Abstract—Using airborne full-waveform LiDAR metrics derived by 3-D tree segmentation, this study estimated single tree’s diameter at breast height (DBH) and stem volume (STV). Four regression models were used, including multilinear regression and three up-to-date regression models (i.e., least square boosting trees regression, random forest, and ε-support vector regression) from the machine learning field. This study aimed to comparatively evaluate these regression models in predicting DBH and STV at single-tree level and find some clues to regression model’s selection. The study sites were located in the Bavarian Forest National Park, Germany, a mixed temperate mountain forest. Our comparisons were performed across different tree species types (coniferous and deciduous) and foliage conditions (leaf-on/leaf-off seasons). The importance of predictor variables was also examined. Experimental results revealed that the best accuracy from machine learning methods outperformed the multilinear model by 1.5 cm for DBH and 0.18 m$^3$ for STV in terms of rmse. Through comparative analysis, our work provided some clues to the performance variation of regression models for extracting 3-D tree parameters.

Index Terms—Airborne full-waveform LiDAR, diameter at breast height (DBH), machine learning, prediction, single trees, stem volume (STV).

I. INTRODUCTION

R ECENT advances in full-waveform LiDAR technology provide a higher point density to represent a detailed vertical profile of vegetation and its reflectivity characteristics. The development of new approaches to estimate forest structural parameters at an individual tree level utilizing LiDAR data has become an important research issue. Three-dimensional approaches for the individual tree detection (ITD) satisfactorily tackle the segmentation problem when compared to using only the conventional crown height model (CHM) [1], [2]. Reference [1] has demonstrated that a 3-D segmentation technique for LiDAR point cloud data resulted in enhanced overall detection rate of single trees in a temperate forest (particularly >20% increase of accuracy at the lower forest layers). Here, the ITD approach adopting species-specific models may have advantages over the area-based approach (ABA) with respect to retrieving accurate forest inventory attributes in mixed stands [1]. Full-waveform LiDAR data render new possibilities to reconstruct tree objects. It will be interesting to find out how ITD methods from full-waveform LiDAR data can contribute to the accuracy of forest inventory results based on different regression methods.

Recent studies have made several efforts to extract various forest structural parameters like diameter at breast height (DBH) and stem volume (STV) via both ITD and ABA approaches. Reference [3] reported the estimation of STV and DBH in a boreal forest based on random forest and achieved relative rmse values of 38% and 21%, respectively, based on 26 point cloud features rather than waveform LiDAR metrics. Reference [4] showed that the support vector regression model is of similar accuracy with the multiple regression models, but they are more robust regarding the prediction of forest stand parameters. Reference [5] combined ITD measurements with ABA for estimating forest variables by $k$-MSN method, which could reduce field measurement cost greatly. However, it underestimated the plot-level forest parameters by 2.7%–9.2%. Reference [6] incorporated growth competition index as a predictor variable for the prediction of stem diameter and volume of old-aged forest stands using LiDAR metrics and multiple linear regression. RMSE values of 8.7 cm and 0.91 m$^3$ were obtained, respectively. References [7] and [8] used multiple returns LiDAR data to estimate tree-level STV by features from CHM-based segmentation.

However, inconsistent study conditions (e.g., forest and LiDAR data conditions, and used metrics) make it difficult to compare various regression models. A few studies have conducted their comparative evaluations. It will be useful and essential to analyze and compare the performance of prediction models in different situations. This letter aimed to comparatively evaluate regression models (including multilinear regression, least square boosting decision trees, random forest, and ε-support vector regression) for prediction of DBH and STV.
at single tree’s level based on waveform LiDAR metrics. The used LiDAR metrics of single trees include tree height (TH), crown area (CA), crown height (CH), and crown volume (CV), which were extracted based on the methods in [1] for 3-D segmentation of single trees and species classification.

This letter is organized as follows. Section II briefly describes the used prediction methods. After demonstrating the experimental design and its results, Section III is dedicated to analyzing and comparing the prediction models with respect to different tree species types (coniferous and deciduous) and foliage conditions (leaf-on/leaf-off seasons). The conclusion is drawn in Section IV.

II. PREDICTION METHODS

Given “training” samples \((y_i, x_i)_{i=1}^N\) of known \((y, x)\) values, the goal of prediction models is to derive the transform functions \(y_i = f(x_i)\) with some rules like achieving the minimized prediction errors, minimizing both the structural error and model complexity, and so on, where \(N\) is the number of samples, \(x_i = (x_{i1}, x_{i2}, x_{i3}, x_{i4})^T\) is the vector of predictors (TH, CA, CH, and CV) for the \(i\)th sample, and \(y_i\) is the response variables (DBH or STV). The four prediction models used and compared in this letter are described as follows.

