Problem 4
Cisco’s 12000 series routers are produced in three factories $F_1, F_2$ and $F_3$ and shipped from the factories to two distribution centers $D_1$ and $D_2$. Let $c_{ij}$ the transportation cost (in $/router) from factory $F_i$ to distribution center $D_j$, for $i = 1, 2, 3$ and $j = 1, 2$. Let also $p_i$, $i = 1, 2, 3$, the production cost (in $/router) at factory $F_i$. We have

$$C = (c_{ij}) = \begin{bmatrix} 10 & 100 \\ 80 & 120 \\ 60 & 50 \end{bmatrix}, \quad p = (p_1, p_2, p_3) = (1100, 1400, 1250).$$

The monthly capacity at each factory $i$ is denoted by $u_i$ and the demand at distribution center $j$ is denoted by $d_j$, for $i = 1, 2, 3$ and $j = 1, 2$. We have

$$u = (u_1, u_2, u_3) = (800, 1170, 1000), \quad d = (d_1, d_2) = (1500, 1200).$$

Production should not exceed capacity at each factory and demand must be met at each distribution center.

(a) Formulate as a linear programming problem the problem of devising a monthly production and transportation plan that minimizes the total production and transportation cost. You want to decide the number of routers produced at factory $F_i$ and shipped to distribution center $D_j$ for $i = 1, 2, 3$ and $j = 1, 2$. Use CPLEX or any other LP solver to solve the problem and obtain sensitivity information.

(b) If the demand at $D_1$ increases by 100 routers, how is the optimal production and transportation cost affected?
(c) What is the impact on the total cost if we insist on producing one router at factory $F_3$ and shipping it to distribution center $D_1$?

(d) Suppose the capacity at $F_1$ increases to 1600. What is the impact on the optimal cost? If you cannot find the cost exactly calculate bounds on the impact in the cost. Justify your answer.

(e) Suppose that the transportation cost from $F_1$ to $D_1$ increases by $30. What is the impact on the optimal cost? What is the new optimal production and transportation plan?