Temptation[1]

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Current Draft
April 2011

[1]We thank Eddie Dekel, David Dillenberger, Drew Fudenberg, Jawwad Noor, Philipp Sadowski, Rani Spiegler, and Norio Takeoka for helpful comments and Deborah Goldschmidt for expert proofreading. This research was supported by grants from the National Science Foundation.

Abstract

This survey discusses decision–theoretic models of agents who seek to constrain and regulate their own future behavior. The common theme is that the decision makers' future utility is affected by unwanted temptations. The wish to eliminate temptations from future option sets creates a preference for commitment. The surveyed models explore the relationship between temptations and other psychological phenomena, such as self-control, self-deception, guilt or shame.

1 Introduction

This survey discusses decision–theoretic models of agents who seek to constrain and regulate their own future behavior. The common theme is that the decision makers' future utility is affected by unwanted temptations. The wish to eliminate temptations from future option sets creates a preference for commitment. The surveyed models explore the relationship between temptations and other psychological phenomena, such as self-control, self-deception, guilt or shame.

An experimental literature in psychology and economics has approached these phenomena with different experimental designs, broadly classified into experiments that document *choice reversals* and experiments that document a *preference for commitment.* The first approach confronts subjects with a particular choice at distinct decision dates. If choices at the time of consumption are affected by unwanted temptation then we would expect the choice at the time of consumption to differ from the same choice made at an earlier stage.

A large experimental literature, surveyed in Frederick, Loewenstein and O'Donoghue (2002), examines such reversals for intertemporal choices. Specifically, subjects are asked to choose between a smaller period t reward and a larger period $t+\tau$ reward. Subjects tend to prefer the smaller reward when t is small but the larger reward when t is large. If the subjects' behavior is stationary, this evidence implies that the same intertemporal trade-off is resolved differently depending on when the decision is made.[1]

Casari (2009) examines the connection between choice reversals and a preference for commitment. Let $e(t)$ be the option that yields \$100 in $t+2$ days and let $l(t)$ be the option that yields \$110 in $t+k$ days. Casari confronts subjects with a choice between $e(t)$ and $l(t)$. A person exhibits a choice reversal if – for a suitable choice of k – there are $t_1 < t_2$ such that $e(t_1)$ is chosen over $l(t_1)$ and $l(t_2)$ is chosen over $e(t_2)$. Consistent with choice-reversal literature (Rachlin and Green (1972)), 65% of subjects exhibit a choice reversal.

Casari presents those subjects that reverse their choices with the option to commit. Specifically, let $l_x(t)$ be the option that yields \$110 $- x$ in $t+k$ days. Casari asks his

[1] Noor (2009a) argues that there are good reasons that subjects' preference may not be stationary and thus the experimental evidence may be not reflect a preference reversal.

subjects to choose between commitment to $l_x(t_2)$ and an option that requires the subject to choose between $l(t_2)$ and $e(t_2)$ in $t_2 - t_1$ days. Of those subjects that exhibited a choice reversal, 61.5% choose the commitment option when $x = 0$ while 17.9% choose the commitment option when $x = 2$. Thus, for a majority of subjects, choice reversal is indeed associated with a preference for commitment. At the same time, for a significant minority no preference for commitment can be documented. Thus, while choice reversals are associated with a desire to commit, not all subjects who reverse themselves prefer commitment.

Ashraf, Karlan and Yin (2006) conduct a field experiment to determine demand for a savings product that offers commitment. Customers could choose between an amount-based savings goal or a date-based savings goal. If the former was chosen, savings could only be withdrawn once the amount saved exceeded the goal. If the latter was chosen, savings could only be withdrawn after a specified date. The interest rate paid on the commitment accounts was identical to the interest paid on regular accounts. The new savings product was chosen by 28.8% of the subjects and was associated with a significant increase in savings over the control group. The authors find that for women the probability of choosing the commitment product is correlated with preference reversals but for men this relationship is statistically insignificant.

Thaler and Benartzi (2004) report a field experiment following the introduction of a new retirement savings program, *Save More Tomorrow*. With this program, employees commit to allocating a proportion of the next salary increase to their retirement savings. Thaler and Benartzi report that 78% of employees who were offered the program joined and that 80% of those remained in the program through at least four salary increases. The savings rate of participants increased as a result of participation in the program.

The goal of this survey is to discuss decision theoretic models that accommodate behavior such as the one described in these experiments and analyze related phenomena such as self-control or self-deception.

1.1 A Preview

Kreps (1979) introduced a two period model of the decision making with choice of a menu in the first period and choice from the menu in the second period. In this survey, we discuss research that uses Kreps' framework to analyze temptation and related psychological phenomena.

In a standard setting there is no need to study decisions in both periods. Second period choice maximizes a utility function and first period choice simply maximizes the induced value of menus. Therefore, it is enough to specify a utility function over consumption and menu utility follows as the utility of the maximal element. The choice over menus is useful if we want to study phenomena that the consumption choice alone cannot reveal. Specifically, it is useful to study choice affected by temptations.

Temptations are items that the agent would want to exclude from the menu and, thus, are the source of the agent's preference for commitment. Consider a decision maker who struggles to control his calorie consumption. Second period choice reveals a standard maximizer of a utility function that favors high-calorie meals. The menu choice, by contrast, reveals an agent who avoids menus containing high-calorie meals. The menu choice allows us to distinguish this agent from someone who simply enjoys high-calorie meals.

The presence of temptations facilitates the analysis of a variety of psychological phenomena, such as self-control, belief-distortions, self-deception, guilt or shame. As an illustration, suppose there are two consumptions a and b and three possible menus, $\{a\}, \{b\}$ and $\{a, b\}$. Let $\succeq_1$ be a preference that describes (period 1) menu choice and let $\succeq_2$ be a preference that describes (period 2) choice from the menu. If $\{a\} \succ_1 \{a, b\}$ then b is a temptation. In this case, the agent would rather commit to a menu that offers only a than choose from a menu that offers both a and b. Suppose commitment is not possible and the agent must choose from $\{a, b\}$ in the second period. If the agent chooses b then he succumbs to temptation. If the agent chooses a then he exercises self-control. In this case, the fact that the choice a from $\{a, b\}$ is worse than the choice a from $\{a\}$ reveals a cost of self-control. Next, suppose that the agent succumbs to temptation and chooses b from $\{a, b\}$. If $\{b\} \sim_1 \{a, b\}$ then the agent, in period 1, anticipates succumbing to temptation and does not value the availability of a. If, on the other hand, $\{a, b\} \succ_1 \{b\}$ then the agent values the availability of a even though he will never choose it. We interpret this as self-deception: the agent believes he might (or will) choose a even though this never happens.

We confine our discussion to work in the Krepsian revealed preference tradition, that is, models that takes choice behavior as a primitive. There is a related literature that takes a particular psychological conflict as its starting point and develops an optimization model to analyze this conflict. For example, work by Thaler and Shefrin (1981), Fudenberg and Levine (2006) and Salant, Silverman and Ozdenoren (2010) falls into this latter category. While the models are related, the nature of the theoretical contributions are quite different and therefore it is difficult to discuss both approaches in a single survey. Our narrow focus facilitates a more detailed discussion of the reviewed work.

The paper is organized as follows. We begin with a two period version of the standard expected utility model. The two period analysis of the standard model is unnecessarily complicated but useful as a pedagogical device to illustrate the setting. Next, we introduce the self-control model of Gul and Pesendorfer (2001). That model contains the Strotz (1955) model of changing tastes as a special case but generalizes it to define and analyze self-control. Noor and Kopylov (2009) and Kopylov (2009) introduce and analyze self-deception into the self-control model. We present Kopylov's model and refer to it as the κ-self-control model. The two period setting is useful for its simplicity but applications often require a fully dynamic model. We present the recursive self-control model (Gul and Pesendorfer (2004), Krusell, Kuruscu and Smith (2010), Noor (2009c)) as a tractable specification for infinite horizon decision problems.

In section 7, we turn to extensions that relax the restrictions on the choice from menus. We discuss models of nonlinear self–control costs due to Fudenberg and Levine (2006, 2010a, 2010b) and Noor and Takeoka (2010a, 2010b) and show how they are consistent with a variety of choice anomalies, including the Allais paradox. Next, we discuss models that permit random choice from menus (Dekel, Lipman, and Rustichini (2009), Stovall (2010), and Dekel and Lipman (2010)) and, finally, we consider infinite-horizon models that allow nonstationary behavior (Gul and Pesendorfer (2007)). The concluding section discusses a number of other uses of preferences over menus to capture phenomena such as regret (Sarver (2008)), costly contemplation (Ergin and Sarver (2010)), and ambiguity (Olszewski (2007), Ahn (2008), and Gajdos, Hayashi, Tallon, and Vergnaud (2008)).

2 A Two Period Model

We begin with a two period expected utility model. Consumption takes place in period 2 only. In period 1, the agent takes an action that affects the set of alternatives available in period 2. Thus, we can model the period 1 problem as a choice among menus with the interpretation that, in period 2, the agent must pick an alternative from the menu chosen in period 1. The period 2 choices are consumption lotteries.

Let $\mathcal{C}$ be a compact metric space and let $\Delta(\mathcal{C})$ be the set of all measures on the Borel $\sigma-$algebra of $\mathcal{C}$. We endow $\Delta(\mathcal{C})$ with the topology of weak convergence and let Z be the set of non-empty, compact subsets of $\Delta(\mathcal{C})$. We endow Z with the topology generated by the Hausdorff metric.[2] Define $\alpha x + (1-\alpha)y := \{p = \alpha q + (1-\alpha)r : q \in x, r \in y\}$ for $x, y \in Z, \alpha \in [0,1]$. The set $\mathcal{C}$ represents the possible period 2 consumptions, $\Delta(\mathcal{C})$ represents the consumption lotteries and Z represents the objects of choice in period 1.

The primitives of the model are two binary relations; the binary relation $\succeq_2$ on $\Delta(\mathcal{C})$ describes the agent's ranking of consumption lotteries in period 2 and the binary relation $\succeq_1$ on Z describes the agent's ranking of menus in period 1. The following axioms are standard assumptions that are assumed to hold throughout most of this survey.

Axiom 1 (Preference Relation)**.** $\succeq_1, \succeq_2$ *are complete and transitive binary relations.*

Axiom 2 (Independence)**.** $x \succ_1 y$ *and* $\alpha \in (0,1)$ *implies* $\alpha x + (1-\alpha)z \succ_1 \alpha y + (1-\alpha)z$.

Dekel, Lipman and Rustichini (2001) and Gul and Pesendorfer (2001) interpret Axiom 2 as a combination of the usual independence axiom together with the assumption that the agent is indifferent as to the timing of resolution of uncertainty. Consider an extension of the decision-maker's preferences to the set of lotteries on Z. If $x \succ_1 y$ and if the usual independence axiom holds then a lottery that yields x with probability α and z with probability $1-\alpha$ is preferred to a corresponding lottery with outcomes y or z. Note that, in this case, the randomization between x and z or y and z occurs prior to the choice in period 2. Next, consider the same lotteries but the randomization occurs *after* the period 2 choice. The menus $\alpha x + (1-\alpha)z$ and $\alpha y + (1-\alpha)z$ contain all the possible contingent plans and, therefore, describe the situation if the randomization between x and z or y and z is performed after the choice. Thus, if the agent satisfies the usual independence axiom then she will prefer a lottery with outcomes x or z over the same lottery where

[2]The Hausdorff distance between x and y is

$$d_h(x,y) = \max\{\max_x \min_y d(p,q), \max_y \min_x d(p,q)\}$$

where d is a metric that generates the weak topology.

x is replaced with y. If, in addition, the agent satisfies the standard indifference to the timing of resolution of uncertainty then she will satisfy Axiom 2.

The next axiom connects first and second period preferences. It requires that adding an alternative can only increase the utility of a menu if this alternative is chosen in the second period.

Axiom 3 (Sophistication)**.** *If $x \cup \{p\} \succ_1 x$ then $p \succ_2 q$ for all $q \in x$.*

This and related axioms that connect first and second period choice are due to Noor (2009c) and Kopylov (2009a). In section 5, we discuss a model that weakens sophistication to analyze agents that are prone to self-deception.

Next, we state two continuity axioms that will be used below. Axiom 4* requires that upper and lower contour sets of $\succeq_1$ are closed and the lower contour sets of $\succeq_2$ are closed.