A. Multilinear Regression (Linear)

DBH and STV can be estimated through (1) deployed in [1]. This is a standard approach to be compared with other three state-of-the-art machine learning methods

\[
f(x_i) = a_0 + a_1 x_{i1} + a_2 x_{i2} + a_3 x_{i3} + a_4 x_{i4} + a_5 x_{i1}^2 + a_6 x_{i2}^2 + a_7 x_{i3}^2 + a_8 x_{i4}^2 + e_i,
\]  

(1)

Although the formula seems nonlinear, we solve this equation using the linear least square estimation. Hence, it is referred to as the multiple linear regression (herein linear) method.

B. Least Square Boosting Trees Regression (Boosting)

Boosting regression is a strategy of combining several weak learners into a strong one [9]. Regression tree is a sequence of rules which derive the feature space’s partitions that get similar values for a response variable. It derives boosting trees regression by taking the regression tree as the weak learner. The least squares (LS) based boosting trees regression [9] was used in this paper. Its basic idea is to iteratively fit the prediction residuals by the regression tree until it produces minimized LS error in the loss function. To avoid overfitting, shrinkage rate \(v\) is used to reduce the impact of each additional tree. Assuming that \(M\) regression trees \(T_i(x) (i = 1, 2, \ldots, M)\) are needed, LS-based boosting trees regression (herein boosting) is as follows.

1) Initialization: set \(\bar{y} = (y_i)_{i=1}^N/N\) as initial prediction \(f_0(x_i)\).
2) For \(m = 1\) to \(M\) do:
   a. \(\hat{y}_i = y_i - f_{m-1}(x_i) (i = 1, 2, \ldots, N)\);
   b. obtaining regression tree \(T_m(x)\) fitting the \(\hat{y}_i\) best;
   c. \(f_m(x) = f_{m-1}(x) + vT_m(x)\)

End for.

Typically, \(v\) is 0.1 or smaller. When doing predictions, (2) is iteratively performed with \(m = 1, 2, \ldots, M\), and \(f_M(x)\) is the final prediction value. To do accurate regression, model parameters \((M, v)\) must be selected appropriately. In this letter, the best \((M, v)\) were derived through grid searching method with \(M \in [25, 200]\), \(dM = 25\) and \(v \in [0.05, 1.0]\), \(dv = 0.05\) using fixed incremental values based on the minimized prediction error.

C. Random Forest Regression (RF)

The random forest (herein RF) [10] is a widely used ensemble learning method to perform the classification or regression task [3], [11]. For regression, the RF is constructed by growing \(M\) regression trees \(T_i(x) (i = 1, 2, \ldots, M)\), which are aggregated as \(f(x) = f_{RF}(x) = \sum_{i=1}^{M} T_i(x)/M\) to achieve more accurate prediction. Given the variance of single regression tree as \(\sigma^2\), the variance of the RF is formulated as [10]

\[
\text{Var}(f_{RF}(x)) = \rho \sigma^2 + (1 - \rho)\sigma^2/M
\]  

(3)

where \(\rho\) is the correlation coefficient between regression trees in the RF model. Using (3), it can be deduced that a low variance of the prediction model can be achieved by choosing a large \(M\) and by controlling the correlation coefficient \(\rho\) between any two trees to be minimized. Thus, two specific ways are used in the construction of regression trees to reduce \(\rho\) by enhancing randomness: first, bootstrapping (random sampling with replacements) is adopted for the selection of training samples for growing single regression trees; second, \(\rho\) (predefined) predictor variables are selected randomly to achieve the best split at each node of the given growing regression tree. The increase of prediction error for the modified and original out-of-bag data can be viewed as a measure to determine the importance of variables in the parameter prediction, since the used features are randomly permuted and selected at each node in the process of growing regression trees. Like the boosting method, the best RF model parameters \((M, \rho)\) were derived by grid searching method with \(p = 2, 3\) and \(M \in [25, 200]\), \(dM = 25\) to obtain the best prediction.

