SE/EC/ME 724 Advanced Optimization Theory and Methods

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Lecture 1: Outline

1. Administrative stuff.
2. Main topics to be covered.
3. Some Applications.

Main Topics

- Theory and algorithms for **Nonlinear Programming (NLP)**.
  - Optimality conditions.
  - Algorithms.
  - Convergence.
  - Sensitivity.
  - Duality.
- Large scale methods such as incremental gradient methods.
- Modern Convex Optimization
  - Semidefinite programming.
  - Conic Quadratic Programming.
  - Robust Linear Programming.
- Stochastic approximation algorithms.

What is NLP ?

$$\begin{array}{ll} \min & f(\mathbf{x}) \\ \text{s.t.} & \mathbf{x} \in X \end{array}$$

- $f : \mathbb{R}^n \to \mathbb{R}$, continuous, usually differentiable.
- $X \subset \mathbb{R}^n$, usually closed.

Special Cases:

- $X \equiv \mathbb{R}^n$: **unconstrained problems**.
- f linear, X a polyhedron: **linear programming (LP)**.
- f convex, X convex: **convex optimization problem**.

What do we seek ?

- Characterization of minima
  - Necessary conditions.
  - Sufficient conditions.
  - Lagrange multiplier theory.
  - Sensitivity.
  - Duality.
- Solution by (iterative) algorithms
  - Iterative descent.
  - Approximation methods.
  - Dual and primal-dual methods.

The Modern View

The traditional view: Linear is **easy**, nonlinear is **hard**.

"... In fact, the great watershed in optimization isn't between linearity and nonlinearity, but convexity and nonconvexity."

R. Tyrrell Rockafellar,
"Lagrange multipliers and optimality,"
SIAM Review 35 (1993), 183–238.

Some Applications

- Control and optimization in communication systems:
  - Routing.
  - Scheduling, e.g., transmission scheduling in wireless and ad-hoc networks.
  - Pricing and resource allocation.
  - Network/topology design.
  - Anomaly detection.
- Optimizing production and inventory policies in manufacturing systems and supply chains.
- Neural network training and applications in recent approximate dynamic programming techniques.
- Pattern recognition and classification.
- Applications of semidefinite programming in combinatorial optimization, control theory, design of chips, the derivation of performance bounds in controlled flexible manufacturing systems.

Some Applications (cont.)

- Optimal portfolio selection in the finance industry.
- Estimation and system identification.
- Stability analysis in dynamical systems.
- Synthesis of filters and antennae systems.
- Optimal control problems (e.g., rocket launching).
- Convex duality and its applications in large deviations theory.
- Applications of nonlinear duality in microeconomic theory (e.g., pricing, equilibrium models).
- Game theory.
- Traffic assignment and equilibrium in transportation networks.
- Drug design and molecular docking.
- Protein folding.