Axiom 4*. *The sets $\{z \in Z : z \succeq_1 x\}$, $\{z \in Z : x \succeq_1 z\}$ and $\{p \in \Delta(\mathcal{C}) : p \succeq_2 q\}$ are closed.*

Axioms 1 and 4* imply that $\succeq_1$ can be represented by a continuous utility function and that $\succeq_2$ can be represented by an upper semicontinous utility function. Axiom 4, below, weakens 4* and allows for representations of $\succeq_1$ that fail continuity but are upper semicontinuous.

Axiom 4.

(a) *(Upper Semi-Continuity) The sets $\{z \in Z : z \succeq_1 x\}$ and $\{p \in \Delta(\mathcal{C}) : p \succeq_2 q\}$ are closed.*

(b) *(Lower von Neumann-Morgenstern Continuity) $x \succ_1 y \succ_1 z$ implies $\alpha x + (1-\alpha) z \succ_1 y$ for some $\alpha \in (0, 1)$.*

(c) *(Lower Singleton Continuity) The sets $\{p : \{q\} \succeq_1 \{p\}\}$ are closed.*

Axiom 4b is "one half" of the familiar continuity assumption used in the von Neumann-Morgenstern theorem. The other half is implied by Axiom 4a. Axiom 4c implies that the period 1 preference restricted to singleton menus can be represented by a continuous utility function.

A utility function $W : Z \to \mathbb{R}$ represents $\succeq_1$ if $x \succeq_1 y$ if and only if $W(x) \geq W(y)$. A lottery utility $u : \Delta(\mathcal{C}) \to \mathbb{R}$ represents $\succeq_2$ if $p \succeq_2 q$ if and only if $u(p) \geq u(q)$. A lottery utility is linear if $u(\alpha p + (1-\alpha)q) = \alpha u(p) + (1-\alpha)u(q)$.

A standard expected utility maximizer evaluates each menu by the lottery that yields the largest expected utility. That is, there is a linear, continuous lottery utility $u : \Delta(\mathcal{C}) \to \mathbb{R}$ that represents $\succeq_2$ and the utility W defined as

$$W(x) = \max_{x} u(p) \tag{1}$$

represents $\succeq_1$. We say that a utility is a *standard utility* if it satisfies (1) above and we refer to a preference pair that can be represented by a standard utility as a *standard pair.*

If $(\succeq_1, \succeq_2)$ is standard and $x \succeq_1 y$ then adding the menu y to x has no effect on the value of the menu. The following axiom, introduced by Kreps (1979), captures this.

Axiom S (Standard)**.** *$x \succeq_1 y$ implies $x \cup y \sim_1 x$.*

We say that $\succeq_1$ is non-degenerate if $x \succ_1 y$ for some $x, y \in Z$. A pair $(\succeq_1, \succeq_2)$ is non-degenerate if $\succeq_1$ is non-degenerate.

Theorem 1. *The non-degenerate pair $(\succeq_1, \succeq_2)$ satisfies Axioms 1-4 and S if and only if it is a standard pair.*

Source: Kreps (1979) contains the analogue of Theorem 1 in a setting without lotteries. Theorem 1 follows as a corollary to Theorem 2 below.

The lottery utility of a standard preference represents the agent's ranking of singleton menus. Put differently, u represents the agent's ranking over menus when she commits to a consumption lottery in period 1. We refer to this utility function as the *commitment utility.* Standard agents evaluate menus according to their best alternative for the commitment utility and make second period choices to maximize this utility.

3 Temptation and Self-Control

The experimental and empirical work discussed in the introduction gives examples of situations where economic agents benefit from commitment. Thus, this literature documents instances where agents strictly prefer a menu $y \subset x$ over x. This behavior violates Axiom S above and is therefore inconsistent with standard preferences. The temptation model (Gul and Pesendorfer (2001)), introduced below, provides a theory that accommodates and interprets such behavior.

Let $W : Z \to \mathbb{R}$ be a utility function such that

$$W(x) := \max_{p \in x}[u(p) + v(p)] - \max_{p \in x} v(p) \tag{2}$$

for some continuous, linear functions $u : \Delta(\mathcal{C}) \to \mathbb{R}$ and $v : \Delta(\mathcal{C}) \to \mathbb{R}$. We refer to utility functions that satisfy (2) above as *self-control utilities* and to a preference that is represented by a self-control utility as a self-control preference. Each self-control utility is characterized by two lottery utilities, u and v. The utility u (the *commitment utility*) describes how the agent ranks singleton menus, that is, menus where the agent is committed to a choice in period 2. The lottery utility v (the *temptation utility*) measures how tempting an alternative is in the second period. When $u(p) > u(q)$ but $v(q) > v(p)$ then $W(\{p\}) > W(\{p, q\})$. Thus, the presence of the temptation q leads to a strict preference for the smaller menu $\{p\}$ in the first period. If commitment is not possible, and the agent is stuck with the menu $\{p, q\}$ then, in the second period, she can either give in to temptation and choose q or exercise self-control and choose p. The agent will choose the former if

$$u(q) + v(q) > u(p) + v(p)$$

and the latter if this inequality is reversed. When the agent exercises self-control, the term

$$v(p) - \max_{\{p,q\}} v(p) = v(p) - v(q)$$

captures the cost of self-control.

We say that $\succeq_1$ is a self-control preference if it can be represented by a utility function W that satisfies (2) above. The pair $(\succeq_1, \succeq_2)$ is a self-control pair if $\succeq_1$ is a self-control preference with temptation utility v and commitment utility u and $\succeq_2$ is represented by the lottery utility $u + v$.

The self-control cost depends on the magnitude of v relative to u . If we multiply v by a large positive number k, the self-control cost grows. Fix any $\epsilon > 0$ and let p be the choice and q the most tempting alternative. For k sufficiently large, we must have $v(q) - v(p) \leq \epsilon$ and hence the second period choice is near-optimal for the temptation utility v. In the limit case, the agent never exercises self-control and always maximizes the temptation utility. This limit case is described by a utility W such that

$$W(x) := \max_{p \in x} u(p) \text{ subject to } v(p) \geq v(q) \text{ for all } q \in x \tag{3}$$

where u, v are continuous, linear lottery utilities. We refer to utilities that satisfy (3) above as Strotz utilities and to a preference $\succeq_1$ that is represented by a Strotz utility as a Strotz preference. Strotz (1955) introduces the idea that decision makers may have a preference for commitment, that is, exhibit a strict preference for a smaller set of options. In his model, second period preference lexicographically first maximizes the temptation utility and then maximizes the commitment utility. That is, $p \succeq_2 q$ if and only if

$$v(p) > v(q) \text{ or } v(p) = v(q) \text{ and } u(p) \geq u(q)$$

In the first period, the agent anticipates this behavior and evaluates menus according to the commitment utility implied by the second period choice. Strotz refers to this behavior as *consistent planning*. We say that $(\succeq_1, \succeq_2)$ is a Strotz pair if $\succeq_1$ is represented by a Strotz utility and $\succeq_2$ reflects consistent planning.

If a preference $\succeq_1$ can be represented by a self-control utility or a Strotz utility then it satisfies Set Betweenness:

Axiom 5 (Set Betweenness)**.** $x \succeq_1 x \cup y \succeq_1 y$.

Set Betweenness captures decision makers who can rank alternatives according to how tempting they are and who are not made worse off when alternatives are added to the menu that are less tempting than an existing alternative. To see this connection, consider two menus x and y. If the choice from $x \cup y$ and the most tempting alternative in $x \cup y$ are both in x then the addition of y to x does not affect the agent's utility and we have $x \cup y \sim_1 x$. If the most tempting alternative from $x \cup y$ is in y then and the choice is in x then $x \succeq_1 x \cup y \succeq_1 y$. In either case, Set Betweenness is satisfied.

Theorem 2. *The binary relation $\succeq_1$ satisfies Axioms 1, 2, 4 and Set Betweenness if and only if it is a Strotz preference or a self-control preference.*

Source: Theorem 2 is Theorem 3 from Gul and Pesendorfer (2001)

The preference $\succeq_1$ is improper if $x \subset y$ implies $x \succeq_1 y$. An improper preference never benefits from additional options. Improper self-control preferences have a commitment utility u and a temptation utility v such that u is constant or $u = \alpha v + \beta$ for $\alpha \leq -1$. We say that $\succeq_1$ is proper if it is not improper and we say that $(\succeq_1, \succeq_2)$ is a proper pair if $\succeq_1$ is proper. If the preference is improper, sophistication (Axiom 3) is trivially satisfied and hence places no constraint on period 2 behavior. The following theorem shows that the period 2 preference of a proper pair must maximize $u + v$ if it is a self-control pair or must lexicographically maximize v then u if it is a Strotz pair.

Theorem 3. *The proper pair* $(\succeq_1, \succeq_2)$ *satisfies Axioms* $1-4$ *and Set Betweenness if and only if it is a Strotz pair or a self-control pair.*

Source: Theorem 3 extends Theorem 2 to second period choice. If u is not an affine transformation of v, Theorem 2 is a special case of Theorem 2.4 in Kopylov (2009). The cases where u is an affine transformation of v but the preference is proper can be proved using a similar argument.

We say that the preference has a (u, v) representation if it can be represented by a Strotz or a self-control utility with commitment utility u and temptation utility v. Both self-control preferences and Strotz preferences include standard preferences as a special case. This happens if $v = \alpha u + \beta$ for some $\alpha \geq 0, \beta \in \mathbb{R}$ or if u is a constant function. At the other extreme, there are preferences that have *maximal preference for commitment*; that is, for all non-singleton x there is $y \subset x$ such that $y \succ_1 x$. This happens if and only if $v = \alpha u + \beta$ for some $\alpha < 0, \beta \in \mathbb{R}$. We say that a preference relation is *regular* if it has some preference for commitment but does not have maximal preference for commitment. Hence, if (u, v) represents $\succeq$ then $\succeq$ is regular if and only if neither u nor v is constant and v is not an affine transformation of u.

Using the representation, it is straightforward to show that self-control preferences satisfy the stronger continuity Axiom 4*. By contrast, Strotz preferences may fail Axiom 4* (but satisfy Axiom 4.) To see this, consider a menu $\{p, q\}$ such that $v(p) = v(q)$ but $u(p) > u(q)$. In that case, $W(\{p, q\}) = u(p)$. If we perturb q so that $v(q) > v(p)$ then $W(\{p, q\}) = v(q)$. Hence the preference violates Axiom 4*. Moreover, it is straightforward to show that every regular Strotz preference violates Axiom 4*. Therefore, a regular preference with representation (u, v) is either a self-control preference or a Strotz preference but not both.

Theorem 4 shows that regular preferences have a unique (u, v) representation.

Theorem 4. *Suppose* (u, v) *represents the regular preference* $\succeq_1$*. If* $\succeq_1$ *is a self-control* *preference then* (u', v') *also represents* $\succeq_1$ *if and only if* $u' = \alpha u + \beta_u$ *and* $v' = \alpha v + \beta_v$ *for some* $\alpha > 0$ *and* β_u, $\beta_v \in \mathbb{R}$. *If* $\succeq_1$ *is a Strotz preference then* (u', v') *represents* $\succeq_1$ *if and only if* $u' = \alpha_u u + \beta_u$ *and* $v' = \alpha_v v + \beta_v$ *for* α_u, $\alpha_v > 0$ *and* β_u, $\beta_v \in \mathbb{R}$.

Source: Theorem 4 is Theorem 4 in Gul and Pesendorfer (2001).

We say that q *tempts* p if $\{p\} \succ_1 \{p, q\}$. Thus, under Set-Betweenness, q tempts p if the agent would rather commit to p than to q and has a preference for commitment at $\{p, q\}$. If the preference has a (u, v) representation then q tempts p if and only if $u(p) > u(q)$ and $v(q) > v(p)$. In this sense, v measures temptation. Commitment allows the agent to eliminate temptations before they can affect her utility. By contrast, self-control describes a response to the presence of temptations at the consumption stage. The agent exercises *self-control* at the consumption stage (period 2) if she does not choose the most tempting alternative. Define $c(x, \succeq_2)$ to be the second period choice from x. That is,

$$c(x, \succeq_2) = \{p \in x |\, p \succeq_2 q, \forall q \in x\}.$$

Note that $c(\cdot, \succeq_2)$ is well defined if $\succeq_2$ satisfies Axioms 1 and 4.