D. Support Vector Regression (SVR)

Based on the structural risk minimization principle, the SVR generates a good generalizability and robustness against outliers. It has recently drawn remote sensing community’s attention and been used for the estimation of forest attributes [4] and biomass [11]. In this letter, \(\varepsilon\)-SVR [12] with radial basis function kernel \(e^{-\gamma ||x_i - x||^2}\) is used, in which \(\varepsilon\)-insensitive function \(e_i = |y_i - f(x_i)| - \varepsilon = \max(0, |y_i - f(x_i)| - \varepsilon)\) is used as loss function, where \(\varepsilon\) is a predefined nonnegative value. The basic idea for the \(\varepsilon\)-SVR is the following: for training data \((y_i, x_i)\), if \(|y_i - f(x_i)| > \varepsilon\), it is taken as a support vector, and a loss rising linearly with \(|y_i - f(x_i)| - \varepsilon\) should be associated with the modeled estimates; otherwise, it is not a support vector, and “no loss” meaning “no penalty” should be imposed to the estimates. Consider that \(f(x_i)\) is linear with form \(f(x_i) = \omega x_i + b\), where \(\omega = (\omega_1, \omega_2, \omega_3, \omega_4)\) and \(b\) are parameters to be estimated. SVR aims to obtain suitable \(\omega\) and \(b\) through finding the tradeoff between the complexity (or flatness) of
functions and the amount of training mistakes (or fitness) on support vector samples. Its good generalization ability can be achieved by adjusting the penalty constant \( C \). If \( f(x_i) \) is nonlinear, this can be solved by mapping input features with a nonlinear function \( \phi(x_i) \) and by using the kernel function \( K(x_i, x_j) \) to express dot production \( \phi(x_i) \cdot \phi(x_j) \), which is a convenient solution to SVR’s optimization problem. A more detailed solution to SVR can be referred to [12]. To derive the best SVR estimations, the adjustable parameters (i.e., \( C, \varepsilon, \gamma \)) must be selected appropriately. This letter adopted grid search and \( k \)-fold cross validation to select the best parameters.

### III. Experiments and Analysis

#### A. Data Materials and Experimental Design

Experiments were conducted with two flight campaigns over the Bavarian Forest National Park, Germany, in both leaf-on and leaf-off seasons. We selected 18 sample plots with an area size between 1000 and 3600 m\(^2\) from two major test sites containing sub-alpine spruce forest, mixed mountain forest, and alluvial spruce forest as the three major forest types. Reference data for all valid trees (DBH > 7 cm) have been collected for 688 Norway spruces, 812 European beeches, 70 fir trees, 71 Sycamore maples, 21 Norway maples, and 2 lime trees. Tree parameters including the DBH, total TH, stem position, and tree species types were surveyed and georeferenced by the GPS, tachometry, and “Vertex III” system. The STVs of the reference trees were computed from DBH, TH, and species-specific parameters. We updated the reference data concurrently for the LiDAR data of these two flights. The characteristics of the individual sample plots and descriptive statistics of the field trees are referred to [1].

Using all correctly detected trees and the corresponding extracted tree features including TH, CA, CH, and CV by the methods in [1], our experiments were performed to compare regression models for predicting DBH and STV at a single-tree level: all of the detected trees were divided into four groups according to foliar conditions (leaf-off/leaf-on seasons) and tree species types (coniferous and deciduous), regression models were applied to each group of trees sequentially and separately, and fivefold cross validation was applied with the number of training and test samples as described in the following table (see Table I).

#### B. Prediction of DBH and STV

Four models for predicting DBH and STV, respectively, were derived using methods described in Section II and data materials depicted in Section III-A. The agreements between the reference and predicted DBHs and STVs from the four regression models are, respectively, shown in Figs. 1 and 2 (dataset I = leaf-on; dataset II = leaf-off). To validate the model performance and stability in a more objective way, we applied fivefold cross validations, and the corresponding results are shown in Tables II and III.

The achieved results can be compared with previous studies in terms of rmse%. For instance, [3] used the RF and discrete

