is unable to make a choice. Put differently, we use words to communicate: there is only an answer to the question of the minimum height of a tall person if we decide to use the word in such a way. Inherently, then, it is indeed a game theory problem.

3 Vagueness and Bounded Rationality

As noted numerous times in the preceding section, it may be possible to understand the prevalence of vague terms in a model which incorporates a different kind of bounded rationality than has been studied in the extant literature. In this section, I first try to say why existing models of bounded rationality will not help. I then sketch three possible directions in increasing order of ambitiousness (and vagueness!).

A key point is that vagueness requires a different way of thinking about information and what it is. In most models of bounded or unbounded rationality, information takes the form of an event in some state space. That is, when an agent receives information, what he learns is always modeled as the fact that the state of the world lies in some set. The learning may be incomplete and even systematically flawed, but the ultimate conclusion takes this form. In this sense, what is learned is precise.¹³ I don’t know how we can mathematically represent vague knowledge, but I believe that this is what is called for.

Turning to some possible approaches, first, I think that a natural intuition is that vagueness is easier than precision, for the speaker, listener, or both. Intuitively, for the speaker, deciding on which precise term to use may be harder than using a vague term. For the listener, information which is too specific may require more effort to analyze. With vague language, perhaps one can communicate the “big picture” more easily. Several of the points mentioned in the previous section can be thought of as versions of this approach. This requires a different model of information processing than anything in the literature. There is no model in the literature of what vague information

¹³There are models where information is treated in a purely syntactic fashion — that is, where information consists of simply a set of formulas in some logical language. See, for example, Lipman [1999] or Feinberg [2005a, 2005b]. However, even in these models, the agent’s view of the information is translated into some kind of subset of a state space.

is or how it is processed.

An approach which seems more difficult still would be to focus on the relationship between vagueness and unforeseen contingencies. That is, when I use a word, I do not know all the possible situations where you would use my information and hence I might want to “hedge” my bets and be vaguer.¹⁴ A very concrete example of this comes from contract theory. Instead of attempting to specify in exact terms what a party to the contract is supposed to do, contracts often use vague terms such as “taking appropriate care” or “with all due speed.” If agents fear that circumstances may arise that they currently cannot imagine, then they may wish to avoid being too precise in order to avoid being trapped later. Instead, they require the other party to respond to unexpected circumstances “appropriately,” relying on the hope that the meaning of this word will be sufficiently clear *ex post*.¹⁵ While there has been more work in recent years on this topic, I think it fair to say that there is still no reasonably plausible model of unforeseen contingencies which could be used to analyze such contracts. Dekel, Lipman, and Rustichini [1998] is a somewhat dated but detailed review of the literature; Lipman [2008] is briefer but more recent.

To understand the third suggestion, let us return to one comment I made in the previous section. I noted that if 1 did not know the height of the person he is describing to 2, he should form a subjective probability distribution over it and communicate this distribution precisely to 2. A natural reply is that people don’t form precise subjective beliefs and hence cannot communicate them. If one takes the Savage view of subjective beliefs, one must interpret this reply as saying that agents do not have precise preferences or, perhaps, that agents do not truly “know” their own preferences. If we think of preferences over, say, flavors of ice cream, this sounds ridiculous. If we think of preferences over state-contingent sequences of commodity bundles over one’s lifetime, it seems obviously correct. In 1967, Savage described the problem as “the utter impracticality of knowing our own minds in the sense implied by the theory.” He went on to comment

You cannot be confident of having composed the ten word telegram that suits you best, though the list of possibilities is finite and vast numbers of possibilities can be eliminated immediately; after your best efforts, someone may suggest a clear improvement that had not occurred to you.

¹⁴Pierce [1902] seems to *define* vagueness in terms of unforeseen contingencies, saying “a proposition is vague when there are possible states of things concerning which it is *intrinsically uncertain* whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed by the proposition . . . because these questions never did . . . present themselves . . .” [italics in original].

¹⁵This idea bears more than a passing resemblance to the Grossman–Hart–Moore approach to incomplete contracts. See Hart [1995] on this approach and Dekel, Lipman, and Rustichini [1998] for a discussion of the connection between it and formal models of unforeseen contingencies.

Put differently, the vastness of even very simple sets of options suggests it is ludicrous to think a real person would have well defined ideas, much less well behaved preferences, regarding the set. I think this is more than saying preferences are incomplete in reality since the key is that some options may not even been recognized as such.

In short, it is not that people have a precise view of the world but communicate it vaguely; instead, they have a vague view of the world. I know of no model which formalizes this. I think this is the real challenge posed by the question of my title.

4 Postscript

The foregoing is essentially the paper I wrote 25 years ago and have not seriously updated in 15 years. One might wonder why I didn't update it, instead using this postscript to clarify some points and comment on the more recent literature. If I were to rewrite the main body of the paper instead, it would arguably do a disservice to those who read and wrote about previous versions. Later readers would find some of their comments puzzling if I change the paper they were commenting on.