Definition 1. *The pair* $(\succeq_1, \succeq_2)$ *exercises self–control at* x *if for every* $p \in c(x, \succeq_2)$ *there is* $q \in x$ *such that* q *tempts* p.

The following theorem shows that three notions of self-control are equivalent: (i) our definition of self-control as an individual's ability to resist temptation; (ii) a definition based on preferences over sets; (iii) a revealed preference definition based on the observation that an agent with self-control may prefer y to x even though at time 2 the same choice is made from the menus y and x.

Theorem 5. *Suppose the preference* $(\succeq_1, \succeq_2)$ *has a* (u, v) *representation. Then, the following three statements are equivalent:*

(i) $(\succeq_1, \succeq_2)$ *exercises self-control at* x.

(ii) $y \succ_1 x \succ_1 z$ *for some* y, z *with* $y \cup z = x$.

(iii) *There exists* $y \subset x$ *such that* $y \succ_1 x$ *and* $c(x, \succeq_2) = c(y, \succeq_2)$.

Source: Theorem 5 is a slightly modified version of Theorem 2 in Gul and Pesendorfer (2001). ▌

We are unaware of any experimental or empirical study that seeks to document self-control. Part (iii) of Theorem 5 shows that instances of self-control can be identified as situations where the agent has a preference for commitment even when there are no choice reversals. Thus, Theorem 5 (iii) provides a testable prediction of self-control.

4 Examples of Temptations

4.1 Tempting Consumption

Motivated by Strotz (1955), Phelps and Polak (1968) formulate the quasi-hyperbolic discounting model and Laibson (1997) applies it to consumption choices. In a two period setting, it is a special case of (u, v) preferences, and, as Laibson (1997) observes, it can accommodate the above described choice reversals. Let $\mathcal{C} = \{(c_2, c_3) \in [0, n]^2\}$ be pairs of consumption for dates 2 and 3. The minimal consumption is zero and the maximal consumption is n. Assume that $(\succeq_1, \succeq_2)$ is a Strotz pair with commitment utility $u(c_2, c_3) = w(c_2) + \delta w(c_3)$ and temptation utility $v(c_2, c_3) = w(c_2) + \beta\delta w(c_3)$. Then, the consumption $(0, l)$ is chosen over $(s, 0)$ at date 2 if $\beta\delta w(l) > w(s)$. By contrast, in period 1 the agent chooses $\{(0, l)\}$ over $\{(s, 0)\}$ if $\delta w(l) > w(s)$. Thus, second period choice discounts period 3 consumption at a rate $\beta\delta$ while first period choice (assuming commitment) discounts period 3 consumption at a rate δ. The observed choice reversals occur if $\delta w(l) > w(s) > \beta\delta w(l)$.

Note that quasi-hyperbolic agents have a preference for commitment if and only if they exhibit a choice reversal. Thus, agents who do not reverse their preferences do not exhibit a preference for commitment. The possibility of self-control creates a wedge between preference for commitment and preference reversals and allows for the possibility that agents seek commitment even though they do not reverse their choices. To see this, let $(\succeq_1, \succeq_2)$ be a self control pair with $u(c_2, c_3) = w(c_2) + \delta w(c_3)$ and $v(c_2, c_3) = \sigma(w(c_2) + \beta\delta w(c_3))$. If $\delta w(l) > w(s) > \beta\delta w(l)$ and σ is above some threshold $\bar{\sigma}$ – the self-control cost is large – the agent exhibits choice reversals and a preference for commitment. If σ is below the threshold – the self-control cost is small – there is no preference reversal in the example above but the agent prefers commitment to $(0, l)$ over the choice between $(s, 0)$ and $(0, l)$.

Benhabib, Bisin, and Schotter (2009) present experimental evidence on choice reversals. They argue that the quasi-hyperbolic model is less successful at addressing their experimental findings than a model that includes a small fixed cost of delay, in addition to standard exponential discounting. In their model, the consumption $(0, l)$ is preferred to $(s, 0)$ at date 2 if $\delta w(l) - f > w(s)$ for some constant f. The temptation model is flexible enough to allow a similar specification. Assume that $(\succeq_1, \succeq_2)$ is a self-control pair with commitment utility $u(c_2, c_3) = w(c_2) + \delta w(c_3)$ and a temptation utility v such that

$$v(c_2, c_3) = \alpha w(\min\{b, c_2\})$$

Thus, the temptation utility v is satiated once period 2 consumption exceeds b. Suppose the agent compares the consumptions $(0, l)$ and $(s, 0)$ with $s < l$. In period 1, $\{(0, l)\}$ is preferred to $\{(s, 0)\}$ if $\delta w(l) \geq w(s)$ while in period 2 the consumption $(0, l)$ is preferred to $(s, 0)$ if

$$\delta w(l) + \alpha w(0) > w(s) + \alpha w(\min\{b, s\})$$

If $s \geq b$, then this inequality simplifies to

$$\delta w(l) - \alpha[w(b) - w(0)] > w(s)$$

and thus implies a fixed cost of delay as in the specification used by Benhabib et al.

Intertemporal choices are not the only choices associated with a preference for commitment. Wertenbroch (1998) examines the effect of quantity discounts for products that can be classified as more or less virtuous. In one study, Wertenbroch compares purchase intentions of consumers for a new brand of potato chips, alternatively described as "fatty" or "fatfree."[3] Consumers could choose a single bag or a package containing three bags of potato chips. Purchase intentions for the chips described as fatty are found to be less sensitive to quantity discounts than purchase intentions for the chips described as fatfree. Wertenbroch interprets this finding as an indication that consumers forgo quantity discounts to store less of the vice-good at home. To fit this setting into our two period

[3] In the experiment subjects are asked to state their demand for a new brand of potato chips. The same product is alternatively described as 25% fatty or 75% fatfree and thus Wertenbroch's experiment is really concerned with a framing effect. To fit this into our setting, we must interpret the chips described as fatty as a different product than the chips described as fatfree.

model, interpret the purchase decision as the period 1 choice of items to be stored in the pantry. In period 2, the agent picks an item from the pantry and consumes it. If the temptation utility favors vice-goods and the commitment utility favors virtuous goods then the temptation model can accommodate behavior that forgoes quantity discounts for vice goods but not for virtuous goods.

The discounting examples and the supermarket examples share the feature that the date 1 preference, $\succeq_1$, is more inclined to take the virtuous action (through commitment) whereas the date 2 preference $\succeq_2$ is more inclined to choose vice-goods. Put differently, these are examples where temporal distance (Noor (2009)) is a force toward virtuous behavior.[4] The following subsection shows that factors other than the passage of time can be responsible for the distinction between the two stages in temptation models. It also provides an example that behavior contaminated by temptation may be more virtuous than the commitment behavior.

4.2 Social Norms and Shame

Dillenberger and Sadowski (2008) use a temptation model to address the following evidence.[5] When subjects must divide a given surplus (as in a dictator game) they tend to act altruistically. At the same time, at an *ex ante* stage that is not observed by the recipient, some subjects are willing to accept a reduced surplus if the opportunity to act altruistically is take away.

Dana, Cain and Dawes (2006) conduct the following experiment. A subject can choose between a \$9 payment and a dictator game in which \$10 must be divided between the subject and a recipient. If the former option is chosen, the recipient receives no payoff and no information. Otherwise, the recipient is made aware of the dictator game, and is paid according to its outcome. In the experiment 11 of the 40 dictators (28%) choose the \$9 payment.

[4]Noor (2009) provides a normative theory based on the idea that temporal distance is a force towards virtuous behavior.

[5]The model developed by Sadowski and Dillenberger does not use lotteries and therefore employs a different axiomatization than the temptation model above. Their representation shares similarities with self-control utilities but is not a special case of self-control utilities. Saito (2011) develops a model of shame using lotteries.

This experiment is an example of behavior where observation by the recipient of altruism, rather than temporal distance from consumption, leads to different behavior in the two stages. Here, the earlier decision is less virtuous. The experiment illustrates behavior of people who give money when they meet a beggar but cross the street to avoid the encounter. The temptation model addresses such behavior by specifying a v that is more altruistic than the commitment utility u. The decision in the dictator experiment is governed by the more altruistic date-2 preference $\succeq_2$ (represented by $u + v$) while the decision whether to play the dictator game or choose the \$9 payment is governed by the menu preference $\succeq_1$ (represented by W). In the terminology of Dillenberger and Sadowski (2008), a person experiences *shame* if she chooses an allocation that is less altruistic than is optimal for v. Thus, shame in this setting is analogous to self-control in a setting with intertemporal choices. Despite the different connotations, self-control and shame are both instances where the decision maker finds it costly to resist unwanted pressures at the decision stage. The decision of the two subjects who chose the \$9 option but intended to give 0 in the dictator game reveals that they expect to incur a cost of shame in the dictator game. Their expected choice was selfish but nevertheless they were willing to receive a smaller payment to eliminate the pressure (temptation) to act altruistically.

4.3 Tempting Beliefs

Epstein (2006), Epstein and Kopylov (2007) and Epstein, Noor and Sandroni (2010) examine situations where temptation takes the form of a belief-distortion. These papers address experimental and field evidence that suggests violations of Bayesian updating or that subjects choose probabilities that are not independent of the choice situation.

To illustrate these models, we first consider a version of the two period model in which the period 2 choices are Anscombe-Aumann acts. Let $\mathcal{C}$ be a compact metric space, let S be a finite set of states, let $M = \Delta(\mathcal{C})^S$ be the set of Anscombe-Aumann acts and let Z be the set of non-empty compact subsets of M. In period 2, the agent has a preference $\succeq_2$ on M and in period 1 the agent has a preference $\succeq_1$ on Z. The set M is a mixture space and therefore Theorems 2-4 apply to this setting without change. Thus, the pair $(\succeq_1, \succeq_2)$ has a (u, v) representation if the preferences satisfy the conditions of Theorem 3.

The commitment utility $u : M \to \mathbb{R}$ and the temptation utility $v : M \to \mathbb{R}$ are continuous and linear and therefore there are continuous lottery utilities $u_s, s \in S$ and $v_s, s \in S$ such that for any $f \in M$,

$$u(f) = \sum_{s\in S} u_s(f_s)$$
$$v(f) = \sum_{s\in S} v_s(f_s)$$

The following two assumptions (due to Epstein (2006) and Epstein and Kopylov (2007)) imply that temptation and commitment utilities differ only in their priors over the states: Identify each constant act with its element of $\Delta(\mathcal{C})$. Then,

$$x \succeq_1 y \Rightarrow x \sim_1 x \cup y \text{ if } x, y \subset \Delta(\mathcal{C})$$

This assumption says that $(\succeq_1, \succeq_2)$ is a standard pair when restricted to constant acts. Furthermore, if $\{f(s)\} \succeq_1 \{g(s)\}$ for all s then f is better than g irrespective of the probabilities of states and, therefore, g cannot tempt f. For such acts we have:

$$\{f, g\} \sim_1 \{f\} \succeq_1 \{g\}$$

These assumptions imply that there is a continuous, linear lottery utility $w : \Delta(\mathcal{C}) \to \mathbb{R}$, priors $\alpha \in \Delta(S), \beta \in \Delta(S)$ and a weight $b \geq 0$ such that

$$u(f) = \sum_{s\in S} \alpha(s)w(f(s))$$
$$v(f) = b\sum_{s\in S} \beta(s)w(f(s))$$

Thus, the commitment and temptation utilities differ only in their priors α and β. If (u, v) is a self-control preference then the date-2 behavior maximizes expected utility for the prior

$$\gamma = \frac{\alpha + b\beta}{1 + b}$$

whereas if commitment is possible at date 1 then the agent commits to an act that maximizes expected utility for the prior α. Fix the lottery utility w and let $W(\cdot|\alpha, \beta, b)$ be the self-control utility corresponding to the parameters (w, α, β, b). That is,

$$W(z, \alpha, \beta, b) = \max_{f\in z} \left[\sum_S (\alpha(s) + b\beta(s))\, w(f)\right] - \max_{f\in z} \left[\sum_S b\beta(s)w(f)\right]$$

Epstein (2006) uses the model above to analyze agents who process information in ways that violate Bayes' rule. To this end, Epstein adds uncertainty in period 1. At the end of period 1, information about the state-2 probabilities arrives in the form of a signal $t \in T$ (finite). Period 1 choices are signal-contingent menus (of AA-acts). Let $\mathcal{F} = \{F : T \to Z\}$ be the collection of signal contingent menus. Period 1 preferences can be represented by a linear utility function $V : \mathcal{F} \to \mathbb{R}$ so that

$$V(F) = \sum_T \rho(t) W(F(t)|\alpha_t, \beta_t, b_t)$$

where $W(\cdot|\alpha_t, \beta_t, b_t)$ is a self-control utility as described above with parameters $(w, \alpha_t, \beta_t, b)$ and ρ is a probability on T. The probabilities α_t and β_t can be interpreted as updates of the probabilities of date-2 states. The probability α_t is the conditional probability associated with the commitment utility and β_t is the conditional probability associated with the temptation utility. The parameter b_t determines the weight of β_t for the period-2 preference.

