

Pricing in Multiservice Loss Networks: Static Pricing, Asymptotic Optimality, and Demand Substitution Effects

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Various Parameters for the Examples in Section IX

A. Networks without Substitution Effects

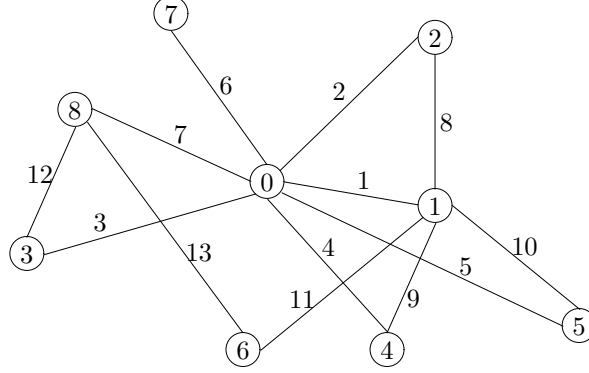


Figure 1: A 9-node, 13-link network with 59 service classes. The link capacities are $C_1 = 840, C_2 = 420, C_3 = 420, C_4 = 420, C_5 = 420, C_6 = 300, C_7 = 420, C_8 = 420, C_9 = 420, C_{10} = 420, C_{11} = 420, C_{12} = 210, C_{13} = 420$.

i	Nodes (Links)	r_i	$\lambda_i(u_i)$	μ_i	i	Nodes (Links)	r_i	$\lambda_i(u_i)$	μ_i
1	1, 0 (1)	1	$300 - 120u_1$	2	2	2, 0 (2)	1	$400 - 120u_2$	2
3	2, 1 (8)	1	$200 - 40u_3$	2	4	3, 0 (3)	1	$250 - 50u_4$	2
5	5, 0 (5)	1	$400 - 80u_5$	2	6	5, 1 (10)	1	$300 - 60u_6$	2
7	5, 0, 3 (5, 3)	1	$100 - 16u_7$	2	8	5, 1, 4 (10, 9)	1	$200 - 50u_8$	2
9	6, 1, 0 (11, 1)	1	$200 - 40u_9$	1	10	6, 1 (11)	1	$200 - 40u_{10}$	1
11	6, 1, 5 (11, 10)	1	$150 - 30u_{11}$	1	12	7, 0, 1 (6, 1)	1	$300 - 40u_{12}$	1
13	8, 0 (7)	1	$300 - 40u_{13}$	1	14	8, 0, 1 (7, 1)	1	$300 - 40u_{14}$	1
15	8, 0, 2 (7, 2)	1	$200 - 40u_{15}$	1	16	8, 0, 4 (7, 4)	1	$100 - 20u_{16}$	1
17	1, 0 (1)	2	$100 - 10u_{17}$	4	18	2, 0 (2)	2	$80 - 8u_{18}$	4
19	2, 1 (8)	2	$100 - 8u_{19}$	4	20	3, 0 (3)	2	$100 - 10u_{20}$	4
21	3, 0, 1 (3, 1)	2	$80 - 10u_{21}$	4	22	4, 0 (4)	2	$120 - 12u_{22}$	2
23	4, 1 (9)	2	$120 - 10u_{23}$	2	24	4, 1, 2 (9, 8)	2	$100 - 12u_{24}$	2
25	5, 0 (5)	2	$80 - 8u_{25}$	2	26	5, 1 (10)	2	$80 - 10u_{26}$	2
27	5, 1, 2 (10, 8)	2	$100 - 8u_{27}$	2	28	5, 3 (5, 3)	2	$80 - 8u_{28}$	2
29	5, 1, 4 (10, 9)	2	$80 - 10u_{29}$	2	30	6, 1, 0 (11, 1)	2	$80 - 6u_{30}$	1
31	6, 1 (11)	2	$100 - 10u_{31}$	1	32	6, 1, 2 (11, 8)	2	$100 - 8u_{32}$	1
33	6, 8, 3 (13, 12)	2	$100 - 12u_{33}$	1	34	8, 0, 1 (7, 1)	2	$80 - 8u_{34}$	2
35	8, 0, 2 (7, 2)	2	$60 - 6u_{35}$	2	36	8, 0, 5 (7, 5)	2	$60 - 8u_{36}$	2
37	8, 6 (13)	2	$100 - 10u_{37}$	2	38	1, 0 (1)	4	$40 - 4u_{38}$	1
39	2, 0 (2)	4	$60 - 5u_{39}$	1	40	2, 1 (8)	4	$80 - 12u_{40}$	1
41	3, 0 (3)	4	$40 - 4u_{41}$	1	42	3, 0, 1 (3, 1)	4	$40 - 5u_{42}$	1
43	3, 0, 2 (3, 2)	4	$40 - 6u_{43}$	1	44	4, 0 (4)	4	$60 - 3u_{44}$	1
45	4, 1 (9)	4	$60 - 4u_{45}$	1	46	4, 1, 2 (9, 8)	4	$60 - 4u_{46}$	1
47	5, 0 (5)	4	$40 - 4u_{47}$	1	48	5, 1 (10)	4	$40 - 2u_{48}$	1
49	5, 0, 4 (5, 4)	4	$50 - 5u_{49}$	1	50	6, 1, 0 (11, 1)	4	$50 - 5u_{50}$	4
51	6, 1 (11)	4	$50 - 4u_{51}$	4	52	6, 1, 2 (11, 8)	4	$60 - 4u_{52}$	4
53	6, 8, 3 (13, 12)	4	$60 - 6u_{53}$	4	54	6, 1, 5 (11, 10)	4	$30 - 3u_{54}$	2