I hope this postscript will clarify what I didn't communicate well 25 years ago. The subsequent literature, notably including Égré and Spector's [2024] paper in this issue, have helped clarify the issues and helped me think better about what I was trying to say.

My point had been that the models I knew could not give the answer that seemed "obviously" correct — being vague is useful, even when trying to communicate clearly and effectively. My point was not that this should lead us to rethink whether vagueness is useful but should lead us to be more creative in thinking about ways to formalize this insight in useful models.

To make this point, I needed to do two things. First, I needed to define "a useful model." Second, I needed to demonstrate that a sensible notion of vagueness could not be fit into then-available versions of such models.

For my purposes as a game theorist, "useful" did not include a more purely logical or linguistic based approach. I have enjoyed reading this literature (see, e.g., Kennedy [2019]) and believe it provides ideas that can help test the extent to which our models of interactive communication have generated phenomena which adequately describe vagueness. But I didn't (and still don't) see where this literature contains the model of interactive communication I seek.

While I don't wish to overstate this point as I waver on it over time, philosophically,

I question whether words *can* mean anything aside from the way we use them to communicate. So when Égré and Spector write “Lipman’s approach basically fails to distinguish between what we might call ‘making *vague use* of a word’, and ‘making use of a *vague word*’” I agree. If meaning is solely derived from use, there is no distinction between these things. I think this is the point where Égré and Spector’s perspective and mine most diverge.

In short, between my interest in studying strategic communication and my philosophical leanings, I defined “a useful model” to mean a game-theoretic one.

To demonstrate that we can’t understand vagueness using standard game-theoretic tools, I have to show that there is no formal definition of vagueness within standard models that fits our conception of vagueness well enough. If meaning comes from usage, then this requires something about the equilibrium meaning of certain messages to have properties we associate with vagueness.

It seems self-evident that pure strategies do not fit this purpose. With a pure strategy, the speaker uses a certain word, say “tall,” if and only if the set of observations she makes are within a certain precisely defined set, say X . Thus the equilibrium meaning of “tall” is “my observation is in the set X .” In this sense, the word “tall” has a precise equilibrium meaning, not a vague one.

On the other hand, mixed strategies do have at least some properties matching our conception of vagueness. If my likelihood of describing a person as “tall” is strictly increasing in their height, then my listener can’t know exactly the height of the person I’m describing or be sure what set of heights I am referring to when I say they are tall. Indeed, while Égré and Spector argue that mixed strategies can’t give a proper treatment of vague terms, their formulation is (for my purposes) equivalent to a mixed strategy. They treat a speaker as describing someone as tall if their height is above a certain random variable. For my purposes, it doesn’t matter if that random variable is something the speaker draws for themselves or something that is drawn independently in the world. Either way, the probability the speaker says “tall” is increasing in the height of the person being described and conveys noisy information about that height.

I did not intend to suggest that mixed strategies are the “right” definition of vagueness. My point is that this definition fits in certain respects, but what I showed was that it fails to give us the right conclusions about vagueness. Hence, ultimately, it doesn’t seem to be the definition we should use, at least not without some other ingredient to avoid the negative conclusions I reached.

To the extent I disagree with some of Égré and Spector’s conclusions, I think these disagreements are about the divergence in perspective noted above. For example, they discuss a mathematician randomly fluctuating between writing “ $x \leq y$ ” and “ $y \geq x$.”

They suggest that my definition would say that “ $x \leq y$ ” and “ $y \geq x$ ” are vague.

Their conclusion is a consequence of taking my thoughts about meaning in equilibrium and applying them to an objective, external notion of meaning. For me, whether these words are vague or not is an equilibrium statement about how the words are used by the speaker in a specific equilibrium of a specific game. If the mathematician’s strategy is to randomize between these two expressions when he means that x is a larger number than y , then it seems natural to expect that, in equilibrium, the listener/reader will entirely understand the meaning. On the other hand, if the mathematician is trying to develop a new language and for some strange reason replaces “John is tall” with “ $x \leq y$ ” and “The table is large” with “ $y \geq x$,” these terms would be vague and not equivalent to one another.

Despite this difference in our perspectives, I see my main conclusion and the main conclusion of Égré and Spector to be very much in line with one another. In my conclusion, I wrote that “vagueness requires a different way of thinking about information and what it is” and that “I don’t know how we can mathematically represent vague knowledge, but I believe that this is what is called for.” Égré and Spector introduce a different way of describing the information conveyed by a statement, one which relies on an external notion of meaning or, as Égré and Spector put it, “on a semantic characterization of the notion of vagueness.”

