Problem Set 1

(Convexity and Optimality conditions for unconstrained optimization)

Due: Monday, February 5, 2018

Note: I will be referring to the textbook as [Ber].

Problem 1

(a) Let $\mathbf{X}$ be a positive definite matrix. Show that $\log\det\mathbf{X}^{-1}$ is a convex function of $\mathbf{X}$. *Hint:* Show that $f(t) = \log\det(\mathbf{X}_0 + t\mathbf{H})^{-1}$ is a convex function of the scalar t.

(b) Let $\mathbf{X}$ be a symmetric matrix. Show that its maximum eigenvalue $\lambda_{\max}(\mathbf{X})$ is a convex function of $\mathbf{X}$.

Problem 2

Consider a scalar function $h(\mathbf{x}, \mathbf{y})$, where $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^m$, and assume that it is convex in $\mathbf{x}$ and $\mathbf{y}$. Show that $f(\mathbf{x}) = \inf_y h(\mathbf{x}, \mathbf{y})$ is convex in $\mathbf{x}$.

Problem 3 (Convexity in the right hand side of the constraints)

Let C be a convex subset of $\Re^n$ and consider the convex functions $f : \Re^n \to \Re$ and $g : \Re^n \to \Re^m$. Consider the following function $\phi : \Re^m \to \Re \cup \{\pm\infty\}$ on the extended real line

$$\phi(y) = \inf_{\substack{g(x) \leq y \\ x \in C}} f(x).$$

(a) Show that $\phi(\cdot)$ is convex.

(b) Show that it is monotonically non-increasing.

Problem 4: Exercise 1.1.1 of [Ber]

Problem 5: Exercise 1.1.3 of [Ber]

Problem 6: Exercise 1.1.4 of [Ber]

Problem 7: Exercise 1.1.8 of [Ber]