MA 226 Quiz 6 – B

Please show your work.

1. (5 pts) Consider the two systems of differential equations

   (i) \[
   \frac{dx}{dt} = 0.3x - 3xy \\
   \frac{dy}{dt} = -2y + 0.1xy
   \]

   (ii) \[
   \frac{dx}{dt} = 0.3x - 0.1xy \\
   \frac{dy}{dt} = -0.1y + 2xy
   \]

   One of these systems refers to a predator-prey system with very lethargic predators – those who seldom catch prey but who can live for a long time on a single prey (for example, boa constrictors). The other system refers to a very active predator that requires many prey to stay healthy (such as a small cat). The prey in each case is the same. Identify which system is which and justify your answer.

   (i) Active predator, each interaction has a small impact for predator as seen in the \(+0.1xy\) term. Predator needs many prey and has large impact on prey population as seen in the \(-3xy\).

   (ii) Lethargic predator, interactions have large positive effect on predator but small negative effect on prey because there are few interactions.
2. (5 pts) A harmonic oscillator equation for $y(t)$ is given as

\[
(* \quad \frac{d^2y}{dt^2} + 6 \frac{dy}{dt} + 7y = 0)
\]

Using the guess and check method find two nonzero solutions that are not multiples of one another.

Let $y(t) = e^{st}$

$\Rightarrow \quad s^2 e^{st} + 6se^{st} + 7e^{st} = 0$

$e^{st} \left( s^2 + 6s + 7 \right) = 0$

$s = -6 \pm \sqrt{36 - 4 \cdot 7} \frac{2}{2}$

$s = -6 \pm \frac{\sqrt{8}}{2}$

$s = -3 \pm \sqrt{2}$

Let $y_1(t) = e^{(-3+\sqrt{2})t}$

Let $y_2(t) = e^{(-3-\sqrt{2})t}$

check:

\[
\frac{dy_1}{dt} = (-3+\sqrt{2})e^{(-3+\sqrt{2})t}
\]

\[
\frac{d^2y_1}{dt^2} = (-3+\sqrt{2})^2 e^{(-3+\sqrt{2})t}
\]

Plug into $(*)$

\[
(-3+\sqrt{2})^2 e^{(-3+\sqrt{2})t} + 6(-3+\sqrt{2})e^{(-3+\sqrt{2})t} + 7e^{(-3+\sqrt{2})t} = 0
\]

\[
(-3+\sqrt{2})^2 + 6(-3+\sqrt{2}) + 7 = 0
\]

$0 \cdot e^{(-3+\sqrt{2})t} = 0$