Existence and Uniqueness Theorem

Let \( \frac{d\bar{y}}{dt} = F(t, \bar{y}) \)
be a system of differential equations.
Suppose that \( t_0 \) is an initial time
and \( \bar{y}_0 \) is an initial value. Suppose
that the function \( F \) is continuously
differentiable. Then there exists an \( E \) and \( \bar{y}(t) \) defined for \( t_0 - \varepsilon < t < t_0 + \varepsilon \)
such that
\[
\frac{d\bar{y}}{dt} = F(t, \bar{y}) \quad \text{and} \quad \bar{y}(t_0) = \bar{y}_0
\]
Moreover, for \( t \) in this interval,
this solution is unique.

Consequences of Uniqueness for Autonomous Systems

1) The solution curve for a single solution
cannot loop back and intersect itself
unless the solution is periodic, and the
solution curve is a simple closed curve.

2) Solution curves for two different
solutions cannot intersect unless they
sweep out the same curve.
Example: \( \frac{dx}{dt} = x^2 + 1 \)

\( (x(0), y(0)) = (0, 0) \)

\( \frac{dy}{dt} = 1 \)