<table>
<thead>
<tr>
<th>Foliar conditions</th>
<th>NO. of coniferous trees</th>
<th>NO. of deciduous trees</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Training</td>
</tr>
<tr>
<td>Leaf-off</td>
<td>349</td>
<td>280</td>
</tr>
<tr>
<td></td>
<td>409</td>
<td>327</td>
</tr>
<tr>
<td>Leaf-on</td>
<td>334</td>
<td>267</td>
</tr>
<tr>
<td></td>
<td>379</td>
<td>303</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Different group of trees</th>
<th>Linear</th>
<th>Boosting</th>
<th>RF</th>
<th>SVR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leaf-off coniferous</td>
<td>6.45</td>
<td>6.38</td>
<td>6.45</td>
<td>6.29</td>
</tr>
<tr>
<td></td>
<td>(14.91%)</td>
<td>(14.75%)</td>
<td>(14.91%)</td>
<td>(14.52%)</td>
</tr>
<tr>
<td>Leaf-off deciduous</td>
<td>5.81</td>
<td>5.56</td>
<td>5.61</td>
<td>5.81</td>
</tr>
<tr>
<td></td>
<td>(23.18%)</td>
<td>(16.52%)</td>
<td>(18.62%)</td>
<td>(18.48%)</td>
</tr>
<tr>
<td>Leaf-on coniferous</td>
<td>7.05</td>
<td>6.24</td>
<td>6.36</td>
<td>6.08</td>
</tr>
<tr>
<td></td>
<td>(16.38%)</td>
<td>(14.49%)</td>
<td>(14.77%)</td>
<td>(14.10%)</td>
</tr>
<tr>
<td>Leaf-on deciduous</td>
<td>7.19</td>
<td>7.19</td>
<td>7.35</td>
<td>7.02</td>
</tr>
<tr>
<td></td>
<td>(20.02%)</td>
<td>(20.00%)</td>
<td>(20.46%)</td>
<td>(19.53%)</td>
</tr>
</tbody>
</table>
LiDAR data to achieve the prediction accuracy with rmse% of 21.4% and 45.8% (38% in the best cases), respectively, for DBH and STV in a Finland boreal forest. They also demonstrated that the RF was robust for the estimation of single tree’s DBH and STV when compared to linear models. Reference [13] implemented k-most similar neighbor (herein k-MSN) predicting forest structural attributes with rmse% ranging from 12.9%–17.5% and 30.1%–44.3% for DBH and STV, respectively, in a boreal forest. Reference [6] achieved good predictive power with rmse% of 13.6%–18.0% for DBH and of 26.3%–37.8% for STV due to the presence of larger dominant trees. Regarding rmse, our results (in Tables II and III) across different regression models, the superiority of the machine learning methods over the linear models was more distinct for the STV prediction than for the DBH prediction. This demonstrated that such machine learning methods could even better predict complex tree attributes. We expected that the structural features of single trees (CV, CA, and CH) should contribute more to the prediction of STV than that of DBH. However, our results were opposite according to the feature importance assessment in the following Section III-D. This may help in explaining why the prediction accuracy for DBH was better than that for STV to some extent.

According to Tables II and III, the superiority of the machine learning methods over the linear method could be observed more distinctly for the STV prediction than for the DBH prediction. This demonstrated that such machine learning methods could even better predict complex tree attributes. We expected that the structural features of single trees (CV, CA, and CH) should contribute more to the prediction of STV than that of DBH. However, our results were opposite according to the feature importance assessment in the following Section III-D. This may help in explaining why the prediction accuracy for DBH was better than that for STV to some extent.

By analyzing $R^2$ in Figs. 1 and 2 for the training samples in different cases, the machine learning methods were shown to have much better ability in explaining the relationship between DBH (or STV) and prediction features; the DBH’s prediction better agreed with the reference data than that of the STV. By comparing the prediction accuracy difference between cross validation and training samples’ fitting, the $SVR$ got the biggest one, followed by the RF. Thus, it was shown to have the most tendency to overfitting and the least robustness to the outliers among the used methods in this letter. The $boosting$ could be taken as the most robust to outliers as it achieved the least accuracy changes compared to the other two machine learning models.

A process for model parameter selection is needed for the used machine learning methods. The $SVR$ suffered from the most complicated training complexity as three parameters (i.e., $C, \epsilon, \gamma$) had to be selected through brute grid searching with computing intensive optimization algorithms, which caused the largest computation burden to $SVR$. $boosting$’s parameter selection process was more complicated than that of the RF as the $v$ in the $boosting$ took more searching steps than the $p$ in the RF in our experiments according to Section II. Thus, the $boosting$ suffered more computation burden than the RF in model training. In contrast, almost no computation burden was on the $linear$ because of its simple computation.

### C. Model Performance Comparison

From the comparisons of accuracies (Figs. 1 and 2 and Tables II and III) across different regression models, the $SVR$ yielded the best overall accuracy for the prediction of both DBH and STV. The RF and $boosting$ worked equally well, and both are better than the $linear$. The fivefold cross validation results showed that the best accuracy from the machine learning methods outperformed the $linear$ model by 1.5 cm for DBH and 0.18 m$^3$ for STV in terms of rmse, which generally indicate the superiority of modern machine learning methods. However, the accuracy differences between $SVR$ and other models (especially other machine learning models) were not always distinct.

