Lecture 23: Outline

- Practical Guidelines for solving LPs.
- Case study: The fleet assignment problem.
Practical Guidelines for LPs

- Exploit sparsity.
- Start with a “good” basis in simplex.
- Use of dual simplex.
- Use of interior point methods. Simplex or interior point method?

Fleet assignment

Given fleet and flight schedule assign aircrafts to flights.

**Fleet information**

- $\mathcal{F}$: set of fleets.
- $S(f)$: # of aircraft in fleet $f \in \mathcal{F}$. 
Fleet assignment (cont.)

**Schedule information**

- \( \mathcal{C} \): set of cities served
- \( \mathcal{L} \): set of flights
- \((o, d, t) \in \mathcal{L} \): (origin, dest., dep. time)
- \( c_{fodt} \): cost
- \( t_{0} \): reference time
- \((t_{1}, \ldots, t_{n}) \): time partition
- \( t^{-}, t^{+} \)
- \( t(f, o, d) \): time that fleet \( f \) does \( o-d \)
- \( O(t_{0}) \): set of flights on the air in \([t_{0}, t_{0}^{+}]\)
- \( \mathcal{H} \): set of “required through” pairs of flights (to be served by same fleet)

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**Formulation**

**Objective:** Minimize cost

**Decision variables**

\[
x_{fodt} = \begin{cases} 
1 & \text{if fleet } f \text{ for } (o, d, t) \\
0 & \text{otherwise}
\end{cases}
\]

\( y_{fot} = \# \text{ of aircraft from } f \text{ on ground in } o \text{ during } [t, t^{+}] \)

\( z_{fot} = \# \text{ of aircraft from } f \text{ that arrive in } o \text{ at } t \)
Formulation (cont.)

\[
\begin{align*}
\min & \sum_{f \in \mathcal{F}} \sum_{(o,d,t) \in \mathcal{L}} c_{fodo} x_{fodo} \\
\sum_{f \in \mathcal{F}} x_{fodo} &= 1 \quad \forall (o,d,t) \in \mathcal{L} \\
z_{fot} + y_{fot} - \sum_{d \in \mathcal{C}} x_{fodo} - y_{fot} &= 0 \quad \forall f, o, t \\
x_{fodo} - x_{fodo'} &= 0 \quad \forall f \in \mathcal{F}, ((o,d,t),(d',d',t')) \in \mathcal{H} \\
\sum_{(o,d,t) \in O(t_0)} x_{fodo} + \sum_{o \in \mathcal{C}} y_{fot_0} &\leq S(f) \quad \forall f \in \mathcal{F} \\
z_{fot} &= \sum_{\{(d,o,\tau) \in \mathcal{L} | \tau + t(f,d,o) = t\}} x_{fdo\tau} \\
x_{fodo} &\in \{0,1\}, \ y_{fot} \geq 0, \ y_{fot} : \text{integer}
\end{align*}
\]

Algorithm

- Preprocessing.
- Perturb all costs.
- Dual simplex or path following on LP relaxation.
- Remove perturbation.
- Reoptimize with original cost.
- Round variables above 0.99 to 1.
- Preprocessing.
- Dual simplex.
- Branch and bound.