As an illustration, consider the decision on whether or not to buy flood insurance. At the end of period 1, the agent observes whether $(t = 1)$ or not $(t = 2)$ there is a flood in a neighboring community. At the end of period 2, the agent's house may $(s = 1)$ or may not $(s = 2)$ be flooded. When choosing among signal-contingent menus, the agent believes the flood risks in his own and in the neighboring communities are independent and therefore the signal $t \in \{1, 2\}$ provides no information about the likelihood of a flood. Thus, $\alpha_1 = \alpha_2$ and the agent will choose the same insurance irrespective of what happens in the neighboring community if she can commit to an insurance plan prior to the signal arrival. If commitment is not possible and a flood in the neighboring community is observed, the agent is tempted to increase her assessment of the likelihood of a flood $(\beta_1(1) > \alpha_1(1))$. Upon observing signal $t = 1$, period 2 choices maximize expected utility with the flood probability

$$\gamma_1(1) = \frac{\alpha_1(1) + b\beta_1(1)}{1 + b} > \alpha_1(1)$$

As a result, the agent will buy additional insurance after observing the flood in the neighboring village. If no flood is observed in the neighboring community, then the agent experiences no such temptation $(\alpha_2 = \beta_2)$. Assuming that α_s presents the correct Bayesian update of the flood probabilities, the updating bias in this example illustrates the *hot hand fallacy.* The agent overreacts to information and buys more insurance after observing a flood in the neighboring community. As Epstein (2006) illustrates, this model is equally capable of describing an under-reaction to information.

There is evidence that economic agents systematically violate Bayes' rule and the temptation model can accommodate those violations. The temptation model also predicts that belief distortions are associated with a preference for commitment. To date, evidence of such a connection is missing.

5 Self-Deception and Guilt

The temptation model above assumes that agents are sophisticated, that is, adding an alternative to a menu is only of value if this alternative is chosen in the second period. The sophistication axiom (Axiom 3) captures this idea. A paper by Della Vigna and Malmendier (2006) on health club contract choice and attendance suggests that decision makers may violate this assumption.

To illustrate the connection between contract choice and sophistication, consider the following stylized example. Customers of a health club can choose between a monthly contract and a trial contract that allows the customer to attend the club at most k times. If a denotes the number of club attendances then the monthly contract corresponds to the menu $x = \{a| \, 0 \leq a \leq 30\}$ while the trial contract corresponds to the menu $y = \{a| \, 0 \leq a \leq k\}$ with $k < 30$. Sophistication requires that if the customer strictly prefers the monthly contract x to the trial contract y then, in period 2, she must choose $a > k$ if the monthly contract was chosen. Thus, health club attendance must be greater than k under the monthly contract.

Della Vigna and Malmendier (2006) examine the contract choice and attendance of the customers of 3 US health clubs. Among the contract choices, customers can pay \$10 per visit or choose a monthly contract for \$70. On average, customers on a monthly contract attend the health club approximately 4 times per month and pay roughly \$17 per attendance. Prior to the termination of the contract, there are, on average, 2.5 contract periods without attendance. Before cancelation, unsubsidized[6] customers spend, on average, \$187 for club membership without attendance. (Della Vigna and Malmendier

[6]Some customers' health club attendance is subsidized and these customers are excluded from these calculations.

(2006), p. 710). Among these findings, the behavior of customers 2-3 months prior to canceling the contract is the strongest evidence of a violation of the sophistication axiom. In this case, customers of health clubs pay for options they do not use.

Motivated by this evidence, Kopylov and Noor (2009) and Kopylov (2009a) relax the assumption of sophistication in the temptation model. The analysis below follows Kopylov (2009a). The utility $W : Z \to \mathbb{R}$ is a κ-self-control utility if there are u, v such that

$$W(x) := \max_{p\in x}[u(p) + v(p)] - \max_{p\in x} v(p) + \kappa \max_{p\in x} u(p)$$

for some continuous, linear functions $u : \Delta(\mathcal{C}) \to \mathbb{R}$ and $v : \Delta(\mathcal{C}) \to \mathbb{R}$ and some constant $\kappa \in (-1, \infty)$. As before, the utility u (the *commitment utility*) describes how the agent ranks singleton menus, that is, menus where the agent is committed to a choice in period 2. The lottery utility v (the *temptation utility*) measures how tempting an alternative is in the second period. First period choice of a $\kappa-$self-control agent maximizes the utility W above while second period choice maximizes $u + v$.

What distinguishes κ-self-control utilities from self-control utilities is the term

$$\kappa \max_{p\in x} u(p)$$

If $\kappa > 0$ the model captures agents who are prone to self-deception. Let x be the stylized monthly contract described above and let y be the contract that allows k attendances. Let $a^* > k$ be the u-maximal number of attendances from contract x and let $a \leq k$ be the $u + v$ maximal attendances from contract x. Then, a κ-self control agent attends the gym less than k times with either contract but nonetheless strictly prefers the monthly contract. We can interpret this behavior as self-deception: at the contract choice, the agent behaves as if there is some chance that she might maximize the commitment utility in period 2. However, the actual behavior in period 2 maximizes $u + v$.[7]

Kopylov's model also allows the possibility that $-1 < \kappa < 0$. In that case, the model captures agents who may strictly prefer the smaller menu y over x in the above health club example so as to avoid guilt associated with not exercising more than k.

The pair $(\succeq_1, \succeq_2)$ is a $\kappa-$self-control pair if $\succeq_1$ is represented by a $\kappa-$self-control utility with parameters u, v, κ and $\succeq_2$ maximizes $u + v$. As we argued above, $\kappa-$self-

[7]Alternatively, the agent may be described as partially naive. Kopylov (2010) uses the term "perfectionism" to describe this behavior.

control pairs may violate sophistication. However, those preferences will satisfy the following weaker version of sophistication.

Axiom 6 (Weak Sophistication). *If $x \cup \{p\} \succ_1 x$ then $p \succ_2 q$ for all $q \in x$ or $\{p\} \succ_1 \{q\}$ for all $q \in x$.*

Weak sophistication says that if the addition of an alternative strictly increases period 1 utility then this alternative must be chosen or must be maximal for the commitment preference. If p is not chosen in the second period, then this ranking is an instance of self-deception.

The $\kappa-$self control model also requires a weakening of Set-Betweenness. For example, assume that the menu specifies the number of times the person exercises during the next month. The three levels of exercise are $0, 5$ and 10 with $u(10) > u(5) > u(0)$ and $v(0) > v(5) > v(10)$ and $u(5) + v(5) > u(0) + v(0) > u(10) + v(10)$. Then, if κ is close to zero, it is easy to check that $y = \{0, 5\} \succ_1 x = \{0, 10\}$. However, for $\kappa > 0$ we have $x \cup y \succ_1 y$. Thus, Set-Betweenness is violated. However, the following weakening of Set-Betweenness holds. Let $Z_p = \{x \in Z | \, p \in x, \{p\} \succeq_1 \{q\}, \forall q \in x\}$ denote the set of menus that contain p and for which p is the optimal commitment choice.

Axiom 7 (κ–Set Betweenness). *If $x, y \in Z_p$ then $x \succeq_1 y$ implies $x \succeq_1 x \cup y \succeq_1 y$.*

This above axiom requires Set Betweenness only in situations where the menus share the same optimal commitment choice p. For those menus, the agent's inclination towards self-deception does not affect the rationale for Set Betweenness described above.

Kopylov (2009a) proves analogues of Theorems 2 and 3 for the $\kappa-$self-control model where sophistication and Set-Betweenness are replaced by the weak versions stated above.

Theorem 6. *The binary relation $\succeq_1$ satisfies Axioms $1, 2, 4^*$ and $\kappa-$Set Betweenness if and only if it is a κ-self-control preference.*

Source: Theorem 2.1 in Kopylov (2009a).

We say that $(\succeq_1, \succeq_2)$ is a regular pair if $\succeq_1$ is regular.

Theorem 7. *The regular pair $(\succeq_1, \succeq_2)$ satisfies Axioms $1, 2, 4^*$, $\kappa-$Set Betweenness and Weak Sophistication if and only if it is a $\kappa-$self-control pair.*

Source: Theorem 2.4 in Kopylov (2009a). ∎

To illustrate Theorems 6 and 7, consider the intertemporal choices in Casari's (2009) experiments. In period t, the agent chooses (according to the preference $\succeq_2$) consumptions for periods t and $t+1$. In period 1, the agent chooses (according to the preference $\succeq_1$) among menus of consumption pairs. The pair $(a, b) \in \mathbb{R}^2_+$ represents consumption a in period t and b in period $t+1$. The agent has a κ-self control preference represented by (u, v, κ). Let $u(a, b) = a + b$ and let $v(a, b) = \sigma a$. If $\sigma > .1$ then,

$$u(100, 0) + v(100, 0) > u(0, 110) + v(0, 110) > u(0, 105) + v(0, 105)$$

and

$$W(\{(0, 110)\}) > W(\{(0, 105)\}) > W(\{(100, 0)\})$$

Therefore, this agent exhibits a preference reversal. Sophistication would imply that $W(\{(100, 0), (0, 110)\}) = W(\{(100, 0)\})$ and therefore would also imply

$$W(\{(0, 105)\}) > W(\{(100, 0), (0, 110)\})$$

Thus, the agent should be willing to commit to 105 in period $t + 1$ rather than face the choice between $(100, 0)$ and $(0, 105)$ in period t. However, if $\kappa > 1$ then

$$W(\{(100, 0), (0, 110)\}) > W(\{(0, 105)\})$$

and, therefore, the agent refuses commitment. Self-deception, like self-control, weakens the association of preference reversals and preference for commitment. Self-control identifies instances where agents prefer commitment even when there are no preference reversals. Self-deception identifies instances where there is no preference for commitment even though there are preference reversals.

6 The Recursive Self-Control Model

The two period model of section 3 can be extended to multiple periods with consumption in every period. For a compact metric space X let $\Delta(X)$ be the set of Borel measures on X and let $\mathcal{K}(X)$ be the set of compact subsets of X. Then, define inductively, $Z_1 = \mathcal{K}(\Delta(\mathcal{C}))$ and for $t > 1$

$$Z_t = \mathcal{K}(\Delta(\mathcal{C} \times Z_{t-1})$$

The set Z_t represents $t-$period menus. Each choice $p \in Z_t$ is a probability measure over period-t consumption and period-$t+1$ menus. Gul and Pesendorfer (2004) show the existence of a compact metric space Z_∞ that satisfies the homeomorphism

$$Z_\infty =_{\text{homeo}} \mathcal{K}(\Delta(\mathcal{C} \times Z_\infty))$$

The set Z_∞ represents infinite-horizon menus.

In this setting, the agent must choose a lottery $p \in \Delta(\mathcal{C} \times Z_\infty)$ from some menu z in every period. The chosen lottery yields a consumption ($c \in \mathcal{C}$) in the current period and a menu of choices ($z \in Z_\infty$) for the next period. Thus, the primitive of the model is a preference $\succeq$ on $\Delta(\mathcal{C} \times Z_\infty)$. The recursive setting allows us to describe the agent with a single preference that governs behavior in every period.

Let $u : \Delta(\mathcal{C}) \to \mathbb{R}$ and $v : \Delta(\mathcal{C}) \to \mathbb{R}$ be lottery utilities, and let $\beta \in [0, 1), \delta \in (0, 1)$ be two discount factors. Given (u, v, δ, β) we define $U : \Delta(\mathcal{C} \times Z) \to \mathbb{R}, V : \Delta(\mathcal{C} \times Z) \to \mathbb{R}$ and $W : Z \to \mathbb{R}$ such that

$$\begin{aligned} U(p) &= \int [u(c) + \delta W(z)] dp \\ V(p) &= \int [v(c) + \beta W(z)] dp \\ W(z) &= \max_{p \in z} [U(p) + V(p)] - \max_{p \in z} V(p) \end{aligned} \tag{4}$$

The preference $\succeq$ is a *recursive self-control preference* with parameters (u, v, β, δ) if

$$p \succeq q \text{ if and only if } U(p) + V(p) \geq U(q) + V(q)$$

where U, V satisfy (4) above. Recursive self-control preferences are stationary and time-separable extensions of the self-control preferences described above. Noor (2010) shows that for each u, v, δ, β there is a U, V, W that satisfies (4) and hence the above system of equations is well–defined.