i	Nodes (Links)	r_i	$\lambda_i(u_i)$	μ_i	i	Nodes (Links)	r_i	$\lambda_i(u_i)$	μ_i
55	7, 0 (6)	4	$60 - 3u_{55}$	4	56	7, 0, 2 (6, 2)	4	$40 - 4u_{56}$	2
57	7, 0, 3 (6, 3)	4	$20 - 4u_{57}$	2	58	7, 0, 4 (6, 4)	4	$30 - 3u_{58}$	2
59	8, 0 (7)	4	$60 - 6u_{59}$	2					

Table 1: The services provided by the network of Figure 1.

Figure 1 depicts a network with 9 nodes and 13 links. The network provides 59 classes of services, the parameters are listed in table 1.

B. Networks with Substitution Effects

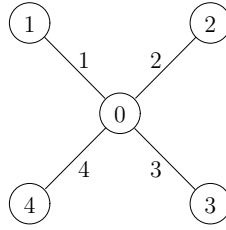


Figure 2: A 5-node, 4-link network with 12 service classes. The link capacities are $C_1 = 90, C_2 = 100, C_3 = 110, C_4 = 120$.

The network in Figure 2 provides 12 classes of services, but between class 1 and 7, 2 and 8, there exist substitution effects (see Table 2).

Class i	Nodes (Links)	r_i	$\lambda_i(\mathbf{u})$	μ_i
1	1, 0, 2 (1, 2)	1	$150 - 75u_1 + u_7$	5
2	1, 0, 3 (1, 3)	1	$150 - 60u_2 + u_8$	5
3	1, 0, 4 (1, 4)	1	$250 - 125u_3$	5
4	2, 0, 3 (2, 3)	1	$140 - 40u_4$	5
5	2, 0, 4 (2, 4)	1	$300 - 150u_5$	5
6	3, 0, 4 (3, 4)	1	$150 - 35u_6$	5
7	1, 0, 2 (1, 2)	5	$6 - u_7 + 0.1u_1$	1
8	1, 0, 3 (1, 3)	5	$7 - 1.2u_8 + 0.1u_2$	1
9	1, 0, 4 (1, 4)	5	$6.6 - 0.8u_9$	1
10	2, 0, 3 (2, 3)	5	$6 - 0.6u_{10}$	1
11	2, 0, 4 (2, 4)	5	$6 - 0.6u_{11}$	1
12	3, 0, 4 (3, 4)	5	$6 - 0.5u_{12}$	1

Table 2: The services with demand substitution effects provided by the network in Figure 2.