Overall, the prediction accuracy of DBH was better than that of STV under identical conditions due to the more complex relationships between the predictor features and STV. The foliage condition has less influence on the estimation of DBH than STV, which can be induced from the accuracy difference between the leaf-on and leaf-off conditions. With regard to the influences of tree species types, coniferous trees granted better prediction results than deciduous trees: the complex branching structure of deciduous trees might lead to difficulties in the predictions when compared to coniferous ones. Comparing the influence of foliage conditions on the different models, our results showed that the performance of the $linear$ varied more strongly with leaf-on and leaf-off states than the machine learning methods. For the machine learning predictions on deciduous trees, the leaf-off was more favorable than the leaf-on season, which might indicate the fact that structural information about trees can be better captured by LiDAR in the leaf-off condition. The results of applying the $linear$ method were somewhat opposite to the machine learnings, implying that the $linear$ provides less generalization ability with respect to tree crown saturation. The difference in prediction accuracies between the leaf-off and leaf-on conditions was not significant as expected for the machine learnings, which could be credited to the high point density of the data sets on one hand and to the better generalization ability of machine learning methods on the other hand.

### D. Feature Importance Assessment by RF Regression

The RF provides a way to assess the feature’s importance to model prediction accuracy [3], [11]. The relative importance of

### Table III

<table>
<thead>
<tr>
<th>Different group of trees</th>
<th>Linear</th>
<th>Boosting</th>
<th>RF</th>
<th>SVR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leaf-off coniferous</td>
<td>0.6539 (24.54%)</td>
<td>0.6163 (23.13%)</td>
<td>0.6072 (22.78%)</td>
<td>0.6005 (22.53%)</td>
</tr>
<tr>
<td>Leaf-off deciduous</td>
<td>0.8929 (49.26%)</td>
<td>0.7406 (40.75%)</td>
<td>0.7346 (40.43%)</td>
<td>0.7077 (38.95%)</td>
</tr>
<tr>
<td>Leaf-on coniferous</td>
<td>0.7867 (29.92%)</td>
<td>0.6319 (24.03%)</td>
<td>0.7330 (21.87%)</td>
<td>0.6312 (24.00%)</td>
</tr>
<tr>
<td>Leaf-on deciduous</td>
<td>0.8531 (44.71%)</td>
<td>0.8320 (43.60%)</td>
<td>0.8279 (43.38%)</td>
<td>0.8168 (42.80%)</td>
</tr>
</tbody>
</table>
laser-derived features to estimate DBH and STV is depicted in Figs. 3 and 4 in terms of mean squared errors (MSE) increase. Features with higher MSE contributed more to the prediction accuracy. Figs. 3 and 4 showed the order of importance in four predictors resembled for both leaf-off and leaf-on conditions. TH predominantly contributed to the prediction of both DBH and STV regardless of the foliage conditions, which was not as expected in [8]. On the other hand, CA, CH, and CV were more responsible to the prediction accuracy of deciduous trees than coniferous trees since deciduous trees have a greater complexity in the vertical structure and it requires more structural variables to predict the DBH and STV. CV was the second most important LiDAR feature in most cases, except for the DBH prediction of deciduous trees under leaf-on condition, whereas the contributions of CA and CH were scattered across different cases.

IV. CONCLUSION

In this letter, a comparative study of four representative regression models for predicting single tree’s DBH and STV has been conducted in a temperate mixed forest with airborne waveform LiDAR data. We have achieved similar or better rmse% compared to that reported in [3], [6], and [13], but complete accuracy comparisons with previous studies could not be done here due to different experimental environments. Comparisons of models were conducted considering the influences of foliage conditions and tree types. When compared to the linear method, the machine learning methods (boosting, RF, and SVR) showed not only better prediction accuracies but also more robustness with respect to species- and foliage-specific conditions. SVR was found as the most accurate in all cases, especially for leaf-off deciduous trees and leaf-on coniferous trees, while with heavier computation burden. Boosting was the most robust to outliers and was the least tending to overfitting. Accuracy differences among different regression models varied with foliage conditions and tree types. We should select the proper regression model based on the integrated consideration of tree species, foliage condition, deployed resources like training complexity, computational time, and accuracy. Thus, the selection of the proper estimation model can only be made available based on the tradeoff among required accuracy, deployed resources, and data properties.

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REFERENCES