The function W is analogous to the value function in dynamic programming problems: it assigns a value to each menu. The function $U + V$ represents the choices in each period. Note that, unlike the two period model, the primitive of the dynamic model does not specify a ranking of menus. However, holding fixed current consumption, the preference $\succeq$ ranks continuation menus and this ranking is represented by the function W. Thus, W is the analogue of the menu utility in the two period case. The functions U and V are the analogues of the corresponding (lower case) functions for the two period case.

A special case of this model with $\beta = 0$ was introduced and axiomatized by Gul and Pesendorfer (2004); the general model is due to Krusell, Kuruscu and Smith (2010) and was axiomatized by Noor (2010).

6.1 Temptation and Taxation

As an illustration of the infinite horizon model, consider a deterministic dynamic economy with a single consumption good and a single capital good in every period. Let $\mathcal{C} = [0, n]$ be the set of possible consumptions. Prices of consumption and capital and the household's income are functions of the aggregate capital stock $\bar{k} \in [0, 1] = K$. The function $G : K \to K$ describes the evolution of aggregate capital over time. The function $m : K \to [0, n]$ is the households labor income (labor is supplied inelastically) as a function of the aggregate capital stock and $r : K \to [0, n]$ is the price of capital as a function of the aggregate capital stock. Both m and r are in terms of current period consumption.

The household with capital endowment k must choose consumptions c and capital holdings k' subject to the budget constraint

$$c + r(\bar{k})k' = m(\bar{k}) + kr(\bar{k})$$

in every period. The consumer's decision problem in this economy can be mapped to a collection of (non-stochastic) infinite horizon menus that depend on the aggregate and individual capital holdings. Let $x : K \times K \to Z_\infty$ be such that

$$x(\bar{k}, k) = \{(c, x(G(\bar{k}), k')) |\, c + r(\bar{k})k' = m(\bar{k}) + kr(\bar{k})\}$$

then $x(\bar{k}, k)$ is the infinite horizon menu for a household with capital holding k in an economy with aggregate capital stock $\bar{k}$.

Krusell, Kuruscu and Smith (2010) analyze competitive equilibria in an economy with standard neoclassical production and recursive self-control utilities. Let (c, z) be a degenerate lottery that yields consumption c and continuation menu z. Krusell et al. use the following parametrization of the self-control utilities:

$$U(c, z) = \ln c + \delta W(z)$$
$$V(c, z) = \gamma \ln c + \beta\delta W(z)$$

Let (γ, β, δ) be the relevant parameters of the model. If $\gamma = 0$ or if $\beta = 1$ then the utility reduces to the standard time-separable logarithmic utility. If $\beta < 1$ then the temptation utility V is less patient than the commitment utility V. As $\gamma \to \infty$ the model approximates the quasi-hyperbolic model. In the two-period model, we were able to analyze the limiting Strotz utility as well. For the recursive model, the optimization problem that defines the Strotz utility typically has no solution[8] and, therefore, Strotz utilities are not well-defined in this case.

Krusell et al. analyze optimal taxation in this model. Specifically, they assume that the government can impose a linear capital tax (subsidy) τ. Tax revenue is redistributed to consumers in the form of a lump-sum. A well-known result from the literature of optimal taxation states that long-run taxes on capital (or capital income) should be zero (Chamley (1985), Judd (1985)). Krusell et al show that this result does not hold when households have recursive temptation utilities. Specifically, they show that the optimal linear capital tax is

$$\tau = \frac{\gamma(\beta - 1)}{1 + \gamma + \frac{\delta}{1-\delta}(1 + \beta\gamma)}$$

When $\gamma = 0$ or $\beta = 1$ this tax is zero. When $\gamma > 0$ and $\beta < 1$ the optimal tax is negative and hence optimal government policy subsidizes investment income.

7 More on Choice from Menus

The assumption that choice from menus maximizes a preference rules out many choice behaviors that can be naturally interpreted and understood in terms of temptation. In this section, we discuss extended versions of the temptation model which address such behavior. These models continue to treat choice of a menu as represented by a revealed preference $\succeq_1$ over menus. However, we now take the behavior under the influence of temptation to be insufficiently regular to be described in terms of a single revealed preference relation. In other words, we consider models in which choice from menus does not satisfy the Weak Axiom of Revealed Preference (WARP). The next subsection considers such models in the simplest possible setting, namely, static, nonstochastic choice. The following subsections consider models in which WARP is violated because choice is stochastic (section 7.2) or subject to dynamic effects (section 7.4) with a digression on

[8] See Peleg and Yaari (1977), Gul and Pesendorfer (2005) for a discussion of this issue

multidimensionality (section 7.3).

7.1 Static Choice

We now assume choice from menus is deterministic and stationary in the sense that we can describe such choices via a *choice correspondence* — that is, a function $c : Z \to 2^{\Delta(\mathcal{C})}$ such that $c(x) \subseteq x$, $c(x) \neq \emptyset$ for all $x \in Z$.

It is well–known that there exists a complete and transitive $\succeq_2$ which rationalizes c if and only if c satisfies the Weak Axiom of Revealed Preference, or WARP.

Axiom 8 (Weak Axiom of Revealed Preference)**.** *If $p, q \in x \cap y$, $p \in c(x)$, and $q \in c(y)$, then $p \in c(y)$.*

Thus if c satisfies WARP, we can work with the revealed preference $\succeq_2$ instead of a choice correspondence and recover the formulation used in section 3.

It is not obvious that we should assume that choice under temptation satisfies WARP. The following example appears in various forms in Fudenberg and Levine (2006), Dekel, Lipman, and Rustichini (2009) and Noor and Takeoka (2010a, 2010b). Consider a menu consisting of a healthy item, denoted h, and a small dessert d. Then d may not be so tempting as to interfere with the decision maker's choice so that he chooses h easily. Thus $c(\{h, d\}) = \{h\}$. However, suppose we add a large dessert D to this menu. Now the decision maker's desire for a sweet dessert may be kindled, leading him to compromise and choose d. Thus $c(\{h, d, D\}) = \{d\}$, so the choice correspondence violates WARP.

Nehring (2006) considers one possible approach to understanding such violations of WARP. He gives an axiomatic characterization of a representation in which temptation is a force that can be controlled by the agent where there is a cost to manipulating the strength of temptation. In this survey, we focus on models that interpret temptation as a fixed force that alters behavior, necessitates self–control or both, though we note that there are interesting connections between the models we focus on and other approaches such as Nehring's.

Fudenberg and Levine (2006, 2010a, 2010b) and Noor and Takeoka (2010a, 2010b) provide versions of the self–control model which maintain the simplicity of the model but enrich it to accommodate such violations of WARP. In the process, they are also able to explain and interpret a large variety of experimental data, including Allais type behavior as well as the preference reversals we discussed in section 1. The basic idea is to keep the essential structure of the self–control model but introduce the minimal nonlinearity needed to generate such violations of WARP.

For simplicity, we focus on the *menu–dependent self–control representation* of Noor and Takeoka (2010b), but the other models of Fudenberg–Levine and Noor–Takeoka have similar properties. Now we represent a pair $(\succeq_1, c)$ by means of a triple of functions (u, v, ψ). As before, u and v are continuous linear functions defined over $\Delta(\mathcal{C})$ where u will represent the commitment preference and v will represent temptation. The function $\psi : \mathbb{R} \to \mathbb{R}_+$ is an increasing function. We require that the function

$$W(x) = \max_{p \in x} \left[u(p) - \psi\left(\max_{q \in x} v(q)\right)\left(\max_{q \in x} v(q) - v(p)\right)\right]$$

represents $\succeq_1$ and

$$c(x) = \text{argmax}_{p \in x} \left[u(p) + \psi\left(\max_{q \in x} v(q)\right) v(p)\right].$$

In other words, the cost of self control incurred by choosing p from x is still related to $\max_{q \in x} v(q) - v(p)$ but is now scaled up by an increasing function of the "level of temptation" in the menu as represented by $\max_{q \in x} v(q)$. Thus the level of temptation shifts the extent to which choice is driven by u or by v.

Clearly, if we add an item to the menu which increases the maximum v, we make the cost of self–control higher. Thus we can generate exactly the violation of WARP described above. To see this concretely, define u and v by

	u	v
h	10	0
d	5	2
D	-4	4

Assume $\psi(a) = a$. Then the choice from the menu $\{h, d\}$ is h if

$$u(h) + \psi(\max\{v(h), v(d)\})v(h) > u(d) + \psi(\max\{v(h), v(d)\})v(d).$$

It is easy to verify that this holds given the specification of (u, v, ψ) above. However, the choice from $\{h, d, D\}$ is d if

$$u(d) + \bar{\psi}v(d) > \max\left\{u(h) + \bar{\psi}v(h), u(D) + \bar{\psi}v(D)\right\}$$

where $\bar{\psi} = \psi(\max\{v(h), v(d), v(D)\})$. It is easy to verify that this is satisfied.

Noor and Takeoka's axiomatization of this model retains all the axioms of the self–control model except for independence. To see why relaxing independence is crucial, recall from section 2 that the justification for independence relies in part on indifference to the timing of the resolution of uncertainty. More specifically, it relies on the hypothesis that the decision maker views the menu $\alpha x + (1-\alpha)y$ as the same as the lottery giving menu x with probability α and y otherwise. As explained earlier, $\alpha x + (1-\alpha)y$ can be interpreted as the set of options available when the decision maker chooses as a function of the outcome of the lottery but makes this choice prior to seeing the outcome. By contrast, the lottery corresponds to making the choice from the menu after seeing the outcome.

The key to temptation, of course, is the idea that what the decision maker chooses in advance and what he chooses faced with the actual situation may be different. Thus it is not clear that this indifference to timing is natural for temptation. For example, suppose $x = \{h, d\}$ where h is a healthy dish and d a tempting dessert and that $y = \{h\}$. Thus $\lambda x + (1-\lambda)y = \{h, \lambda d + (1-\lambda)h\}$ where $\lambda d + (1-\lambda)h$ denotes the lottery giving d with probability λ and h otherwise. If the decision maker chooses from $\lambda x + (1-\lambda)y$, refraining from the tempting lottery $\lambda d+(1-\lambda)h$ may not be difficult since the uncertainty lowers the appeal of the temptation. On the other hand, with the lottery over menus, the decision maker chooses from $\{h, d\}$ with probability $1-\lambda$, where the temptation's impact may be much stronger. Hence it seems plausible that the decision maker may prefer $\alpha x + (1-\alpha)y$ to the lottery over x and y since the choice from this menu is less influenced by temptation.

In particular, if we consider menus x and y with $x \sim y$, it seems plausible that the decision maker would be indifferent between a lottery over x and y versus x for sure. But then we would expect $\alpha x + (1-\alpha)y \succeq x$. That is, it might be more natural to assume that $\succeq$ is quasi–concave or, in terms of the representation, that self–control costs are convex, just as in the Fudenberg–Levine and Noor–Takeoka models.

Like the other models by these authors, the menu–dependent self–control model is capable of generating a variety of seemingly anomalous choice behavior, including Allais type behavior, despite the fact that both the u and v represent expected–utility preferences over lotteries and hence *cannot* rationalize such behavior. To see the idea, first consider Allais behavior. Assume the commitment preference reflects risk neutrality, so

$u(w) = w$, but that the agent is tempted by a "sure thing." For example, assume that the temptation preference prefers \$3000 for sure to \$4000 with probability .8 (0 otherwise), even though the risk neutral commitment preference ranks the latter higher. Given a choice between these two lotteries, the agent chooses the "tempting" sure thing if

$$3000 + \psi(v(3000))v(3000) > (.8)(4000) + \psi(v(3000))(.8)v(4000)$$

where we normalize so that $v(0) = 0$. One version of the Allais paradox is that many agents make this choice, but when faced with the choice between \$3000 with probability .25 (0 otherwise) and \$4000 with probability .2 (0 otherwise), they choose the latter, a pair of choices inconsistent with the independence axiom for preferences over lotteries. This second choice is predicted by the menu–dependent model whenever

$$(.25)(3000) + \psi(.25v(3000))(.25)v(3000) < (.2)(4000) + \psi(.25v(3000))(.2)v(4000).$$

The pair of choices is therefore predicted iff

$$\psi(v(3000)) > \frac{200}{v(3000) - .8v(4000)} > \psi(.25v(3000)).$$

In other words, if u is risk neutral and v prefers the sure–thing, then the Allais choices emerge when the menu–dependence effect is such that ψ is increasing at the "right" rate.