While I find this very interesting, I do have one concern. As Égré and Spector suggest in their conclusion, their account of the value of vague language implicitly relies on an assumption that it is “harder” or “more costly” for the speaker to report their probability beliefs directly. For my purposes, this is a critical point. Suppose a speaker’s strategy is a message as a function of her probability belief. Then if the speaker uses a pure strategy, the equilibrium meaning of any word is that the sender’s belief lies in a particular set. In this sense, it becomes costless to communicate a partition of one’s probability beliefs. My theorem says that this would be better for the speaker and listener than Égré and Spector’s vague language (at least weakly). I agree with what I take to be their view that this theorem would not capture reality very well, but that’s exactly my point.

Blume and Board [2013] introduce private information about language competence in a model intended to enable an adequate game-theoretic treatment of vagueness. Here the speaker has some set of messages she can use and this set is part of her private information. In equilibrium, the speaker communicates precisely in one sense but vaguely in another. To be specific, the speaker communicates precisely about her private information. But her private information combines her knowledge of her language competence with the information about the world that she really wants to convey. As a result, the listener is unable to separate these two aspects of the speaker’s information. Thus the information about the world that the speaker wants to convey can only be interpreted vaguely. While

different from Égré and Spector in many critical details, the spirit of introducing a new source of randomness that affects equilibrium meaning is similar. To my reading, the advantage of Blume and Board’s formulation is that it avoids the critique I offered above about communicating one’s probability beliefs directly. On the other hand, the advantage of Égré and Spector’s formulation is that their semantic account of vague terms seems very natural.

In short, I’m not sure we have the definitive answer to the question I tried to raise, but I believe a number of thought-provoking papers including Blume–Board and Égré–Spector have made significant progress and opened up new and interesting issues to explore.

References

- [1] Blume, A., and O. Board, “Intentional Vagueness,” *Erkenntnis*, **79**, 2014, 855–899.
- [2] Blume, A., and O. Board, “Language Barriers,” *Econometrica*, **81**, March 2013, 781–812.
- [3] Crawford, V., and J. Sobel, “Strategic Information Transmission,” *Econometrica*, **50**, November 1982, 1431–1451.
- [4] De Jaegher, K., “A Game-Theoretic Rationale for Vagueness,” *Linguistics and Philosophy*, **26**, October 2003, 637–659.
- [5] Dekel, E., B. Lipman, and A. Rustichini, “Recent Developments in Modeling Unforeseen Contingencies,” *European Economic Review*, **42**, 1998, 523–542.
- [6] Dow, J., “Search Decisions with Limited Memory,” *Review of Economic Studies*, **58**, January 1991, 1–14.
- [7] Égré, P., and B. Spector, “Explaining Vague Language,” working paper, 2024.
- [8] Feinberg, Y., “Subjective Reasoning — Dynamic Games,” *Games and Economic Behavior*, **52**, July 2005a, 54–93.
- [9] Feinberg, Y., “Subjective Reasoning — Solutions,” *Games and Economic Behavior*, **52**, July 2005b, 94–132.
- [10] Hart, O., *Firms, Contracts, and Financial Structure*, Oxford: Clarendon Press, 1995.
- [11] Keefe, R., and P. Smith, ed., *Vagueness: A Reader*, Cambridge, MA: MIT Press, 1996.

- [12] Kennedy, C., “Ambiguity and Vagueness: An Overview,” in C. Maienborn, K. von Heusinger, and P. Portner, eds., *Semantics – Lexical Structures and Adjectives*, Berlin: De Gruyter Mouton, 2019.
- [13] Lipman, B., “Decision Theory without Logical Omniscience: Toward an Axiomatic Framework for Bounded Rationality,” *Review of Economic Studies*, 66, April 1999, 339–361.
- [14] Lipman, B., “Unforeseen Contingencies,” in S. N. Durlauf and L. E. Blume, eds., *The New Palgrave Dictionary of Economics*, Palgrave Macmillan, 2008.
- [15] Monderer, D., and L. Shapley, “Potential Games,” *Games and Economic Behavior*, 14, May 1996, 124–143.
- [16] Peirce, C.S., “Vague,” in J.M. Baldwin, ed., *Dictionary of Philosophy and Psychology*, New York: MacMillan, 1902.
- [17] Rubinstein, A., *Modeling Bounded Rationality*, Cambridge, MA: MIT Press, 1998.
- [18] Sainsbury, R.M., “Concepts without Boundaries,” in R. Keefe and P. Smith, eds., *Vagueness: A Reader*, Cambridge, MA: MIT Press, 1996.
- [19] Savage, L. J., “Difficulties in the Theory of Personal Probability,” *Philosophy of Science*, 34, 1967, 305–310.
- [20] Wright, C., “Further Reflections on the Sorites Paradox,” in R. Keefe and P. Smith, ed., *Vagueness: A Reader*, Cambridge, MA: MIT Press, 1996.