This effect can also explain some experimental evidence relating choice reversals and risk preferences. Keren and Roelofsma (1995) and Weber and Chapman (2005) present evidence that making all rewards probabilistic dampens the preference reversal effect. To show how menu–dependent self–control explains these observations, we first give a rationalization of the preference reversal phenomena in the self–control model.[9] Suppose that the commitment preference evaluates consumption of c in period t by $\delta^{t-1}w(c)$, while the temptation preference evaluates it by $(\beta\delta)^{t-1}w(c)$ where $\beta \in (0, 1)$ and $w(0) = 0$. Consider an agent who prefers a small reward s today rather than a large reward ℓ tomorrow, but reverses this preference if both are pushed forward t periods. This behavior is consistent with the self–control model if

$$\beta\delta w(\ell) < w(s) < \delta w(\ell)$$
$$2w(s) > \delta(1 + \beta)w(\ell)$$
$$\delta^t(1 + \beta^t)w(\ell) > \delta^{t-1}(1 + \beta^{t-1})w(s).$$

[9] We discussed other rationalizations using the (u, v) model in section 4.1, but this version will prove useful to illustrate the menu–dependent model.

It is easy to see that $w(s) < \delta w(\ell)$ implies the latter inequality must hold for t sufficiently large.

The self–control model cannot, however, accommodate the evidence that the preference reversal is less likely if the rewards are probabilistic. To see the point, suppose we replace s with receiving s with probability α, 0 otherwise, and replace ℓ similarly. Since the commitment and temptation utilities in the self–control model are linear, the model still predicts the preference reversal. That is, all the inequalities above would still hold if we replace s and ℓ with the corresponding lotteries.

In the menu–dependent self–control model, we have the preference reversal with deterministic rewards if

$$\beta\delta w(\ell) < w(s) < \delta w(\ell)$$
$$(1+\psi_1)w(s) > \delta(1+\psi_1\beta)w(\ell)$$
$$\delta^t(1+\psi_2\beta^t)w(\ell) > \delta^{t-1}(1+\psi_2\beta^{t-1})w(s)$$

where $\psi_1 = \psi(w(s))$ and $\psi_2 = \psi(\beta^{t-1}\delta^{t-1}w(s))$. Again, the other inequalities imply that this last inequality must hold for t sufficiently large. However, once we make the rewards probabilistic, there is no preference reversal if

$$(1+\psi_3)w(s) < \delta(1+\beta\psi_3)w(\ell)$$

where $\psi_3 = \psi(\alpha w(s))$. Since ψ is increasing, we have $\psi_1 > \psi_3$, so this is consistent with the preference reversal for deterministic rewards as long as

$$\psi_1 > \frac{\delta w(\ell) - w(s)}{w(s) - \beta\delta w(\ell)} > \psi_3.$$

Thus with an appropriately chosen randomization, the preference reversal disappears and the agent always prefers the larger, later reward, as observed by Keren and Roelofsma (1995) and Weber and Chapman (2005).

The menu–dependent self–control model may also yield a new perspective on the *compromise effect*, first noted by Simonson (1989). The compromise effect refers to the effect on choices between two options, say A and B, from adding a third option which leads one of the original two to act as a compromise. For example, suppose A and B are two apartments for rent where A is closer to the decision maker's workplace, but B is in better condition. Thus neither option dominates the other. Now suppose a third option C is added which is further from the subject's workplace than B but also in better condition than B. Then B is in the middle on both attributes and thus is, in a sense, a natural compromise. Simonson showed that adding such a third option could shift choice systematically toward option B.

It is easy to see that the small dessert–large dessert example above follows *almost* the same scenario. The small dessert d is less healthy than the healthy option h but presumably tastes better. The large dessert D is even less healthy and even more tasty, thus making d a natural compromise.[10] However, the behavior differs from the classic compromise effect in that the latter is *symmetric*, while the effect in Noor and Takeoka's model is not. That is, the menu–dependent self–control model would not predict that adding an option H which is healthier and less tasty than h would shift choice from $\{h, H, d\}$ toward h. We know of no evidence for or against such an asymmetric version of the compromise effect.

Finally, Noor and Takeoka (2010b) note the possibility of extending this model further to incorporate a broader range of menu dependence effects. In particular, they discuss a representation of the form

$$W(x) = \max_{p \in x} \left[u(p) - \psi(x) \left(\max_{q \in x} v(q) - v(p) \right) \right]$$

where the scalar giving the effect of the menu on self–control costs is a general function of the menu, rather than depending on the menu only through $\max_{q \in x} v(q)$. One nice feature of such a representation is that it would allow representations along the lines of Benhabib and Bisin (2005). In their model, the decision maker is a Strotz agent unless the gain in utility from self control exceeds some threshold, in which case the decision maker exerts complete self control. One version of this idea would be to use the representation above where

$$\psi(x) = \begin{cases} \infty, & \text{if } \max_{p \in x}[u(p) + v(p)] - \max_{q \in x} v(q) - \beta \leq \max_{p \in B_v(x)} u(p); \\ 1, & \text{otherwise.} \end{cases}$$

In other words, the decision maker appears to be Strotzian if the utility gain from self–control is below some threshold and looks like a self–control agent otherwise.

[10] See Chandrasekher (2010) for an alternative temptation interpretation of the compromise effect.

7.2 Random Choice

One simple reason why WARP may fail is that choice may be random. In this case, it is not appropriate to represent choice via a deterministic choice correspondence. While moving to a model of random choice is more complex, there are several reasons it can be useful to do so.

We illustrate with two examples drawn from Dekel, Lipman, and Rustichini (2009). First, let b denote broccoli, c chocolate, and p potato chips. Then for a dieting agent, the ranking $\{b\} \succ_1 \{b, c\} \succeq_1 \{b, p\} \succ_1 \{b, c, p\}$ seems very natural. (The roles of $\{b, c\}$ and $\{b, p\}$ could be reversed without important changes.) This just says that committing to one's diet is best, having one snack available is worse, and that having two snacks available is still worse. Yet this ranking is impossible in the (u, v) model. To see this, recall from Theorem 2 that both Strotz and self–control preferences must satisfy Set Betweenness — that is, $x \succeq_1 y$ implies $x \succeq_1 x \cup y \succeq_1 y$. But then Set Betweenness requires that if $\{b, c\} \succeq_1 \{b, p\}$, we must have $\{b, c\} \succeq_1 \{b, c\} \cup \{b, p\} = \{b, c, p\} \succeq_1 \{b, p\}$.

How should we think about our intuition that two snacks could be strictly worse than either one individually? One hypothesis, which we return to in section 7.3, is that it is harder to resist two fattening snacks together than it would be to resist either one separately. Another hypothesis is uncertainty. If the agent is not sure what kind of temptations will affect him, then having more options available increases the probability of succumbing to temptation. Thus uncertainty, while not obviously relevant to the initial pattern of choice, becomes one simple way to understand it.

In the second example, we return to the example of section 7.1 with the healthy dish h, the small dessert d, and the large dessert D. It seems quite plausible for a dieting agent to have the preference $\{h, d\} \succ_1 \{d\}$ and $\{h, d, D\} \succ_1 \{h, D\}$. The former simply says that the agent prefers a chance at maintaining her diet. The latter says that the small dessert could be a valuable compromise option when tempted by the large one.

Again, the (u, v) model does not allow this preference. To see this, note that Sophistication says that since $\{h, d\} \succ_1 \{d\}$, we must have $h \succ_2 d$. Also, by Sophistication, $\{h, d, D\} \succ_1 \{h, D\}$ implies $d \succ_2 h$, a contradiction.

As noted in section 7.1, we can interpret this example by hypothesizing that there is no coherent revealed preference $\succ_2$ since choice from menus violates WARP. Alternatively, uncertainty gives another way to understand this behavior. With uncertainty, we can interpret $\{h, d\} \succ_1 \{d\}$ as saying that the agent strictly values the option of h since he may choose it in some situations. It may well be true that in any situation in which the agent would choose h from $\{h, d\}$, he is very capable of resisting temptation and so d is not helpful in $\{h, d, D\}$. But with uncertainty, there may be other possible situations, including some where he cannot resist temptation so easily and would find d a valuable option in $\{h, d, D\}$. In short, uncertainty in *ex post* choice is a handy hypothesis for understanding natural patterns in *ex ante* choice.

To formalize these ideas, we replace the choice correspondence c with a *random* choice correspondence C. That is, $C : Z \to 2^{\Delta(\Delta(\mathcal{C}))}$ where $\Delta(\Delta(\mathcal{C}))$ denotes the set of probability measures over $\Delta(\mathcal{C})$. We require $C(x) \neq \emptyset$ for all x and that for every $\rho \in C(x)$, we have $\rho(x) = 1$.

We begin by considering representations of $\succeq_1$ only. Naturally, one could add uncertainty to either a Strotz model of temptation or a self–control model and we consider both possibilities. Surprisingly, there is a sense in which the random Strotz model nests the random self–control model and in which the models are equivalent given a Lipschitz continuity restriction.

A *random self–control representation* is a pair (u, ν) where $u : \Delta(\mathcal{C}) \to \mathbb{R}$ is a linear, continuous utility and ν is a measure over linear, continuous utilities with the property that the utility $W : Z \to \mathbb{R}$ defined by

$$W(x) = \int \left\{ \max_{p \in x} [u(p) + v(p)] - \max_{p \in x} v(p) \right\} \nu(dv)$$

represents $\succeq_1$. This is exactly the self–control representation modified so that v is uncertain *ex ante*.

Similarly, a *random Strotz representation* is a pair (u, μ) where u is a linear continuous utility, μ is a measure over linear, continuous utilities, and the utility $W : Z \to \mathbb{R}$ defined by

$$W(x) = \int \max_{p \in B_v(x)} u(p) \, \mu(dv)$$

represents $\succeq_1$ where

$$B_v(x) = \{p \in x \mid v(p) \geq v(q), \quad \forall q \in x\}.$$

This is exactly the Strotz representation but modified so that the v is uncertain *ex ante*.[11]

We have the following result due to Dekel and Lipman (2010).

Theorem 8. *If $\succeq_1$ has a self–control representation or a random self–control representation, then it has a random Strotz representation. In particular,*

$$\max_{p \in x}[u(p) + v(p)] - \max_{q \in x} v(q) = \int_0^1 \max_{p \in B_{v+Au}(x)} u(p)\, dA,$$

a random Strotz representation where control is by utility function $v + Au$ where $A \sim U[0,1]$.

In other words, any choice of a menu which can be explained by the self–control or random self–control representation can also be explained by random Strotz. Thus it is the combination of choice of a menu and choices from menus that distinguishes the models.

We showed earlier that a Strotz preference, hence a random Strotz preference, may fail to be continuous, while a self–control preference and hence a random self–control preference must be continuous. Thus the converse to Theorem 8 is false. On the other hand, Dekel and Lipman present a partial converse. We say that $W : Z \to \mathbb{R}$ is Lipschitz continuous if there exists $K \in \mathbb{R}$ such that for every $x, y \in Z$,

$$W(x) - W(y) \leq K d_h(x, y),$$

where $d_h(x, y)$ denotes Hausdorff distance (see footnote 2 for a definition). Then[12]

Theorem 9. *If $\succeq_1$ has a Lipschitz continuous random Strotz representation, then it has a random self–control representation.*

Dekel and Lipman prove this result by showing that certain axioms for $\succeq_1$ implied by the existence of a Lipschitz continuous random Strotz representation imply in turn the existence of a (Lipschitz continuous) random self–control representation. This second

[11] Caplin and Leahy (2006) discuss a version of this model and prove some continuity properties for it. Chatterjee and Krishna (2009) prove a representation theorem for a special case of the random Strotz model.

[12] To be precise, Dekel and Lipman prove this result for the case where $\mathcal{C}$ is finite, not the more general case considered elsewhere in this paper where $\mathcal{C}$ is a compact metric space.

step generalizes a result due to Stovall (2010) who proves the analogous result for the case where the measure ν has finite support. The properties of $\succeq_1$ Dekel and Lipman use are Axioms 1, 2, and 4, an axiom which ensures Lipschitz continuity due to Dekel, Lipman, Rustichini, and Sarver (2007), and a weakening of Set Betweenness which Dekel and Lipman call Weak Set Betweenness[13]:

Axiom 9 (Weak Set Betweenness)**.** *Suppose that for every $p \in x$ and $q \in y$, we have $\{p\} \succeq_1 \{q\}$. Then $x \succeq_1 x \cup y \succeq_1 y$.*

To see the intuition, note that the hypothesis of the axiom is that the decision maker prefers committing to any option in x over any option in y. In this sense, every element of x is better than any element of y. Hence it seems natural to conclude that $x \succeq_1 y$. To see why $x \cup y$ is naturally ranked between x and y, note that it can be thought of as taking x and adding inferior elements to it, creating a set worse than x, or as taking y and adding superior elements to it, creating a menu better than y.

We say that $(\succeq_1, C)$ has a random self–control representation (u, ν) if (u, ν) represents $\succeq_1$ as defined above and if for every menu x, $\rho \in C(x)$ if and only if there exists a measurable $p^* : \mathrm{supp}(\nu) \to x$ such that

$$\rho(E) = \nu\left(\{v \in \mathrm{supp}(\nu) \mid p^*(v) \in E\}\right), \text{ for all measurable } E \subseteq x$$

and

$$u(p^*(v)) + v(p^*(v)) \geq u(q) + v(q), \quad \forall q \in x.$$

Similarly, we say that $(\succeq_1, C)$ has a random Strotz representation (u, μ) if (u, μ) represents $\succeq_1$ and if for every menu x, $\rho \in C(x)$ if and only if there exists a measurable $p^{**} : \mathrm{supp}(\mu) \to x$ such that

$$\rho(E) = \mu\left(\{v \in \mathrm{supp}(\mu) \mid p^{**}(v) \in E\}\right), \text{ for all measurable } E \subseteq x$$

and

$$u(p^{**}(v)) = \max_{p \in x} u(p) \text{ subject to } v(p) \geq v(q), \quad \forall q \in x.$$

Dekel and Lipman show that, except for the special case of no temptation, $(\succeq_1, C)$ cannot have both a random self–control representation and a random Strotz representation. They demonstrate this by showing an interesting relationship between the choice

[13]This axiom was originally discussed by Dekel, Lipman, and Rustichini (2009). Stovall (2010) uses a different formulation which he shows is equivalent given the other axioms.

from menus exhibited by a random self–control and random Strotz representation of the same $\succeq_1$.

Theorem 10. *Suppose $(\succeq_1, C_1)$ has a random self–control representation and $(\succeq_1, C_2)$ has a random Strotz representation. Then the behavior under C_2 is "worse" than that under C_1 in the sense that for every x, for every $\rho_1 \in C_1(x)$ and $\rho_2 \in C_2(x)$, we have $\{\int p\, \rho_1(dp)\} \succeq_1 \{\int p\, \rho_2(dp)\}$. If $\succeq_1$ does not have a standard representation, then this comparison is strict for some x, ρ_1, and ρ_2.*

The intuition for this result is simple. Consider a menu $\{p, q\}$ and suppose that $\{p\} \succ_1 \{p, q\} \succ_1 \{q\}$. Suppose $\succeq_1$ has a (nonrandom) self–control representation and hence has a random Strotz representation. Because $\{p, q\} \succ_1 \{q\}$, Sophistication implies that $p \succ_2 q$, so p is chosen from $\{p, q\}$ in C_1. Thus the self–control representation explains the fact that $\{p, q\} \prec_1 \{p\}$ entirely by the cost of self control. On the other hand, if $(\succeq_1, C_2)$ has a random Strotz representation, the fact that $\{p\} \succ_1 \{p, q\} \succ_1 \{q\}$ implies that both p and q are chosen with positive probability from $\{p, q\}$. That is, the fact that $\{p, q\} \prec_1 \{p\}$ is explained not by control costs but by a positive probability of "bad" behavior. Put differently, a random self–control model explains why x is worse than the best commitment from x partly by succumbing to temptation and partly by self–control costs. Since random Strotz has only the former behavior to explain the same ranking, it must predict more giving in to temptation.

Note that these comments assume that we consider representations where the decision maker does not engage in self–deception. For example, the Kopylov representation of $\succeq_1$ discussed in Theorem 6 is a random self–control representation when $\kappa > 0$. Given $\succeq_1$ only, we cannot distinguish these models. Instead, we need to see whether choice from menus is representable by a $\succeq_2$ or the appropriate random choice correspondence to determine which of these models describes the complete behavior.

7.3 Multidimensionality

Many of the behaviors which can be interpreted in terms of random temptation can alternatively be understood in terms of multidimensional temptation. For example, consider the broccoli–chocolate–potato chips example of the previous subsection where $\{b\} \succ_1 \{b, c\} \succeq_1 \{b, p\} \succ_1 \{b, c, p\}$. Random temptation models would rationalize this by having (at least) two distinct temptations with positive probability, one temptation under which c is more tempting than p and one for which p is the more tempting. Alternatively, we could hypothesize that two distinct temptations, one preferring c, one preferring p, *simultaneously* affect the agent.

It is not hard to see that these two hypotheses could be distinguished by observations of choices from menus. If the correct explanation is random temptation, then we would expect to see random choice. On the other hand, if both temptations affect the agent simultaneously, there is no reason to predict random choice. What is less obvious is that, while these two hypotheses seem to have very similar effects on *ex ante* preferences, they are distinguishable in some cases.

Because of the natural connection to random temptation, we follow our discussion of random temptation in the previous subsection with a discussion of multidimensional temptations in this one. Because this model is one in which choice from menus does satisfy WARP and so can be represented by a single revealed preference, it does not fit with the other models discussed in this section, so it is something of a digression.

Dekel, Lipman, and Rustichini (2009) consider two modifications to the self–control model. First, they introduce randomness as discussed in the previous subsection. Second, they introduce multidimensional self–control costs. For simplicity, we focus only on the latter in this subsection. Given some integer J and continuous, linear utility functions over lotteries u and $v_1, \ldots, v_J$, define a utility function $W : Z \to \mathbb{R}$ by

$$W(x) = \max_{p \in x} \left[u(p) + \sum_{j=1}^{J} v_j(p) \right] - \sum_{j=1}^{J} \max_{p \in x} v_j(p).$$

We say that $(\succeq_1, \succeq_2)$ is a *multidimensional self–control preference* if $\succeq_1$ is represented by W and $\succeq_2$ is represented by $u + \sum_j v_j$. The interpretation is that the decision maker is subject to numerous temptations at the same time. Dekel, Lipman, and Rustichini show that this representation is characterized by the same axioms which characterize the self–control representation discussed in section 3 except that Set Betweenness is replaced by

Axiom 10. $\succeq_1$ *satisfies* Positive Set Betweenness *if* $x \succeq y$ *implies* $x \succeq x \cup y$.

In other words, only the "top half" of Set Betweenness is required.[14]

[14]Dekel, Lipman, and Rustichini also assume that $\mathcal{C}$ is finite and employ a finiteness axiom to be able

It is not hard to show that neither Positive Set Betweenness nor Weak Set Betweenness implies the other. Hence neither the multidimensional self–control nor the random self–control model nests the other. One might expect that the only preferences $\succeq_1$ with both kinds of representation would be those with a self–control representation. Intuitively, the multidimensional self–control representation has no randomness, but the cost of self–control depends on an arbitrary number of v's, while the random self–control representation has randomness over the costs but each possible cost depends on only a single v. Hence it seems natural to say that any preference which can be represented both ways has one cost function which depends on only one v and hence is a self–control preference. One can give counterexamples to show that this is not true.[15] This shows that multidimensionality and random self–control sometimes cannot be distinguished on the basis of preferences over menus alone.

What behavior can be explained by the random temptation models but not multidimensional self control? It is not hard to generate intuitive violations of Positive Set Betweenness in the random Strotz model. To see this, let $x = \{h_1, d_1\}$ and $y = \{h_2, d_2\}$ where the h_i's are healthy dishes and the d_i's unhealthy desserts. Assume $\{h_1\} \sim_1 \{h_2\} \succ_1 \{d_1\} \sim_1 \{d_2\}$, so the healthy dishes are equally good according to the commitment preference and are better than the two desserts which are equally good as one another. Suppose the decision maker is subject to two equally likely temptations, v_1 and v_2, where $v_1(h_1) > v_1(d_2) > v_1(h_2) > v_1(d_1)$ and $v_2(h_2) > v_2(d_1) > v_2(h_1) > v_2(d_2)$. Then the menu x generates a lottery between h_1 (consumed if the temptation is v_1) and d_1 (consumed under v_2), while y corresponds to a lottery between d_2 and h_2. But under the menu $x \cup y$, the choice is a lottery between h_1 (chosen by v_1) and h_2 (chosen by v_2). Hence this is preferred so we have $x \cup y \succ x \sim y$, violating Positive Set Betweenness.

What behavior does the multidimensional self–control model accommodate which the random temptation models cannot? Suppose we have menus x and y which satisfy Positive Set Betweenness but violate Weak Set Betweenness. If these menus are both singletons, so $x = \{p\}$ and $y = \{q\}$, then it is not hard to see that we must have $\{p\} \succeq_1 \{q\} \succ_1 \{p, q\}$. That is, the decision maker would prefer *either* commitment from $\{p, q\}$ to the menu itself. Intuitively, this is a situation of conflicting temptations, where

to restrict the collection of v's to a finite set.

[15] For example, fix any v_1 and v_2 and let $u = v_2 - 3v_1$. It is not hard to show that the multidimensional self–control representation (u, v_1, v_2) is equivalent to the random self–control representation putting probability $1/2$ on each of $2v_1$ and $2v_2$.

q tempts the agent away from p, making it hard to choose p, but similarly the presence of p makes it hard to choose q. For example, if p is a sweet snack and q a salty one, the decision maker may be torn between them, knowing that both is not an appealing option. Arguably, there is an element of regret in this behavior — see Sarver (2008) and Stovall (2010) for discussion.

7.4 Dynamic Effects and Addiction

Another natural reason for choice behavior to violate WARP is the presence of dynamic effects. For an example involving temptation, suppose that consumption is nonstationary because consumption at one date affects self–control and hence choices at later dates. This effect is particularly natural for consumption of addictive substances — indeed, the very definition of addiction seems to necessitate such effects.

In this subsection, we work with the framework of section 6 where Z_∞ is the space of infinite horizon menus which is homeomorphic to the set of lotteries over current consumption and continuation menus. We specialize so that an element of $\mathcal{C}$, the space of consumptions, can be written as a pair (c, d) where d denotes the consumption of drugs (or some other potentially addictive good) and c denotes consumption of all other goods. We generalize the approach taken in section 6 to allow the preference $\succeq$ over $\Delta(\mathcal{C} \times Z_\infty)$ to depend on the state of the agent, denoted $s \in S = [0, 1]$. Thus we now have a family of preferences $\{\succeq^s\}$ over $\Delta(\mathcal{C} \times Z_\infty)$. While one could consider more complex dynamics, for simplicity, we restrict attention to the case where s is given by the previous period's consumption of d.

Gul and Pesendorfer (2007) show how to extend the axiomatization of the recursive self–control model of Gul and Pesendorfer (2004) to this setting in the following way. As in section 6, let $u : \Delta(\mathcal{C}) \to \mathbb{R}$ and $v : \Delta(\mathcal{C}) \to \mathbb{R}$ be continuous and linear lottery utilities and let $\delta \in (0, 1)$ be a discount factor. We assume that v depends only on the marginal over d and write $v(d)$. We also now have an increasing function $\sigma : [0, 1] \to \mathbb{R}_+$ which will represent the effect of the state. Given (u, v, δ, σ), we define $U : \Delta(\mathcal{C} \times Z) \times S \to \mathbb{R}$ and $W : Z \times S \to \mathbb{R}$ analogously to the way we did in section 6. For simplicity, we write out the case of degenerate lotteries only, so

$$U(c, d, x, s) = u(c, d) + \delta W(x, d)$$

$$W(z,s) = \max_{(c,d,x)\in z} [U(c,d,x,s) + \sigma(s)v(d)] - \max_{(c,d,x)\in z} \sigma(s)v(d).$$

(Recall that s is the previous period's d. Hence the W on the right–hand side of the first equation is evaluated at the state d.) The family of preferences $\{\succeq^s\}$ is a recursive addictive self–control preference with parameters (u, v, δ, σ) if $(c, d, x) \succeq^s (c', d', x')$ iff

$$u(c,d) + \sigma(s)v(d) + \delta W(x,d) \geq u(c',d') + \sigma(s)v(d') + \delta W(x',d').$$

In other words, the higher is the previous period's consumption of drugs, the larger is the current state s. Since σ is increasing, this means that past drug consumption influences current choice by putting more weight on the temptation preference v relative to the dynamic commitment preference $u + \delta W$ in decision making.

Alternatively, one could introduce the effect of past consumption in a random Strotz model by having past consumption affect the probability distribution over the v which gets maximized. To see a simple version of this, we use the uniform construction stated in Theorem 8 to rewrite the W above as

$$W(z,s) = \int_0^1 \max_{(c,d,x)\in B_{\sigma v + AU}(z)} U(c,d,x,s)\, dA.$$

Note though that we can renormalize $\sigma v + AU$ to

$$v(d) + \frac{A}{\sigma(s)} U(c,d,x,s).$$

Letting $\tau(s) = 1/\sigma(s)$, we have

$$\begin{aligned} W(z,s) &= \int_0^1 \max_{(c,d,x)\in B_{v+\tau AU}(z)} U\, dA \\ &= \int_0^{\tau(s)} \max_{(c,d,x)\in B_{v+aU}(z)} U\sigma(s)\, da \end{aligned}$$

where the second line follows from the change of variables $a = \tau A$. As s increases, $\sigma(s)$ increases, so $\tau(s)$ decreases. Hence the distribution of A becomes uniform over an interval more tightly concentrated near 0. In other words, the effect of increasing s on the weight on v in the self–control model translates into a higher probability on v's "far away" from u in the random self–control model.[16]

[16] One can rewrite Noor and Takeoka's (2010b) menu–dependent self–control representation as a random Strotz representation where the "level of temptation" in the menu affects the distribution of A in an analogous fashion.

To see the idea more concretely, suppose we have a menu $z = \{(c_1, 0, x), (c_2, 1, x)\}$. That is, the decision maker can either consume no drugs ($d = 0$) and have nondrug consumption of c_1 with continuation menu x or can consume drugs ($d = 1$) with nondrug consumption of c_2 and the same continuation menu x. In the self–control model, the decision maker consumes drugs from this menu in state s if

$$u(c_2) + \sigma(s)v(1) + \delta W(x, 1) > u(c_1) + \sigma(s)v(0) + \delta W(x, 0).$$

Consider a random Strotz representation of the same preference over menus. The representation will have the same W function and would predict that the choice from z in state s is to consume drugs if

$$\sigma(s)v(1) + A\left[u(c_2) + \delta W(x, 1)\right] > \sigma(s)v(0) + A\left[u(c_1) + \delta W(x, 0)\right]$$

where A is a random variable distributed uniformly on $[0, 1]$. Equivalently, the decision maker consumes drugs if

$$\frac{A}{\sigma(s)} < \frac{v(1) - v(0)}{[u(c_1) + \delta W(x, 0)] - [u(c_2) + \delta W(x, 1)]},$$

where we assume both numerator and denominator on the right–hand side are strictly positive for simplicity.

There are two equivalent ways of analyzing this model. First, we can write this as

$$A < A^*(s) \equiv \sigma(s)\frac{v(1) - v(0)}{[u(c_1) + \delta W(x, 0)] - [u(c_2) + \delta W(x, 1)]}.$$

Under this interpretation, the cutoff value of A for which the agent avoids drugs varies with s. In particular, if the agent has consumed drugs in the past, the current state s is higher, so $\sigma(s)$ and hence the cutoff increases, making the probability he consumes again higher. Alternatively, since A is distributed uniformly on $[0, 1]$, $A/\sigma(s)$ is distributed uniformly on $[0, 1/\sigma(s)]$. Thus we can interpret the model as having a state–independent cutoff value of A, but a distribution which depends on whether the agent has consumed drugs in the past. If he consumed drugs in the previous period, $\sigma(s)$ is higher, so the distribution of A is more concentrated toward 0, again implying a higher probability that he consumes again.

While their model is quite different in details, some of the key concepts of Bernheim and Rangel (2004) are similar. In their model, past consumption of drugs increases the probability that the agent enters a "hot state" where he always consumes drugs. This is a random Strotz model where the choice from a menu is made either by u or by a v which cares only about consumption of drugs and where the probability distribution over which "self" has control depends on past drug consumption. The probability distribution is more complex in Bernheim and Rangel, but the idea is similar.

While it is not obvious how to axiomatize it, in principle, one could extend the Gul–Pesendorfer (2007) model to richer dynamics.[17] For example, the state could depend on both the previous period's drug consumption and the previous period's feasible set. For example, we could take $s = \max_{(c,d,x)\in z} v(d) - v(d^*)$ where s is the current state, z is the previous period's menu, and d^* is the previous period's consumption of drugs. In this formulation, the self–control costs today could be higher if a larger amount of self control was exerted in the previous period, reflecting the idea that willpower is a finite stock that can be used up.[18] Alternatively, if we think of self control as a muscle which gets stronger with use, it might be more natural to assume that self–control costs today are lower the more self control was exerted in the previous period.

8 Other Issues

While we have focused on temptation, a number of other interesting extensions of the standard model have been explored using the Krepsian approach. In this section, we briefly discuss a few.

Sarver (2008) uses preferences over menus to study regret — that is, the sense of loss a decision maker experiences when she finds out *ex post* that her choice was not optimal among the set of available options. Naturally, one source of regret could be poor decision making *ex ante*, such as the kind of decisions generated by temptation. To isolate what is novel about regret, Sarver explicitly excludes this source of regret, taking as given that choices are always the best possible *ex ante*. Thus in Sarver's model, the preference over lotteries which governs choice, $\succeq_2$, is the same as the preference over singleton menus, $\succeq_1$. That is, we have $p \succeq_2 q$ if and only if $\{p\} \succeq_1 \{q\}$.

[17] We thank Jawwad Noor for suggesting these ideas.

[18] For related ideas outside the Krepsian tradition, see Ozdenoren, Salant, and Silverman (forthcoming) and Fudenberg and Levine (2010b).

Thus regret does not affect choices from menus, but only affects the choice of a menu. With this in mind, it is easy to understand his main axiom, called Dominance: If $p \succeq_2 q$ and $p \in x$, then $x \succeq_1 x \cup \{q\}$. That is, if the agent would choose p over q, then the only possible effect of having q available is the regret this would generate if the agent were to find out *ex post* that he would have been better off choosing q. Sarver adds this axiom and the Dekel, Lipman, Rustichini, and Sarver (2007) Lipschitz continuity axiom to Axioms 1, 3, and 4 above to obtain what he calls a regret representation where

$$W(x) = \max_{p \in x} \int [u(p) - R(p, x, u)] \; \mu(du)$$

where μ is a measure over continuous, linear utilities and

$$R(p, x, u) = K \left[\max_{q \in x} u(q) - u(p) \right]$$

where $K > 0$. W represents $\succeq_1$ and $\int u \, \mu(du)$ represents $\succeq_2$.

To see the idea, think of the agent choosing from the menu x prior to knowing his tastes — that is, prior to knowing the realization of u. He knows that he will learn u only *ex post* and will suffer regret proportional to the utility loss according to u between the choice he could have made and the choice he did make. Thus $R(p, x, u)$ is his regret given that he chooses p from x and u is the realized utility function. Knowing this, he maximizes his expected utility minus expected regret.

Ergin and Sarver (2010) use a preference over menus to derive a representation of costly contemplation. Analogously to Sarver's regret representation, the model assumes that the decision maker chooses from a menu with an imperfect knowledge of his utility function over lotteries. Instead of postulating that the decision maker regrets choices she later realizes were suboptimal, this representation assumes that the decision maker can expend a utility cost to obtain better information about her utility function. Thus the representation takes the form

$$W(x) = \max_{\mathcal{G} \in \mathbf{G}} \left(\mathrm{E} \left[\max_{p \in x} \mathrm{E}[u \mid \mathcal{G}] \cdot p \right] - \varphi(\mathcal{G}) \right)$$

where $\mathbf{G}$ is a collection of σ–algebras describing the possible information the decision maker could choose to acquire before selecting a lottery from the menu x and φ is a cost function for such information.

The key behavioral property identified by Ergin and Sarver is called Aversion to Contingent Planning or ACP. To see the intuition, suppose that the decision maker faces a lottery over which menu she will choose from, choosing from x with probability λ and from y otherwise. One option for the decision maker would be to make a complete contingent plan prior to observing the outcome of the randomization, deciding what she will pick from x and what she will pick from y. Alternatively, the decision maker could wait until the randomization is determined and then decide what to choose from the relevant menu only. It seems natural to suppose that the latter would be preferred in a world where decision making is difficult.

Formally, this property is stated as convexity of the preference — that is, if $x \succeq_1 y$, then $x \succeq_1 \lambda x + (1 - \lambda)y$. To see the connection to the informal story, recall from our discussion of Independence in section 2 that $\lambda x + (1 - \lambda)y$ is the set of contingent plans where the menu x is received with probability λ and y otherwise. While lotteries over menus are not a formal part of the model, suppose we asked the agent to rank the lottery where she receives x with probability λ and y otherwise when she can wait to see the outcome of the lottery before engaging in contemplation. The intuition suggested above is that this lottery would be preferred to $\lambda x + (1 - \lambda)y$. On the other hand, x would be preferred to the lottery since $x \succeq_1 y$, so the lottery should be ranked between these two menus. Hence x is preferred to the lottery which is preferred to $\lambda x + (1 - \lambda)y$, implying the conclusion of the axiom.

Ergin and Sarver show that such a representation exists if and only if the preference $\succeq_1$ satisfies Axioms 1 and 4*, a weakening of Independence, the Dekel–Lipman–Rustichini–Sarver Lipschitz continuity axiom, and ACP.

Another use of these models takes a different interpretation of menus. Up to this point, we have viewed $\succeq_1$ as a preference over menus where the decision maker will choose from the menu at a later date. However, much of the analysis does not require this interpretation.

To see the significance of this point, consider a seemingly very different situation, namely, ambiguity. Ahn (2008) gives the example of a patient who has to choose between alternative medical treatments. If the patient has no particular medical expertise, it seems reasonable to postulate that she simply considers each treatment in terms of the set of consequences it could have. That is, each treatment is seen as giving a range of probabilities over survival rates, recovery times, costs, etc. — in other words, a set of probability distributions over consequences. Thus a treatment is formally the same as a menu, with the key distinction that Nature, not the patient, will make the later choice from the "treatment menu."

This idea is not new in the literature, though it seems to be less well known than more traditional approaches to ambiguity. See Bossert, Pattanaik, and Xu (2000) for a relatively recent example and citations to the earlier work and Gilboa and Marinacci (2011) for a survey of the more standard approach. The earlier literature, however, did not exploit the structure of lotteries.

Recently, Ahn (2008) and Olszewski (2007) considered preferences over sets of lotteries as a way of modeling ambiguity. While Ahn's representation does not seem to relate in a simple way to the kind of representations we have discussed, Olszewski's representation is an interesting special case of a self–control representation. He axiomatizes a representation of a preference $\succeq_1$ which can be represented by the α–maxmin functional given by

$$W(x) = \alpha \max_{p \in x} u(p) + (1 - \alpha) \min_{p \in x} u(p).$$

It is not hard to see that this is self–control representation where $v = -(1 - \alpha)u$. It is also a random Strotz representation with probability α on $v = u$ and probability $1 - \alpha$ on $v = -u$.

Gajdos, Hayashi, Tallon, and Vergnaud (2008) generalize the notion of a set of lotteries, replacing such a set with an Anscombe–Aumann act and a set of probability distributions over the state space, where the latter is interpreted as information. Given such a pair, one could construct the implied set of lotteries over consequences, though the authors do not assume that the decision maker is indifferent between all pairs generating the same set of lotteries. Despite these differences, their representations are much in the style of random Strotz representations. To see the intuitive link, think of Nature as basing her choice from the menu on some preference which is unknown to the decision maker.

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